

Decision Support System for the hydropower plants management: the MINERVE project

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ABSTRACT: The MINERVE project was initiated for flood management in the Upper Rhone river basin. Its first goal is the prediction of floods. The second one is the optimum management of the complex system of hydropower plants located in the basin in order to diminish the peak discharge in the Rhone River and its tributaries. The MINERVE Interactive Decision Support (MINDS) proposes turbine and bottom outlet preventive operations before the flood peak, for having storage volume in the reservoirs during the peak flow. The *iterative ranking Greedy algorithm* allows successive resolutions for all the hydropower plants in an iterative way until reaching the optimum operations of the whole complex. The *multi-criteria analysis* optimises the start and the end of the preventive operations (turbine and bottom outlet operations) for the whole ensemble hydrological forecasts for a selected hydropower plant.

1 INTRODUCTION

Several flood events during the last decades have caused important disasters in the Upper Rhone River basin in Switzerland and have emphasized the need of dealing with catastrophic inundations. The MINERVE project aims to improve security by reducing damages in this basin (Boillat, 2009). The main objectives are to predict floods in advance for warning and to manage the multi-reservoir system of the existing hydropower schemes in order to gain a better flow control during floods.

The MINERVE system provides hydrological forecasts up to five days (García Hernández et al., 2010). It exploits flow measurements at river gauging stations, operation data from reservoirs and hydropower plants as well as deterministic (COSMO-7 and COSMO-2) and ensemble (COSMO-LEPS) meteorological forecasts from MeteoSwiss. The hydrological model is based on a semi-distributed concept and is completed by rivers and hydraulic structures such as water intakes, reservoirs, turbines and pumps. When the hydrological forecasts are simulated, a report provides the warning level (Notice, Alert or Alarm) at selected control points distributed over the whole basin, being a support to decision-makers for preventive actions.

The hydrological forecasts are also used for the evaluation of priority decisions concerning the safe management of the storage hydropower plants. A tool called MINDS (MINERVE Interactive Decision Support) has been developed for this purpose. Turbine and bottom outlet preventive operations can be proposed to the powerhouse operators, depending on observed discharge at control points, hydrological forecasts and reservoir levels. The goal is to store water inflows in the reservoirs and stopping turbines during the peak flow. Appropriate preventive operations can thus reduce the peak discharges in the Rhone River and its tributaries, limiting or avoiding damages.

Even if these preventive operations can be risky, good understanding and interpretation of the hydro-meteorological forecasts allow the identification of the potentially dangerous floods and the appropriate preventive interventions and manoeuvres.

2 HYDROLOGICAL FORECASTS

2.1 *Meteorological input data*

The MINERVE system exploits probabilistic COSMO-LEPS weather forecast as well as deterministic weather forecasts COSMO-7 and COSMO-2, all of them operated by MeteoSwiss.

COSMO-LEPS is the limited-area EPS (Ensemble Prediction System) developed within the COSMO consortium (Consortium for Small-scale Modeling) and combines the benefits of the probabilistic approach with the high-resolution detail of the model. Twice per day, COSMO-LEPS provides high resolution probabilistic forecasts (horizontal mesh-size of 10 km) based on a 16-member ensemble for central and southern Europe with a lead time of 120 h.

Deterministic forecasts COSMO-7 and COSMO-2 are a support of COSMO-LEPS. The regional COSMO-7 is driven by the global model of ECMWF (European Centre for Medium-Range Weather Forecasts) and covers most of western and central Europe. It is computed on a grid spacing of about 6.6 km and is calculated twice per day for 72 h lead time. The local COSMO-2, driven by COSMO-7, covers the Alpine region with Switzerland located at the center and is computed on a grid spacing of about 2.2 km. It is calculated 8 times a day for a 24 h lead time. Both of them allow the use of the most recent atmospheric conditions observed and the benefit of the short range forecasting.

2.2 *Hydrological model*

The catchment area of the Rhone River has about 5500 km² and has been divided in 239 sub-catchments taking into account all hydraulic structures of dams and hydropower plants. The hydrological concept used to estimate the discharge in the outlet of each sub-catchment is based on the GSM-Socont (Glacier & SnowMelt -SOil CONTRibution) model (García H. et al., 2007).

For the hydrological part of modelling, each sub-catchment is divided into two parts, glacier and non-glacier, both divided in altitude bands. Precipitations and temperatures values are calculated from the meteorological input data for each one of these altitude bands. Then, a snow model follows the temporal evolution of the height and saturation degree of the snow. The snow melt produces an equivalent precipitation starting from a rate of saturation threshold.

In the case of a non-glacier band, this equivalent precipitation supplies the infiltration and the transfer model, composed by two parallel non-linear reservoirs, which produce the slow and fast components of the discharge going to the outlet of the sub-catchment.

In the case of a glacier band, the equivalent precipitation resulting from the snow melt is transferred to the outlet by a linear reservoir. When there is no more snow, a glacier model produces (when temperature is higher than zero) a discharge which is also transferred to the outlet of the sub-catchment by a linear reservoir behaviour.

2.3 *Hydrological simulations*

This semi-distributed hydrological model was built using the hydrological and hydraulic simulation tool Routing System II (García H. et al., 2007; Jordan et al., 2008). This software was designed to simulate the formation and the propagation of free surface flows in a complex system. It allows hydrological and hydraulic modelling by an oriented object approach, according to a semi-distributed conceptual scheme. It takes into account special hydrological processes such as snow and glacier melt, surface and sub-surface flows, routing in reservoirs, water transfer tunnels and rivers as well as the modelling hydraulic structures with valves, gates, water intakes, turbines or pumps.

An evolution of Routing System II tool provides a flood forecast in real-time, coupling the observed measurements and the weather forecast information with the hydrological model. Every time a new weather forecast is provided, the hydrological forecast is updated.

After the first developments of the project with deterministic forecasts until 2006 (Jordan, 2007), the MINERVE system is enhanced since 2008 by implementing new probabilistic forecasts (García H. et al., 2009a) as well as other improvements in other domains (García H. et al., 2009b) with the aim of providing better hydrological forecasts to the decision support system.

3 MINERVE INTERACTIVE DECISION SUPPORT

3.1 Hydraulic model

The hydraulic model of the Wallis and Vaud Cantons, developed for the optimisation tool MINDS (MINERVE Interactive Decision Support), is a simplified model of this complex river basin (Fig. 1). It contains the most important reservoirs RES (triangles), with its bottom outlets and spillways (square dotted lines), hydropower plants HPP (round dotted lines), as well as the main river network (solid lines) with the main control points CP (big circles).

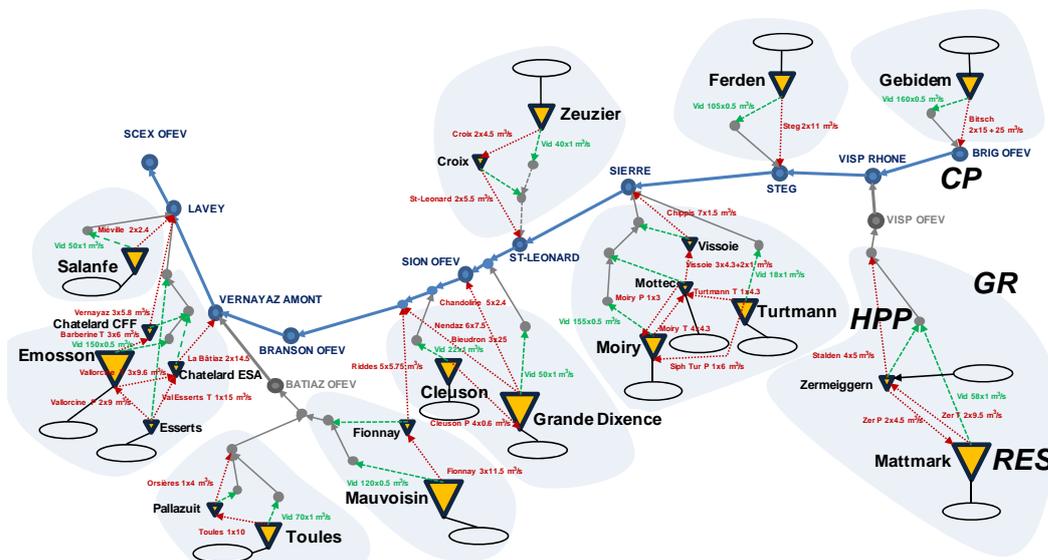


Figure 1. Scheme of the Upper Rhone River catchment with the MINDS model of the hydropower plants.

In this model, the hydropower plants and reservoirs have been divided in independent groups (i.e. without any connexion between them). The characteristics of the groups and their reservoirs are presented in Table 1. Even if reservoirs are generally used to store water, several reservoirs operate without this function, just as a compensation basin. They work as elements where the inflow is derived to other reservoirs or rivers, or where pumping operations can be done.

Table 1. Groups and reservoirs of the MINDS model (P for volume of punctual reservoirs).

Group (GR)	Reservoir (RES)		Vol (Mm ³)	Group (GR)	Reservoir (RES)		Vol (Mm ³)
	Name				Name		
GD	Grande	Dix-	422 · 10 ⁶	EL	Zeuzier	61 · 10 ⁶	P
	ence				Croix		
	Cleuson		28 · 10 ⁶	SAL	Salanfe	43.6 · 10 ⁶	
ESA	Emosson		255 · 10 ⁶	GSB	Toules	27.3 · 10 ⁶	P
	Esserts		P		Pallazuit		
	Chatelard CFF		P	EM	Gebidem	9.6 · 10 ⁶	
	Chatelard ESA		P	KWL	Ferden	2.12 · 10 ⁶	
FMM	Mauvoisin		215 · 10 ⁶				
	Fionnay		P				
KWM	Mattmark		188 · 10 ⁶				
	Zermeiggern		P				
FMG	Moiry		83.3 · 10 ⁶				
	Turtmann		0.844 · 10 ⁶				
	Mottec		P				
	Vissoie		P				

The hydropower plants are also included in the model with their characteristics (discharge capacity, head, installed generation capacity,...). They connect two reservoirs or a reservoir to the river network. When a preventive operation of turbinage or pumping is proposed, the hydropower plant works with the maximum discharge capacity in order to have the best performance possible in terms of time, i.e. reducing the time for preventive operations as much as possible.

Finally, the main control points (CP) of the river network are assumed as the locations where optimisations can be done. The transit times between them are considered as constant.

For all the control points in Table 2, the discharge is given for the flood of October 15, 2000 (Q_{2000} , in m³/s, from which flooding are assumed), together with the extreme expected discharge (Q_{ex}) associated with the expected costs of damages (10⁶ CHF, i.e. Swiss Francs) in the surrounding area.

Table 2. Characteristics of control points and areas related to them.

Control Point (CP)	River	Q ₂₀₀₀	Q _{ex}	Maximum expected damages in the area
Brig OFEV	Rhone	560	750	207.9
Visp OFEV	Visp	190	590	441.0
Visp Rhone	Rhone	760	1380	2835.0
Steg	Rhone	779	1380	560.0
Sierre	Rhone	826	1480	1106.8
St-Léonard	Rhone	859	1520	50.4
Sion OFEV	Rhone	910	1580	896.7
Branson OFEV	Rhone	980	1600	452.3
Batiaz OFEV	Dranses	196	204	56.3
Vernayaz Am.	Rhone	1176	1804	8
Lavey	Rhone	1236	1913	313.16
Scex OFEV	Rhone	1370	2120	1936.44

The simplifications assumed in the model reduce the calculation time to a couple of minutes, without reducing the performance of the system. In fact, transit time has been estimated previously and reservoirs proposed as punctual elements have not enough volume to contribute to the preventive operations or to reduce significantly discharges in the river network. Thanks to these simplifications and the approach explained hereafter, the calculation time for optimisation has been reduced in order to use the software in real-time, updating the forecasts if necessary.

3.2 *Optimisation objectives*

The objective of the system is the minimisation of all the damages and production losses produced in Rhone River catchment area, upstream of the selected control point considered as the objective of the optimisation. All the damages and losses are calculated in economics values for comparison. Firstly, the damages expected in the studied catchment area are taken into account because of the flood. Secondly, the potential production losses in the hydropower plants because the proposed preventive operations are assessed.

Once a control point is selected as downstream objective (the selected point is usually the outlet of the catchment area), the objective function of the system is presented as the minimisation of both the expected damages and the potential costs of the preventive operations upstream of the selected control point. Then, the optimisation of the function searches the optimal sequences of turbine, emptying and pumping in each concerned hydropower plant. Thus, the variables of the system are the start and end for the expected sequences. Consequently, in the case that no damage is expected, the system does not propose any preventive operations.

The costs related to the preventive manoeuvre results, automatically and simultaneously, in a maximisation of the volumes in the reservoirs for the period of the optimisation. The reason is that preventive operations are done when they have an effect in the expected damages, and do not last more than is strictly necessary.

The inputs of the system are the hydrographs in the control points as well as the inflows and initial levels at the reservoirs. The constraints are the usual ones in this case of optimisations, such as the capacity of turbines, pumps and bottom outlets, the correct balance in the reservoirs volume, the emergency rules of reservoirs and the behaviour of the spillway.

To solve this objective function, the problem uses an *Iterative Ranking Greedy algorithm* (IRGA) and a *Multi-Criteria Analysis* (MCA). The IRGA allows the resolution in series for the hydropower plant groups. The MCA minimises the costs based on damages, losses and weights of the forecasts. Different MCA analysis methods have been implemented: Expected Risk, MinMax Regret (Savage, 1951), Hurwicz (Hurwicz, 1950) and Fuzzy logic (Cheng, 1999).

3.3 Damages and cost evaluation

For the estimation of the expected damages ED, the maximum discharge Q_{max} of the simulation period is calculated at each control point k , CP_k . According to Eq. 1, the theoretical discharge where severe flooding occurs (assumed to be the October 15, 2000 flood, Q_{2000}) and the considered extreme discharge Q_{ex} at the same control point are compared to Q_{max} .

If Q_{max} exceeds Q_{2000} , an initial damage ($\delta \cdot ED_{max}$, $\delta \leq 1$) at the area surrounding the control point location is directly produced. The maximum damage ED_{max} is produced when it makes equal Q_{max} and Q_{ex} . The total expected damages are the addition of all the damages upstream of the selected objective location.

$$ED_{CP_k}(a_{i_{coll}}|f_j) = \begin{cases} 0 & \text{if } Q_{max_{CP_k}}(a_{i_{coll}}|f_j) \leq Q_{2000_{CP_k}} \\ \delta \cdot ED_{max_{CP_k}} + (1-\delta) \cdot \left(\frac{Q_{max_{CP_k}}(a_{i_{coll}}|f_j) - Q_{2000_{CP_k}}}{Q_{ex_{CP_k}} - Q_{2000_{CP_k}}} \right)^{1-\lambda} \cdot ED_{max_{CP_k}} & \text{if } Q_{2000_{CP_k}} < Q_{max_{CP_k}}(a_{i_{coll}}|f_j) < Q_{ex} \\ ED_{max_{CP_k}} & \text{if } Q_{max_{CP_k}}(a_{i_{coll}}|f_j) \geq Q_{ex_{CP_k}} \end{cases} \quad (1)$$

where $a_{i_{coll}}$: collection i of preventive operations in all the reservoirs; f_j : forecast j ; δ : initial damage parameter, representing the percentage of initial damages compared to ED_{max} [-]; λ : power damage parameter [-]; Q_{max} : maximum discharge in the whole studied period [m³/s]; Q_{2000} : October 15, 2000 peak discharge, assumed to be the flooding discharge [m³/s]; Q_{ex} : extreme discharge related to maximum damages [m³/s]; ED_{max} : maximum expected damages [CHF].

For the potential preventive operations costs PPOC, installed capacity (P) and energy (E) are calculated depending on the discharge series Q and head H of the hydropower plant h HPP _{h} (Eq. 2, 3). If a reservoir is connected to several hydropower plants, the same preventive operation is provided for all of them.

The potential costs of the preventive operations (energy sale losses) per reservoir r (RES_r) or group g (GR_g) are calculated based on the maximum price the energy could be sold (c_{max}) in the energy market and the current estimated price ($c_{current}$) when preventive operations are realized (Eq. 4, 5). Estimate price depends on time and day of the week and is obviously zero for bottom outlet operations.

$$P_{HPP_h}(a_{i_{RES_r}}) = \rho \cdot g \cdot H_{HPP_h} \cdot Q_{a_{i_{RES_r}}} \cdot \eta_{HPP_h} \quad (2)$$

$$E_{HPP_h}(a_{i_{RES_r}}) = \int_{t=t_a}^{t=t_b} \frac{P_{HPP_h}(a_{i_{RES_r}})}{1000} dt \quad (3)$$

$$PPOC_{RES_r}(a_{i_{RES_r}}) = \sum_{h=1}^{h=u} \int_{t=t_a}^{t=t_b} \frac{P_{HPP_h, HPP_h \in RES_r}(a_{i_{RES_r}})}{1000} \cdot (c_{max} - c_{current}) dt \quad (4)$$

$$PPOC_{GR_g}(a_{i_{GR_g}}) = \sum_{r=1}^{r=h} PPOC_{RES_r, RES_r \in GR_g}(a_{i_{RES_r}}) \quad (5)$$

where ρ : water density, 1000 [kg/m³]; g : gravity, 9.81 [m/s²]; η : plant efficiency [-]; P_{HPP_h} : installed capacity [W]; E : energy [kWh]; $PPOC$: potential preventive operation costs [CHF]; $a_{i_{RES_r}}$: preventive operation i in the reservoir r ; $a_{i_{GR_g}}$: collection i of preventive operations in the reservoirs of the group g .

3.4 Theoretical objective function

The Expected Risk criteria analysis (ERCA) is assumed for presenting the theoretical objective function. The ERCA identifies the ideal preventive operation for the whole ensemble hydrological forecasts based on a risk assessment which depends on expected damages, potential losses and weight of the forecasts taken into account, according to Equation 6.

$$\min \left[\alpha \cdot \frac{1}{n} \sum_{j=1}^{j=n} \left(\sum_{k=1}^{k=p} AD_{CP_k}(a_{i_{collection}} | f_j) \cdot P(f_j) \right) + \beta \cdot \sum_{r=1}^{r=v} PPOC_{RES_r}(a_{i_{collection}}) \right] \quad (6)$$

where α : weight parameter for the expected damages, 1 [-]; β : weight parameter for the potential preventive operation costs, 1 [-]; $P(f_j)$: occurrence probability of forecast j ; n : total number of forecasts; p : total number of control points; v : total number of reservoirs.

Nevertheless, since calculation time increases considerably solving all the variables at the same time, the presented *iterative ranking Greedy algorithm* (Dechter and Dechter, 1989) is also used in order to be able to solve preventive operations reservoir by reservoir for decreasing the calculation time as much as possible for the real-time decision making task.

3.5 Iterative Ranking Greedy algorithm (IRGA)

The IRGA allows solving in series all the hydropower plants. First of all, a hierarchy of priority for the groups' management is defined. When a group is selected, the objective function searches the minimisation of the expected damages in the considered catchment as well as the potential costs of preventive operations in the group.

The hierarchy of the groups is given by the efficiency of their reservoirs for storing water during a flood or by their location from upstream to downstream. Then, a pre-defined rank in the reservoirs of the group provides the position to optimise them. The theoretical objective function (Eq. 6), still assuming the ERCA methodology, becomes the objective function x (because it is related to the reservoir x of the group w) as presented in Equation 7 and 8.

$$\min \left[\alpha \cdot \frac{1}{n} \sum_{j=1}^n \left(\sum_{k=1}^{k=p} ED_{CP_k} (a_{i_{RES_x}} | f_j) \right) + \beta \cdot PPOC_{RES_x, RES_x \in GR_w} (a_{i_{RES_x}}) \right] + \xi \quad (7)$$

$$\xi = \beta \cdot \sum_{r=1}^{r=v} PPOC_{RES_r, r \neq x} \quad (8)$$

The optimisation is obtained by double scanning, searching the start and end for the preventive operations (turbine and bottom outlet operations) for the ensemble of the forecasts in the hydro-power plants linked to the optimised reservoir. Firstly, the optimisation of the turbine sequence is done. Afterwards, if flood damages still occur in the basin, the bottom outlet sequence is optimised.

The first scan searches the sequence of the preventive operations with a bigger calculation step for the start and the end of the sequence (the parameters is pre-defined to four hours, but can be easily changed by the user). Once this solution is found, a second scan searches the optimal solution around this first one. The calculation step is then smaller, normally the same than data coming from hydrographs and inflows (one hour in our case).

This optimisation is carried out for each reservoir of all the groups (Fig. 2). When the preventive operations in the hydropower plants connected to the current reservoir are optimised, the operations of the hydropower plants of the other reservoirs are assumed known and established in advance. The potential costs PPOC of energy losses for the known operations are then ζ , as given in Equation. 8.

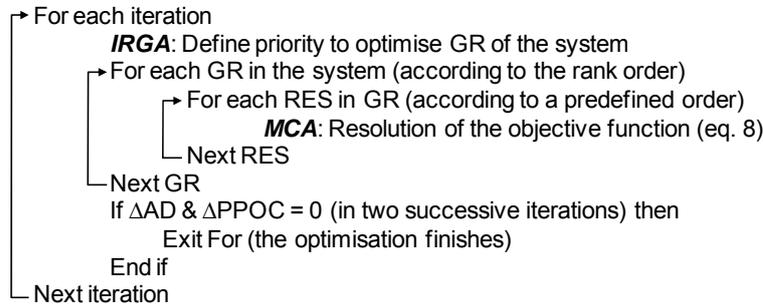


Figure 2. Scheme for the optimisation of the MINDS system.

The optimization is performed several times by iteration until the optimum is found and the results (expected damages in each sector and preventive operations costs in the reservoirs) do not vary anymore). Besides, before the next iteration, the *ranking* of the groups is recalculated and their hierarchy can be changed.

4 RESULTS

The initial results of the system reveal a reduction of the peak discharge and flooding in the Rhone River and its tributaries. The validation of the results is currently tackled but it is already clear that the system has a great performance and could be operated by the crisis task force in the Wallis Canton.

The first page of the MINDS interface (Fig. 3) presents preventive operations in the concerned hydropower plants as well as the control points with overflowing problems. Other pages of MINDS give the benefit of the preventive operations, the potential costs for the hydropower plants or expected damages in the control points, hydrographs with and without the preventive operations etc, always from a probabilistic point of view (e.g. with the help of box plots).

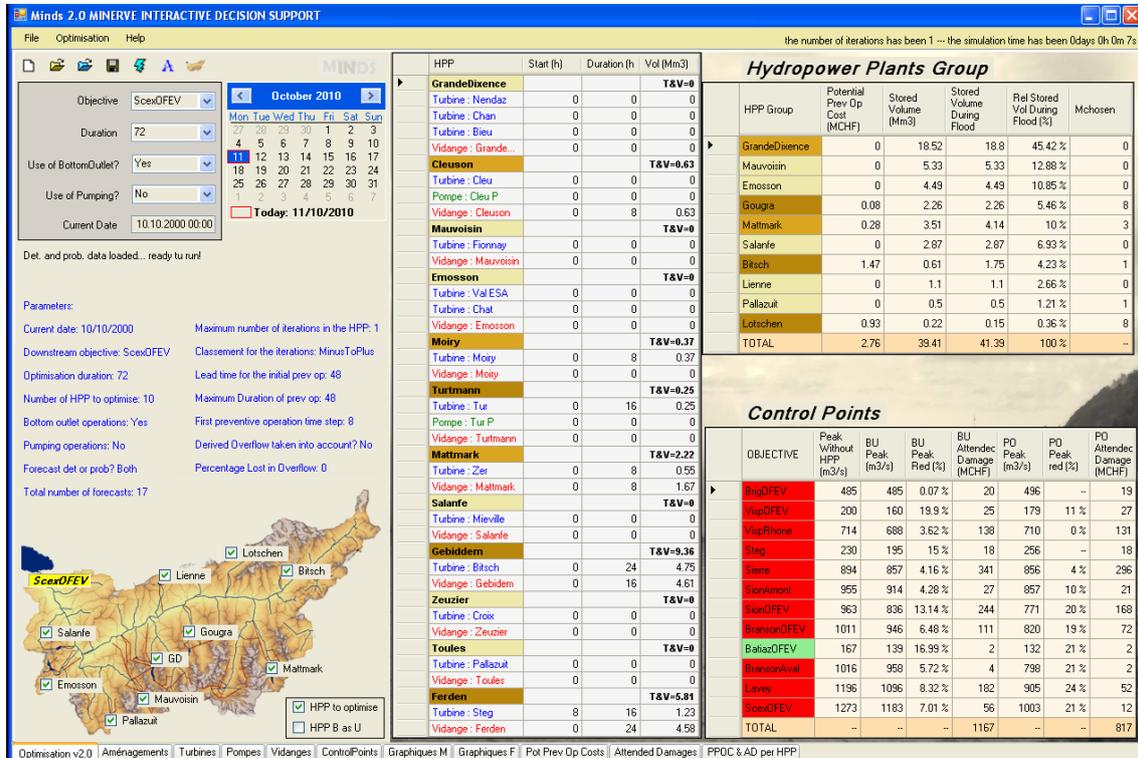


Figure 3. Interface of MINDS.

The MINDS interface which is still under development will give information on the probability damages before and after the optimisation of the preventive operations.

5 CONCLUSIONS

The MINERVE system developed for the Wallis and Vaud Cantons is already operational. It allows simulating the discharge in the river network of the Upper Rhone catchment area since it considers all hydraulic elements of the hydropower plants and dams, preventive turbine operations and water release for flood protection. The flood forecast and management system is the decision-making tool which is used by a crisis task force for limiting flood damages.

The program MINDS (MINERVE Interactive Decision Support) is the main development of the project for preventive operations at reservoirs located in the catchment area. A simple but robust procedure has been implemented for the optimization of this complex river basin.

The reservoirs and hydropower plants are organized in independent groups and are optimised in sequence based on a *Greedy algorithm*. The hydropower plant is optimised regarding its turbine and bottom outlet operations with an objective function which minimises expected damages and potential preventive operations costs upstream of the selected objective location.

The strength of MINDS is its flexibility and adaptability. If a river sector has a reduced flooding threshold because of ongoing construction works, a turbine is under maintenance and/or a bottom outlet gate is not operational, the program is able to recalculate the optimal solution in real-time with the current characteristics of the rivers, reservoirs and hydropower plants.

The preventive operations are then transmitted to the crisis task force which decides whether or not to impose the hydropower plant operators to take actions. In this way, several agreements have been signed between the Wallis Canton and the hydropower plant operators for possible reimbursements when floods do not arrive and the energy sales benefits are reduced.

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