I) **Introduction and summary of the achieved work:**

I.1) ***Preface***

This is the final report of the project entitled "Fractal geometry and its applications in timber construction" (200021-112103). The present research has received a follow-on grant (200020-120037/1). The results presented in this report try to give an accurate understanding of the work accomplished during the past years of the research.

I.2) ***Goals***

This interdisciplinary research project presents a corporation of architects, mathematicians and computer scientists. The team researches new methods for efficient realization of complex architectural shapes. The aim is to develop computer aided solutions which optimize the design and production of free-form surfaces. Therefore, the team worked out a new surface method. The studied method provides new form finding possibilities while satisfying a certain number of material and construction constraints.

I.3) ***Summary of the achieved work***

During the past years, the research was primary concentrated on the study of geometrical modeling methods of complex shapes. The mathematical methods of IFS- and subdivision-modeling have been studied. The aim was to combine the advantages of each of these two methods. Both of the mathematical methods have then been implemented into computer aided design software (CAD). This software modeler consists of several modules which interact with one another. Their specific tasks have been defined according to the following problems:

- Formal control: Topology
- Free control: Modeling scheme and subdivision point editing and Multi resolution control point modeling
- Geometrical constraints: Surface modeling, planar quadrilateral meshes using projective geometry
- Post processing procedures and conversion of the geometric data to constructive elements

The obtained modeling chain was tested and approved by the fabrication of different reduced scale models and prototypes. The geometrical models have been applied to the following architectural object (cf thesis Stotz):

- Fractal shading panels
- Vault construction of screwed timber planks
- Shell structure of quasi-planar quadrilateral timber boards
- Shell structure of planar quadrilateral timber panels
- Hotel lobby Starling, EPFL: Ornamental wall paper
- Timber project exposition table

II) **Presentation of the mathematical model**

The modeled figures are generated according to a particular mathematical model, called CIFS (Controlled Iterated Function System), which allows defining fractal shapes. Based on Barnsley's ISF-formalism, a CIFS description consists in establishing a set of geometrical transformations. These transformations, applied to a certain geometrical element (e.g. a line, single surface, or a volume), generate a set of duplicates of the initial element. This operation is called “subdivision step”. While reiterating subdivision steps on the newly generated elements, we build on automatically a more and more complex set of geometrical elements (cf. figure 1). This way, we obtain a sequence of figures that converge either to fractal shapes or - in some particular cases - to smooth shapes. By using this modeling method, we are able to unify in one single model the till now opposed domains of smooth and fractal shapes.

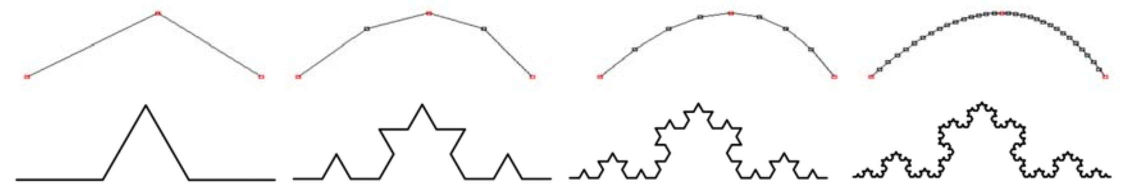

Figure 1: *Iteratively modeled curve: [Top] construction of a Bezier curve using De Casteljau's method; [Bottom] first construction steps of a Von Koch curve.*

III) ***Summary of the results***

We have developed a geometric design method based on generalizations of IFS. We have shown that it is possible to extend the properties of fractal shapes to forms used in classical modeling (e.g Bézier or Splines).

We have developed and studied a new formalism, which we named BCIFS (Boundary Controlled Iterated Function System), which could serve as basis for development of a computer aided design software called modeler.

The resulting figures verify planarity constrains in order to facilitate their physical construction out of planar construction material. Finally, a series of tools has been worked out in order to convert the geometry data into a set of constructional elements, ready for integrated manufacturing.

III.1) ***Topology***

The BCIFS model is based on the description of incidence and adjacency relations between the various figures. Each figure corresponds to a certain cell topology, like the B-Rep (cf. thesis Gouaty and JIG08).

A BCIFS is equipped with an equivalence relation. It establishes the sharing between some sub-cells incident and subdivided. This equivalence relation decomposes into a set of equations of adjacency and incidence. The adjacency equations describe how these sub-cells are interconnected (two edges share the same vertex, two faces share the same edge...).

Each iterative calculation produces a sequence of meshes whose faces-edges-vertices structure corresponds to the subdivisions of the initial cell decomposition.

This model can describe a large variety of topological structures. It can equally well represent classical or fractal topologies. In the case of a classical topology, the topological structure is preserved at each stage of subdivision. In the case of a fractal topology, topological structure is modified at each step, but some structure persists.

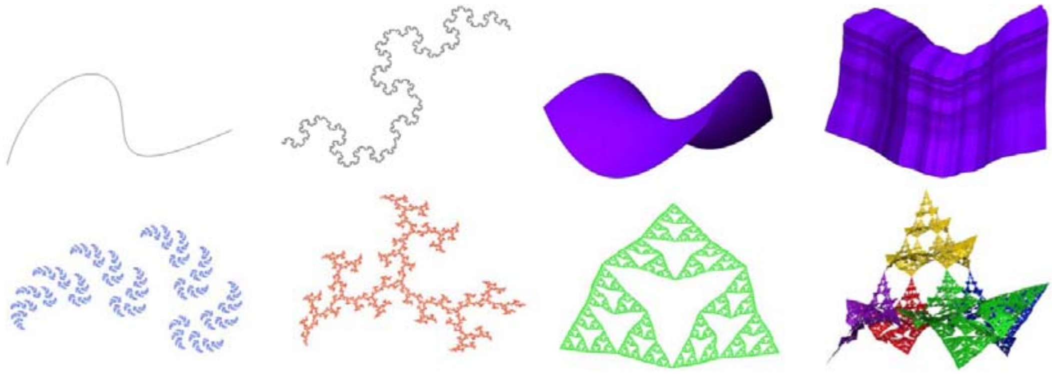

Figure 2: *Examples of classical topologies [Top] and fractal topologies [Bottom]*

III.2) ***Geometry***

Each relationship subdivision or incidence of BCIFS corresponds to a geometric operator represented as a matrix. The control of these operators allows to control the geometry of the figures.

III.2.a) ***Free control***

Our goal was to make the input method of such parameters as intuitive as possible. By associating barycentric space to every figure, we obtain a model similar to the ones commonly used CAD, allowing manipulation of forms through control points. However, in the classical formalisms, the shapes are entirely determined by the control points. In the BCIFS model, shapes are defined by their subdivision operators, which are not predefined. There are two types of parameters:

1) The total distortion obtained by control points (global shape control)
2) The local distortion obtained by subdivision points. Depending on the choice of the subdivision parameters, the resulting figure can be smooth or rough (local shape control).

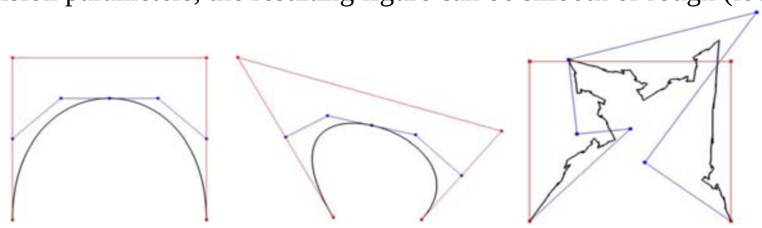

Figure 3: *Subdivision curves can be handled on the one hand by its control points (shown in red), which act on the global aspect of the curve. Subdivision points (shown in blue) act locally on the curves aspect, which changes its "texture" from smooth to fractal.*

The BCIFS formalism has been implemented in software whose input format is explained in the appendix of Gouaty's thesis. As part of the research project, this software has been tested and used by the architect Ivo Stotz PhD to make timber structures. It was also tested by students in architecture in the framework of an architectural design studio.

III.2.b) ***Projective geometry***

The principles of projective geometry are already widely applied in classical CAD-software. They are used for the computation of non uniform rational figures, such as NURBS. The use of the projective geometry gives designers the possibility to assign different weights to the control points. This means that the control points do not longer have only the coordinates {x,y,z} of the three dimensional space ***R3*** but also point weights (w). The figures below illustrate the effect of control point weights on different figures such as smooth Bezier curves, fractal curves, and vector sum surfaces (smooth and fractal).

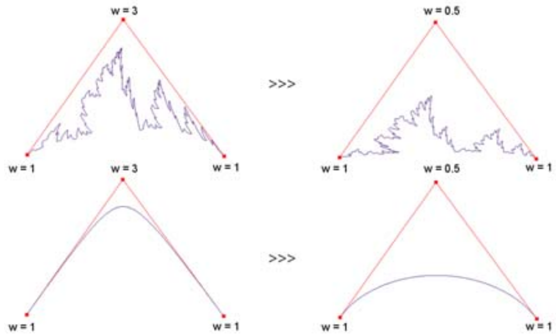

Figure 4: *Influence of point weight editing on curve design; fractal and smooth examples.*

III.3) ***Application to the construction***

III.3.a) ***Geometrical constraints***

In the scope of timber construction, we are interested in modeling 3D surface meshes based on an iterative model inspired from the IFS model (Iterated Function System). The modeled shapes must meet certain properties in order to ensure their physical feasibility.

We have developed a new method for generating planar quadrilateral meshes for the timber construction. We define these meshes as a certain vector sum of two curves. We apply this operator to two IFS, each describing a certain curve. Further we add some techniques from the projective geometry to our model, which will augment the form finding possibilities (cf REFIG09 and IASS09).

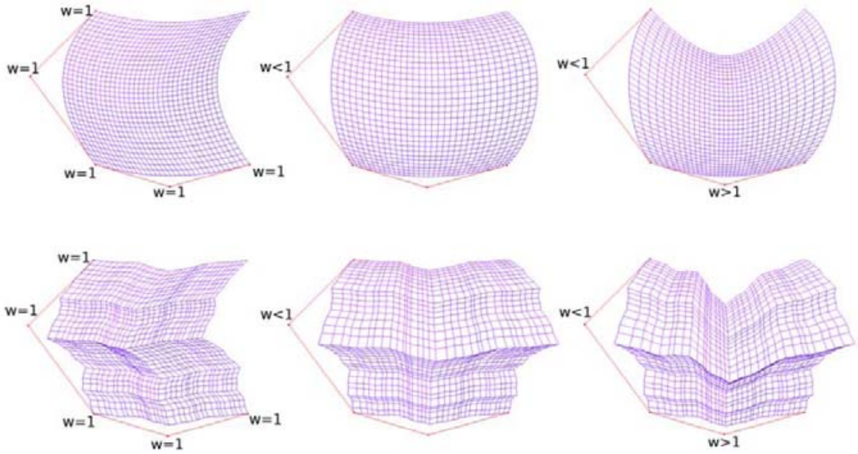

Figure 5: *Influence of point weight editing on surface design; fractal and smooth examples.*

III.3.b) ***Digital production chain of free-form architecture***

In order to realize physical buildings out of discrete virtual geometries, the elements, which constitute the 3D-modells, are replaced by constructional elements. For an iteratively designed curve, the line sections will be substituted by linear constructional elements such as planks or beams. For the case of a discrete surface, we replace its faces by planar constructional elements (panels, plates etc.). The substitution of geometric elements by constructional elements poses a certain number of questions as the geometric figures don't have physical dimensions like thickness.

The procedure to get form the geometry to the machine code has been mainly automated. It is commonly named 'digital chain'. To realize such complex buildings, following work steps are necessary (cf. thesis Stotz):

- Translation of the geometric elements into a set of constructional elements.
- A unique address for each constructional element is necessary for the logistical reason, that the different elements might be assembled in the right place.
- Each element has to be oriented according to the coordinate system of the CNC-machine.
- Automatic generation of the machine code for each element: The material properties, the type of machine and the nature of the cutting tools are of highest importance for integrated production of the elements, which are all different in size and shape.

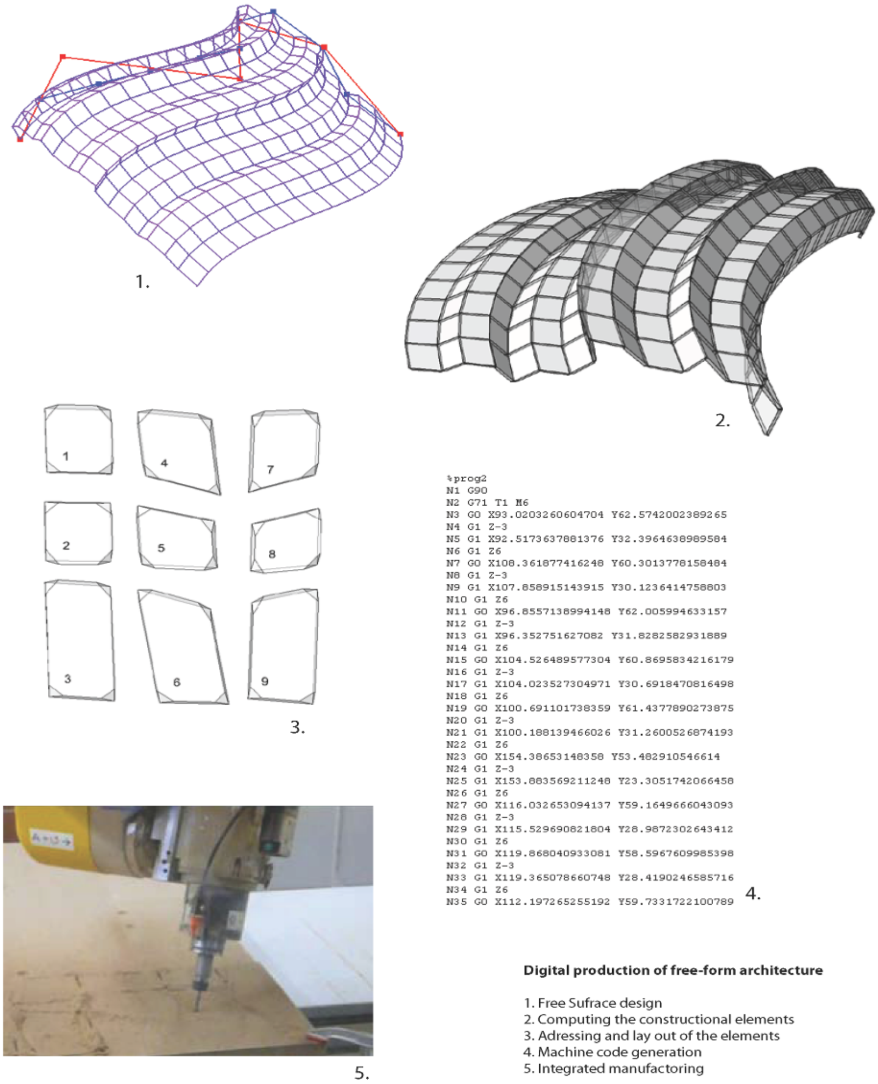

Figure 6: *Schematic and summary representation of the digital chain.*

IV) **Conclusion**

The form finding capabilities of the proposed BCIFS method have been successfully implemented into a CAD-software. The introduction of subdivision points will provide designers unseen design possibilities; giving them a graphical way to act on the subdivision matrices. The fact that the proposed surface method verifies a certain number of topological and geometrical constraints presents an important advantage in comparison to existing surface methods.

The realizations listed in I.3 have shown how efficient our method can be. The planning effort for the production of free-form architecture has been greatly reduced. This will allow an optimization of the production costs. In future, we hope to be able to apply the findings and to develop bigger and more complex objects. Further applications may be suspended ceilings, free-form facades, climbing walls or halls.

V) **Publications**

V.1) ***Publications of the author:***

1. **E. Tosan, I. Bailly-Salins, G. Gouaty, I. Stotz, Y. Weinand, P. Buser** ; *Une modélisation géométrique itérative basée sur les automates* ; GTMG06 ; mars 2006
2. **E. Tosan, I. Bailly-Salins, G. Gouaty, I. Stotz, Y. Weinand** ; *Modélisation itérative de courbes et surfaces : aspect multirésolution* ; GTMG07 ; mars 2007
3. **G. Gouaty, E. Tosan, I. Stotz, Y. Weinand** ; *Un modèle itératif de surfaces pour la construction en bois* ; GTMG07 ; mars 2008
4. **I. Stotz, G. Gouaty, Y. Weinand**; *Iterative surface design for constructions based on timber Panels*; Holzbulletin Holzforschung Schweiz / Heft 1; June 2008
5. **I. Stotz, Y. Weinand**; *IFS-modeling for feasible freeform timber constructions*; World conference of timber engineering, WCTE08; June 2008
6. **I. Stotz, G. Gouaty, Y. Weinand** ; *Iterative geometric design for architecture*. Advances in Architectural Geometry ; Vienna, Austria ; September 13-16 ; p. 121-124 ; ISBN 978-3-902233-03-5 ; 2008
7. **G. Gouaty, Eric Tosan, Ivo Stotz, and Yves Weinand** ; *Un modèle itératif de surfaces pour la construction en bois*. Revue Électronique Francophone d'Informatique Graphique, 3(1):1-12, 2009.
8. I. **Stotz, G. Gouaty, Y. Weinand** ; *Iterative Geometric Design for Architecture*, Journal of the International Association for Shell and Spatial Structures (IASS), vol. 49 (2009) no 1, April n.160, ISSN:1028-365X, p. 11-20 ; 2009

7. **I. Stotz** ; *Iterative geometric design for architecture* ; PhD thesis, Lausanne, 2009, http://library.epfl.ch/theses/?nr=4572

9. **G. Gouaty** ; *Modélisation géométrique itérative sous contraintes* ; PhD thesis, Lausanne, 2009, http://infoscience.epfl.ch/record/142217
10. **G. Gouaty, E. Tosan, I. Stotz, Y. Weinand** ; *Introduction au modèle itératif BCIFS* ; GTMG10 ; mars 2010

V.2) ***Publication, in which our research is explicitly mentioned:***

11. **Y. Weinand** ; *Des géometries complexes entre l'ingénieur et l'architecte* ; matières ; p. 12ff ; PPUR ; 2006
12. **A. Hohler ;** *Modèles fractals dans la construction*; Tracés 17; p. 9f ; SEATU ; September 2006
13. **A. Hohler ;** *Modèles fractals dans la construction*; NIKE-Bulletin 1-2/2007; p. 40ff ; NIKE ; 2007
14. **J. Solt**; *Diskrete Elemente*; TEC21, No. 17-18, April 2008
15. **Y. Weinand** ; Innovative Timber constructions, Journal of the International Association for Shell and Spatial Structures (IASS), vol. 50, no 1, ISSN:1028-365X, p. 111-120 ; 2009

V.3) ***Conferences:***

1. GTMG06; University of Cachan, France, March 2006.
2. GTMG07; University of Valenciennes, France, March 2007
3. GTMG08; Mulhouse, France, March 2008
4. SAH Statusseminar, EMPA-Dübendorf, March 2008
5. WCTE08; Japan; July 2008
6. JIG08; Dijon, France, June 2008
7. SAH Statusseminar, St Loup, March 2009
8. GTMG10; Dijon, France, March 2010

V.4) ***Distinction:***

Distinction for the best Poster at PhD students Day 2006 of the EPFL: *Fractal Geometry (software implementation)*, Gilles Gouaty.