Flexure-based mecano-optical multi-degree-of-freedom transducers dedicated to medical force sensing instruments

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par

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Always try the problem that matters most to you.

— Andrew Wiles

A mathematician is a person who can find analogies between theorems a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories.

— Stefan Banach
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Neuchatel, January 30, 2019

S. F.
Abstract

This thesis develops novel multi-degrees-of-freedom flexure-based force sensors by exploiting white light interferometry. Fabry-Pérot interferometry measurement has nanometric accuracy which yields sub milli-Newton force sensing accuracy. Such force sensing accuracy can be advantageously utilized for medical applications. Using these techniques, this thesis develops the first monolithically fabricated medical instrument having submillimetric diameter, a peeling hook used to treat epiretinal membranes in the eye. The thesis describes the concepts, design, simulation, fabrication and characterization of this type of instrument. The results of this thesis pave the way for new techniques in Minimally Invasive Surgery (MIS).

The study starts with systematization of sensor manufacturing, which is followed by catalogue of proposed sensor topologies. These structures are then dimensioned using Finite Element Analysis (FEA), with emphasis on two medical applications: biopsy needles and intraocular surgery.

Selected designs were manufactured and characterized on a motorized test bench with automatic and repetitive measurement. Sensor calibration methods were developed and tested and characterization methods were also evaluated. Manufacturing and measurement errors were also studied and described.

The test results showed a significant reduction of sensor calibration error in the selected workspace, as compared to industry solutions.

The tool developed in this thesis is an epiretinal membrane peeling hook for minimally invasive intraocular surgery. It was fabricated using Electro Discharge Machining (EDM), and was at the limit of the feature size of this method.

The novel flexure design methods developed in this thesis open new horizons for designing submillimetric tools having complex geometry of the force sensing load cell and thereby significantly advance delicate surgical procedures.

Keywords: Force decomposition, in-vivo force measurement, flexures, compliant mechanisms, Fabry-Pérot interferometry, optical fibers, Epiretinal Membrane peeling, force sensor.
Résumé

Cette thèse développe des nouveaux capteurs de force à plusieurs degrés de liberté fondés sur des guidages flexibles et exploitant l’interférométrie à lumière blanche. L’interférométrie Fabry-Pérot est précise à l’échelle du nanomètre ce qui donne une précision de captage de force supérieure au milli-Newton. Cette précision peut être utilisée de manière avantageuse pour des applications médicales. Cette thèse utilise ces techniques pour développer le premier instrument médical, capable de mesure la force, fabriqué de manière monolithique de diamètre inférieur au millimètre, avec un crochet de pelage pour enlever des membranes épirétinales de l’œil. Cette thèse décrit les concepts, le design, la simulation, la fabrication et la caractérisation de ce type d’instrument. Nos résultats ouvrent la voie à des nouvelles techniques de chirurgie non invasive.

L’étude commence avec la description systématique de la fabrication des capteurs, suivie par un catalogue de propositions de topologies de capteurs. Ces structures sont dimensionnées par éléments finis, avec une attention particulière pour des applications médicales : les aiguilles de biopsie et la chirurgie intraoculaire.

Plusiers designs, parmi les solutions proposées ont été fabriquées et caractérisées sur un banc d’essai motorisé permettant des mesures répétitives automatisées. Des méthodes d’étalonnage ont été développées et testées et des méthodes de caractérisation ont été évaluées. Les erreurs de mesure et de fabrication ont aussi été étudiées et décrites.

Les résultats des tests ont montré une baisse importante de l’erreur d’étalonnage comparée aux solutions industrielles actuelles.

L’outil développé dans cette thèse est un crochet de pelage pour enlever des membranes épirétinales de l’œil. Il a été fabriqué par électro-érosion (EDM) en allant à la limite de précision de cette méthode.

Les nouvelles méthodes de conception développées dans cette thèse ouvrent de nouveaux horizons de design pour des outils à l’échelle inférieure au millimètre ayant une cinématique de captage de force complexe permettant de faire avancer des procédures chirurgicales délicates.

Mots-clés : Décomposition de force, capteurs de force in vivo, guidages flexibles, interféromètres de Fabry-Pérot, fibres optiques, pelage de membrane épirétinienne, capteurs de force
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List of acronyms

CAD ............... Computer Aided Design

cog ............... Center of Gravity

CTI ............... Commision for Technology and Innovation

DOF(-s) ........... Degree(-s) of Freedom

EDM ............... Electro-Discharge machining

FBG ............... Fiber Bragg Grating

FEA/FEM ............ Finite Elements Analysis / Finite Elements Method

FIB ............... Focused Ion Beam

FPI ............... Fabry-Pérot Interferometry

MIS ............... Minimally Invasive Surgery

MRI ............... Magnetic Resonance Imaging

OFS ............... Optical Fiber Sensor

PRBM ............... Pseudo-Rigid-Body-Model

RMS ............... Root Mean Square

RVC ............... Retinal Vein Cannulation

SEM ............... Scanning Electron Microscopy
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1 Goal and Plan of the thesis

1.1 Goal of the thesis

The subject of this thesis is the development of monolithic medical instruments, i.e., which perform their operation with a single piece of material for the base, the force sensitive flexures and the specialized tip.

Existing instruments exploiting this principle are the Endosense ablation catheter [1] and the Sensoptic palpating instrument for measurement of ossicular chain mobility [2]. Both of these have diameter at the millimeter scale.

The goal of this thesis is a feasibility study of miniaturizing technology to sub-millimeter diameters. This could be advantageously applied to Minimally Invasive Surgery (MIS) as well as the systematization of the development process which would reduce the time to market of novel medical instruments.

This goal was achieved by answering the following questions.

• Is it possible to manufacture a tool of sub-millimeter diameter capable of force measurement able to improve MIS?
• Is it possible to construct a generic force sensor model which includes explicit force decomposition taking into account various load cell topologies and multiple fiber location configurations?
• Can sensor accuracy be improved by new unified calibration methods applied to multiple instances of the same sensor design, thereby reducing fabrication time?
Chapter 1. Goal and Plan of the thesis

1.2 Thesis plan

Chapter 2 introduces the role of force sensing in Med-Tech. This is followed by a review of force sensing concepts and recent realizations.

Chapter 3 precisely describes the sensors examined in this thesis and outlines the common elements of measurement systems and load cells developed for medical instruments.

Chapter 4 reviews the load cell topologies evaluated in this thesis, including our novel designs. Each sensor variant is briefly characterized with emphasis on the flexure body and in selected cases safety structures are discussed. Dimensions of the chosen variants are provided along with a description of the research packages dedicated to them.

Chapter 5 introduces the analytical models used to dimension and calibrate sensors. The models give analytic relations between the force components applied to the instrument tip and displacement read by Fabry-Pérot interferometers.

Chapter 6 describes the manufacturing process of our fabricated prototypes.

Chapter 7 describes the test bench developed for sensor characterization.

Chapter 8 presents the FEA and experimental results validating the analytical model.

Chapter 9 presents the VivoForce CTI project, in which developed the first monolithic epiretinal membrane peeling hook. The medical instrument is described from its design up to clinical trials.

Chapter 10 lists the contributions of the thesis along with perspectives for further research.
### 1.3 Timeline

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*Legend: Concept, theory, analysis: Measurement and validation: Application*

Figure 1.1 – Timeline of the thesis with major work packages
2 State of the art

2.1 Force sensing medical instruments

Force sensors are used in various contexts in the medical field. Force measurement provides quantitative feedback between medical instruments and patient tissue during surgery. It is especially valuable for handheld instruments and catheters, as well as for robotic surgical systems. Feedback increases the information obtainable by a surgeon's hand. Sensor miniaturization can provide force information below the human sensory threshold, as proposed in [3]. Force sensing instruments for Minimally Invasive Surgery (MIS) provide information on tissues compliance, something previously possible only during traditional open surgery. Thanks to contact force sensing catheters, tactile information can be extracted from areas not accessible with other tools, thereby increasing the efficiency of ablation, as reported by Kuck [4].

Medical interventions such as MIS lack visual feedback so require alternative imaging methods. Magnetic Resonance Imaging (MRI) is the leading option but it introduces magnetic fields to the operating room which, along with electrically powered instruments such as coagulators, can generate high noise interfering with electrical force sensors. It is therefore preferable to use optical fiber based sensors which allows to keep away the sensitive electronics from the magnetic field and thus allows to be insensitive to such perturbations. There are two leading optical force sensing technologies. One is based on the Fiber Bragg Grating (FBG) and the other, the focus of this study, uses Fabry-Pérot interferometry. The problem with FBG sensors is a lower sensitivity in the principal tool axis, thus requiring higher tool complexity leading to a greater volume. On the other hand, Fabry-Pérot sensors can provide uniform force sensitivity in all three spatial dimensions using a single body force-to-displacement transducer. The intellectual property was protected by Sensoptic SA [5]. The Fabry-Pérot interferometer consists of two partially reflective surfaces spaced micrometers apart. Light rays transmitted through this geometry are split and generate an interferometric pattern, with constructive interference when all the beams are in phase.

One application occurs in palpation operations where the medial tool is held directly by
Chapter 2. State of the art

The surgeon. The tool tip is equipped with a force sensing probe delivering quantitative values. These values unify treatment between surgeons, thereby eliminating the effect of their subjective sensation of applied force. Another field of application are medical robots where force feedback to surgeons is integrated with haptic devices \[6\]. Having this feedback surgeon will be able to perform surgical tasks such as suturing \[7\], with faster cutting and coagulation due to increased precision and dexterity. The goal is to implement force sensing at the tip of the tools used during robotized Minimally Invasive Surgery (MIS) but this requires smaller sensors than are currently available on the automation market. Finally, force sensors are also used in closed loop controls to steer exoskeletons used for rehabilitating patients with motion disorders.

2.2 Force sensors

2.2.1 Introduction to key issues

Sensing of force and torque plays an important role in engineering. Newton's Second Law states that a force \(F[N]\) causes an object with mass \(m[kg]\) to change its velocity with acceleration \(a[m\,s^{-2}]\). This is described mathematically by application of a force vector point at a point. Torque \(\tau[N\,m]\) similarly causes the rotation of an object about an axis. It is described mathematically by the cross-product of the lever-arm \(r[m]\) and the force vector.

\[
F = ma, \quad \tau = r \times F
\]

Force sensing probes range from big units \(10^6 N\) found in robotics and automation, down to the detection of yocto Newton \(10^{-24} N\) forces \[8\]. This range of magnitudes makes it impossible to utilize a single method or even a single principle. Following the terminology of Hunt et al. to describe a measurement system \[9\], the focus of this thesis will be on the elastic elements of load cells. The most common load cells use capacitive or piezoelectric strain gauges. However, this requires either a static electric charge or an electric current supplying the load cell, which can be disadvantageous in certain medical applications, e.g., interference of force measurement during magnetic resonance imaging (MRI). For this reason, our research focuses on interference-optical load cells based on Fabry-Pérot white light interferometers. Not only these have the advantage of resistance to electrical interference, but this type of interferometer also has certain advantages compared to the other optical method used in force measurement, Fiber Bragg Gratings. The Fabry-Pérot method does not add any additional stiffness to the force to deformation transducer.
2.2. Force sensors

Deformation measured by the interferometer is transmitted to the feedback instrumentation. Force is then calculated using Hooke’s Law by considering the stiffness matrix $K$ of the elastic transducer and the measured deformation. The resulting force value can be displayed as text, a graph or other means such as acoustic feedback.

Designing system components for specific applications is a time consuming task, so Fabry-Pérot interferometers are also advantageous because modules are available on the market and feedback instrumentation can be easily adapted to most requirements. On the other hand, there are no guidelines for the design of the elastic transducer, typically its shape is chosen arbitrarily and subsequently optimized either analytically or by FEA. The ultimate goal of this thesis is to provide guidelines for measurement systems based on the requirements defined by the force to be measured. The focus will be on load cells for medical instrumentation having diameter lying between 0.5 mm to 4 mm, as this scale provides safety and miniaturization requirements superior to larger sizes, though our results could be later extended to other ranges.

### 2.2.2 Classification of force sensing concepts

Figure 2.2 – Force sensor designs, PBaki et al [10] (3DOF), a) Primary design of the load cell [11], b) Primary load cell with strain gauges and wiring c) Secondary design of a load cell d) Secondary design of a load cell with strain gauges and wiring
Force sensors in the medical field typically use elastic load cells. These vary from a simple square bar [11], to complex shapes such as the one proposed by Liang et al. [12]. This diversity is due to the different means used in deformation measurement as well as the lack of guidelines for elastic cell design. There are proposals for full 6-DOF elastic load cells based on strain gauges [12] [13], and even certain optical sensors [14]. These can be scaled up, but problems arise in downscaling. Main identified constraint in this case is manufacturing due to the lack of space for complex mechanisms in applications reaching narrow spaces, e.g. required for in-vivo medical testing. The conclusion is that comprehensive design guidelines scalable for specific applications, should not be limited to a single 6-DOF compliant mechanism, but instead a library of such load cells should be provided, starting with 1-DOF elements. This would cover measurement starting from the simplest miniature scale uni-axial force measurements to more complex multi-axial applications.

This library should significantly reduce force sensor development for new applications, these currently require a number of iterations of analysis, manufacturing and validation, see Baki et al.[11] [10]. Reducing the time to market for force sensing tools will have an important impact on the medical field due to wider availability of novel instruments with improved performance.

2.3 Advantages of flexures in the context of force sensing

Our mechanical transducer is based on flexures, i.e., compliant mechanisms [15] [16]. Flexures have the well-known advantages of reducing friction but in our context, their main advantage is their precision due to absence of play. Moreover, we can adjust the stiffness of elementary flexure elements, which are well modeled in [16]. Stiffness of more complicated flexures, as shown in our catalogue was analyzed using FEA.
2.4 Force sensors using Fabry-Pérot interferometry

This thesis will focus on force sensors with measurements done by means of elastic load cells, with deformations measured by Fabry-Pérot white light interferometry. This technology is known and is already present on medical market. It was developed and protected by Sensoptic SA [5]. Already existing applications are: the PalpEar sensor for Ear, Nose, Throat (ENT) and a probe used in a heart catheter commercialized by Endosense SA. The first covers a force range from 0 to 100 gf (≈1N) with a resolution of 3 gf, and accuracy < 5%, according to the manufacturer and by means of a load cell of 4 mm diameter and 10 mm length. Applications currently in development are: a force sensing needle and a force sensing membrane pick for vitreoretinal membrane peeling surgery, described in Chapter 9 of this thesis.

2.4.1 Load cells

Our load cells consist of a single mecano-optical transducer we call the *flexure body* and one or more opto-electrical transducers based on white light Fabry-Pérot interferometry. The thesis focuses on the design and development of the flexure body, while using commercially available opto-electrical transducers.

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Figure 2.4 – a) Force sensing instrument, b) Flexure body, c) Cross-section showing the fiber locations A, B, C d) Fabry-Pérot cavity
Chapter 2. State of the art

Mecano-optical transducer: converting force to a displacement

Figure 2.4a shows a typical force sensing medical instrument with the coordinate system centered at the point of force application. Figure 2.4b shows a detailed section of the flexure body: the base, considered to be a rigid body, is connected to the handle. On the other side is the pointed rigid section with tip adapted to tissue palpation. The compliant mechanism forming the mecano-optical transducer links the tip to the base. It defines force measuring parameters such as force range and the number of DOFs. Different topologies of this compliant structure are proposed.

Opto-electrical transducer: converting displacement to an electrical signal

Figure 2.4d shows the Fabry-Pérot cavity whose length changes with flexure body deformation. When force is applied, the distance between the optical fiber and mirror changes. A distal light source generates an initial ray $R_0$ through an optical fiber of $125 \, \mu m$ diameter. A portion $R_0$ of this ray is reflected back from the end of the fiber, while the remaining portion $R_d$ is reflected by the mirror and returns into the fiber. The interference between $R_0$ and $R_d$ is detected by the interferometer. Displacements ranging from $-5 \, \mu m$ to $+5 \, \mu m$ from the nominal position can be measured with $5 \, nm$ resolution and $50 \, Hz$ acquisition rate. The force is then computed by Hooke's law using the measured displacement and the stiffness matrix of the flexure body.

2.4.2 Implementation of force sensors based on Fabry-Pérot interferometry

Load cell based on extrinsic Fabry-Pérot

The first implementation is shown in Figure 2.4 and consists of a flexure body assembled with an optical fiber and a mirror forming the Fabry-Pérot gap. This implementation is suited to a wide range of medical applications where the mechanical part is separate from optics. The following steps are performed to produce the tool

- Shaping a mechanical tip adapted to medical application (micro-forging, micro machining or EDM)
- Forming the flexure body (EDM)
- Machining grooves for optical fibers (EDM)
- Introduction, alignment and gluing of optical fibers
- Sealing of Fabry-Pérot gap (optional)
- Handle assembly


### Inline F-P cantilevers

Another implementation of a Fabry-Pérot force sensitive instrument consists of a flexure body manufactured directly out of optical fiber. The fiber is also used as the lightguide for measuring displacement and as the flexure deformed by the force applied on cantilever. The main advantage here is the simple and repeatable manufacturing process.

The diameter of the optical fiber is in the 50 to 250μm range, constraining the measurable force range and the tool size. However, only 1-DOF force sensing can be implemented.

Inline Fabry-Pérot cantilevers following this concept are manufactured using FIB or femtosecond laser technology in glass.

The manufacturing steps are:

- Adapting the tip and generating the Fabry-Pérot gap with a single fiber (FIB)
- Hand assembly, or embedding in the casing

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**Figure 2.5** – A scanning-electron microscopy image of the fiber-top cantilever (before the evaporation of the silver layer). Dimensions: length 112 μm, width 14 μm, thickness 3.7 μm. (Quoted from [17].)

**Figure 2.6** – Schematic view of the readout technique. The continuous arrow represents the input light. Dashed arrows represent the light reflected at the fiber-to-air, air-to-cantilever, and cantilever-to-metal interfaces. The shaded area represents the core of the fiber (diameter=9 μm (not to scale). (Quoted from [17].)
Chapter 2. State of the art

2.5 Implementation of force sensors based on Fiber Bragg Gratings

Fiber Bragg Grating (FBG) is another type of OFS used in force-sensitive medical instruments. A Bragg grating is a microstructure made by lateral exposition of a single-mode fiber core to a periodic pattern of light. This generates a periodic modulation of the core index of refraction that creates a resonant structure. As shown in Figure 2.7 broadband light transmitted to the optical fiber is partially transmitted through the grating, while the highest magnitude of reflected portion is around the Bragg wavelength. This wavelength is proportional to the core refraction index and the period of the grating. Changing the period results in different wavelengths read by the OSA. Therefore such structure can be used as a sensor sensitive to strain in axial direction, temperature, pressure, etc. This optical property is used in the instruments shown below [3] [18] [19] [20].

![Figure 2.7 – Schematic view of a Fiber Bragg Grating transducer](image)

2.5.1 Force sensing instrument with integrated Fiber Bragg Grating for retinal microsurgery

Research conducted at John Hopkins University, Baltimore, US led to the development of membrane picks with 3-DOF force sensing capability [3]. Their structure, as shown in Figure 2.8, uses FBG OFS to determine the force applied to the tool tip. To accomplish this task four optical fibers were introduced with FBG, as shown in Figure 2.8 b), c), d). The lack of explicit model to decompose 3-DOF force led to the use of four FBGs. Transverse forces $F_x, F_y$ were modeled using linear implicit model, which use as input, the values from three external FBGs. This allowed to measure forces ranging from 0 to 20mN with resolution of 0.083mN and a maximal error of 1.2mN, which corresponds to 6% of the measurement range. The axial force $F_z$ was modeled using a hybrid approach of linear and Bernstein polynomial models. The best obtained results for $F_z$ ranging from 0 to 20mN, the resolution is 0.41mN and the maximal error is 3.5mN, which corresponds to 17.5% of the measurement range. Identification of model parameters to achieve this performance was performed on a dataset of 138 872 samples.

The performance of this sensor is similar to the performance of the sensor developed in this thesis in terms of transverse force accuracy, but less accurate in terms of axial force accuracy. Moreover multi-part design implies assembly errors, which makes it impossible to use these sensors while relying on a single calibration matrix for multiple sensors manufactured according to the same design.
2.5. Implementation of force sensors based on Fiber Bragg Gratings

The monolithic design approach used in this thesis is therefore another key advantage leading to an explicit model for force decomposition. This allows modeling forces using single model identification based on a few calibration steps (typically 4 to 7 measurements). Moreover, a single calibration is sufficient for multiple instances of the same sensor design.

2.5.2 FBG-based force-sensing micro-forceps for retinal microsurgery

Some MIS operations, like Epiretinal Membrane Peeling favor actuated instruments over passive instruments like the previously presented membrane pick. The tools presented in [19],[21] answer this need, implementing FBG based force sensing on top of micro-forceps.

The first design shown in Figure 2.9 b) [21] consists of three FBGs located evenly on the outer tube of the instrument, while a fourth axial FBG is integrated in the middle tendon, used to actuate the jaws by a piezo-actuator. Results were not satisfactory, due to friction forces.
between the outer tube and the jaws, which corrupted the measurement not allowing for correct axial force $F_{ax}$ decoupling. The second design, shown in Figure 2.9 c) differs from the previous one: it introduces the axial FBG directly on one of the jaws, which remains straight during actuation, while the opposite closes the grip. With this second implementation authors report measurement of forces exerted on the tool tip ranging from 0 to 25mN, with 0.13mN RMS error for transverse forces and 1.99mN for axial force. Maximal errors are not quantitatively reported, but graphical information [21] indicates measurement samples with error greater than 5mN for axial force. An implicit linear model was used to determine transverse forces and an implicit Berenstein polynomial method was used for the axial force.

Figure 2.9 – FBG-Based transverse and axial force-sensing micro-forceps for retinal micro-surgery, B. Gonenc et al. [21].

### 2.5.3 Force-sensing microneedle for assisted retinal vein cannulation

Force sensing brings advantage also during Retinal Vein Cannulation (RVC) procedures. In [20] authors show how force measurement allows assisting surgical procedure such as vein cannulation. The force measurement provides a signal to stop the motion preventing over-puncture, thus allowing the needle tip to remain inside the vein and proceed to drug injection. In this design three FBGs are fixed to the outer tube, as shown in Figure 2.10 b). With measurement range from 0 to 15mN, authors report successful cannulation detection with a peak force of 8.8mN, while the tool was approaching with velocity of 0.3 mm/s and 9.3mN with 0.5 mm/s. The cannulation is done on a chorioallantoic membrane of chicken embryo, which is a valid in vivo model for human RVC. RMS error of this measurement was 0.18 mN for $F_x$ and 0.21mN for $F_y$, with majority of error values below 0.5 mN. Given a resolution of Bragg wavelength detection by interrogator of 1 pm, the achieved force measurement resolution was 0.22 mN. Force decomposition was based on an implicit linear model converting values from three FBGs to $F_x$ and $F_y$. 

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2.6. Rationale for selecting the Fabry-Pérot Interferometry Optical Fiber Sensor category

The analysis of the state of the art shows that use of optical fiber sensors (OFS) brings numerous advantages over conventional sensors [22]. These advantages are common to FPI and FBG:

- Immunity to magnetic interference allowing their use inside medical imaging devices or in connection with surgical robots and electrosurgical instruments (i.e. vessel sealer and divider, monopolar or bipolar cutting or cauterizing tool)

- High resolution to provide force measurement matching or even extending surgeon natural force sensing capabilities with small load cell structure deformation and low overall tool deflection

- High accuracy to provide the information required to perform a safe and controlled surgical gesture and trigger appropriate actions while improving usability, efficiency and repeatability (i.e. force reading accuracy below 0.01 N with sampling frequency greater than 100 Hz)

- Small size to respect constraints of MIS and extend possible application fields of force sensing medical instruments (instrument diameter below 3 mm or even below 1 mm for eye or brain surgery)

- Compatibility with sterilization procedures and surgery environments
• Low cost of goods allowing to manufacture disposable instruments

Selecting FPI instead of FBG allows to determine not only the forces acting at the tool tip, but also the induced displacement (elastic deformation of the flexure body), as explained in Chapters 3 and 5. This advantage is key to some surgical procedures such as the measurement of the stiffness of tissues (e.g. ossicular chain surgery). This feature is considered worth the extra assembly costs and additional sealing required by FPI OFS.
3 Thesis scope

The subject of this thesis is flexure-body mecano-optical transducers dedicated to medical force sensing instruments. The methods are based on the following assumptions:

• The measured force is applied at the specified point on the tool tip, which is relatively distant from the flexure to produce sufficient bending moment.

• Our proposed flexure is rigidly connected to the tool tip

• Displacement of the tool tip is measured using one, two or three Fabry-Pérot interferometers

• Fabry-Pérot displacement is always measured parallel to instrument’s principal axis $Z$. See Figure 3.1.

• The load cell diameter is in the millimeter or sub-millimeter range (in our catalogue it ranges from 0.5 mm to 4 mm).

• Loads applied to the sensor tip are below the plastic deformation limit of the load cell, as it is designed to work only in the elastic domain. More precisely we focus on materials having an elastic domain with major linear subdomain.

This chapter describes in detail our proposed transducers as well as the parameters of the Fabry-Pérot interferometers and explains how their output allows us to calculate sensor tip angular and linear displacement.

3.1 Design methodology

This chapter defines precisely the object of focus of this thesis. The surgical context defines a range of forces while the opto-electrical transducers define a range of allowable displacements,
Chapter 3. Thesis scope

so these imply a range of stiffnesses for the flexure bodies. The flexure body catalogue of Chapter 4 provides candidates that lie within the required stiffness range so the most appropriate is chosen for the required application.

3.1.1 Flexure body mechano-optical transducer

Force sensing is achieved via Hooke's Law, so that force is measured by the displacement it produces, where stiffness is already known. Displacement is measured using an optical transducer, the Fabry-Pérot interferometer.

Flexures have known limitations, e.g., non-linear deflection for large strokes. However, this is not an issue here, as the range of motion of our mechanisms is of order $\pm 5\mu$m and the angles are of order $\pm 0.25^\circ$. It follows that our flexures remain in the elastic domain of their material, guaranteeing repeatability and thus the precision of our instruments.

If not specified otherwise the material used to manufacture our flexures was Medical Grade 5 Titanium.

Flexure-body as mechano-optical transducer

Figure 3.5a shows a typical force sensing medical instrument with the coordinate system centered at the point of force application. Figure 3.5b shows a detailed section of the flexure body: the base, considered to be a rigid body, is connected to the handle. On the other side is the pointed rigid section with tip adapted to tissue palpation. The compliant mechanism forming the mecano-optical transducer links the tip to the base. It defines force measuring parameters such as force range and number of DOFs. Different topologies of this compliant structure are proposed in Chapter 4.

3.1.2 Fabry-Pérot based opto-electrical transducer

Optical fiber sensors (OFS) are divided in two categories: distributed OFS and point OFS. Fabry-Pérot interferometry uses point OFS. More precisely, we will use extrinsic Fabry-Pérot interferometry, as described in [22].

Integration of F-P interferometer into the force sensing tool

This section describes how our measurement is physically realized using the gap between optical fiber parts to provide the information to the analytical model of Chapter 5. Figure 3.1 shows the configuration for translational displacement sensing while Figure 3.2 provides the configuration for rotational displacement sensing.
3.1. Design methodology

The interferometer parameters used in our research were:

- Axial translation measurement lies between +10 µm to +20 µm
- The maximal allowable rotation angle between the two measuring surfaces is 0.5°
- The maximal allowable lateral displacements between the two fibers is 25 µm

The assumptions of the previous section make it easy to calculate translation and rotation, as shown in this section. Translation is computed as illustrated in Figure 3.1B, \( \Delta A_z \) is defined to be the difference between interferometer values at points \( A \) and \( A' \).

![Figure 3.1 – Translation measurement using F-P](image)

Rotation around X axis is computed as shown in Figure 3.2B, the angular displacement is

\[
\theta = \frac{\pi}{2} - \arcsin \left( \frac{\Delta A_z - A_z}{\sqrt{(\Delta A_z - A_z)^2 + A_y^2}} \right) - \arcsin \left( \frac{\sqrt{(\Delta A_z - A_z)^2 + A_y^2} - A_z}{\sqrt{(\Delta A_z - A_z)^2 + A_y^2}} \right)
\]

This equation will be simplified in Chapter 5 using first order approximations.
3.2 Instrument operation and measurement principle

3.2.1 Brief description of the instrument

Developed VivoForce instrument was already described in [23] and will be further highlighted in Chapter 9. We give a summary description here which serves as a generic example of the principles behind our tools.

The VivoForce instrument is inserted into the eye with the force sensing element inside the eye. Standard retinal surgery requires removal of the vitreous gel and replacing it with 20°C water. Since body temperature slowly heats this water, the instrument must operate at a range of 20°C to 34°C. The instrument peels away the epiretinal membrane with peak forces up to 15mN and the force sensor is calibrated to a resolution of 0.03 mN. Peeling is done with a 23 gauges\(^1\) hook. The instrument is operated by the surgeon’s hand and can be easily adapted to robotic surgery. Force sensing is indicated by increasing frequency sounds as force approaches the maximal limit, and a warning sound when above. Force is also recorded in real time and displayed on a screen, so an assistant can inform the surgeon of the measured force.

\(^1\)The gauge refers to the size of the instruments with higher numbers corresponding to smaller instruments (20-gauge = 0.9 mm diameter, 23-gauge = 0.6 mm diameter, 25-gauge = 0.5 mm diameter, 27-gauge = 0.4 mm diameter).
3.2 Instrument operation and measurement principle

3.2.2 Measurement principle (sensor concept)

The force sensing load cell of our instrument consists of a mecano-optical transducer realized by flexures by exploiting the nanometric precision of the Fabry-Pérot interferometer. The intellectual property of this solution belongs to Sensoptic SA [5] and was introduced by the authors in [24].

Figure 3.5 shows the VivoForce instrument and topology of a load cell. The mechanical part has 1-DOF kinematics, measuring the displacement along the $Z$ axis, so the component of force applied on the tip along $X$ axis can be deduced, see Figure 3.5a. We identified this direction as the most crucial for the surgical procedure.

Figure 3.5b shows a detailed section of the flexure body, its base is considered to be rigidly connected to the handle. On the other side is the pointed rigid tip, designed for membrane peeling according to surgeon specifications. The compliant mechanism forming the mecano-optical transducer links the tip to the base.

Since the compliant elements remain within their elastic domain while the force $F_x$ applied along $X$ axis is within the range of 0 mN to 50 mN, this force is proportional to the displacement $d$ measured on the axis of optical fiber, between base and rigid tip. By Hooke’s law $F_x = k(d - d_0)$, where $d_0$ is the distance when no force is applied and $k$ is a constant stiffness (a matrix for > 1-DOF).
Chapter 3. Thesis scope

Converting displacement into an electric signal is done using an opto-electrical transducer exploiting white light interferometry. Figure 3.4d shows the Fabry-Pérot cavity, whose length changes with flexure body deformation. When force is applied, the initial distance between the optical fiber and mirror changes. A distal light source generates an initial ray $R_{in}$ through an optical fiber of diameter 125 µm. A portion $R_0$ of this ray is reflected back from the end of the fiber, while the remaining portion $R_d$ is reflected by the mirror and returns into the fiber. The interference between $R_0$ and $R_d$ is detected by the interferometer. Displacements ranging from -5 µm to +5 µm from the rest position can be measured with a 5 nm resolution and with a 50 Hz acquisition rate. Based on the measured displacement using the stiffness matrix of the flexure body.

Figure 3.5 – a) VivoForce instrument, b) Flexure body, c) Cross-section d) Fabry-Pérot cavity
3.2. Instrument operation and measurement principle

Equation 3.1 is used to calculate the measured force in nominal load conditions (the force is applied at point \( P \) in the \( Y \) direction) in the initial model. Having only small rotations below 0.25\(^\circ\) we assume \( \sin \theta \approx \theta \).

\[
|F| = |F_y| = \Delta A_z K^{rot}_x, \tag{3.1}
\]

where \( \Delta A_z = A'_z - A_z \) and \( P_x, P_y, P_z, A_x, A_y, A_z \) are the coordinates of points \( P \) and \( A \) in

Figure 3.6 – 1-DOF rotational force sensor
Chapter 3. Thesis scope

Cartesian coordinates centered at \( O \), \( K_{\text{rot}}^{x} \) is a constant representing the flexure body’s rotational stiffness and the dimensions of a load cell. \( \Delta A_{Z} \) is the displacement measured by the interferometer with respect to its neutral position when no force is applied on the load cell.

Equation 3.1 is relevant for 1-DOF rotational Fabry-Pérot load cells, where points \( A \) and \( O \) are in the same \( X-Y \) plane, so \( A_{x} = O_{x} = 0 \). Generic solutions for the 1-DOF rotational Fabry-Pérot load cell, where this condition does not have to be met, are shown in Section 5.2.

3.3 Development cycle

Figure 3.7 illustrates the complete development cycle of the sensors considered in this thesis. The process begins with the declaration of needs by medical practitioners, e.g., range and direction of applied forces, tool environment, etc. This allows us to build up a catalogue from which an appropriate sensor can be chosen for a given application. This process is iterative, i.e., when needs are not fulfilled completely, then either some functionality demanded by the surgeon are removed, or else new structures are designed extending the catalogue.

Once the load topology is fixed, then the instrument is dimensioned using the analytical model of Chapter 5 and validated using FEA, checking that the flexures remain within their elastic limit considering a security factor. We also validate the geometric criteria, remaining within \( \pm 0.5^\circ \) angular deflection and \( \pm 5\mu m \) displacement.

When these criteria are fulfilled, we manufacture a prototype and perform full experimental tests to validate the analytical model and FEA calculations. Following this validation of the elastic and geometric criteria, then manufacturing can begin. Otherwise, we return to the dimensioning phase.

Once the product is manufactured, we provide end users with calibration procedures. Finally, clinical trials are performed.
3.4 Other tools based on the same principle

Chapter 4 will introduce other designs of tools based on the above operational principle. Compared to the VivoForce tool, we will present modeling method in Chapter 5 for 1-DOF tools for axial force measurement, as well as 2-DOF and 3-DOF tools capable of measuring axial as well as lateral forces.
This chapter covers the load cell topologies considered in this thesis. The design evolution was driven by the epiretinal membrane peeling application described in Chapter 9.

The first design $V_1$ was initially developed by Sensoptic SA. Further dimensioning using FEA did not succeed in achieving forces at the 15[mN] range with the required accuracy. For this reason, the modified design $V_2$ with symmetrical flexible blades was introduced. It has diameter 4 mm and its sensitivity was useful for peeling force characterization, but not for the

$V_{nx.d}$, or Variant $nx.d$ - Sensor model, where $n$ is a Variant number, common for load cells with the same diameter, considered for the same application. Letter ‘x’ stands for the same topology of the load cell, while digit ‘d’ is linked to the specific set of dimensions of such a load cell.
final instrument, where the transducer must be inserted into the eye through a sub-millimeter diameter canula.

Miniaturization continued with design $V_3$ which had a 2 mm diameter, as well as mechanical stopper, utilized as an advantageous safety structure. This design subsequently evolved into $V_4$ with 3-DOF sensing and diameter 0.5 mm, but was too challenging for manufacturing using EDM.

To overcome this drawback the $V_5$ design was introduced, with 3-DOF force sensing and diameter 0.9mm diameter, which was better adapted to EDM manufacturing and had no additional safety structure. Unfortunately, the price of manufacturing appeared to be too high for this to be marketable.

Finally, a 1-DOF force sensor variant with diameter 0.6 mm was designed and tested by the medical partner.

![Figure 4.2 – Classification of the developed load cell variants, shown with real relative sizes](image-url)
4.1 Variant 1 - \( \Phi 4\text{[mm]} \), 3DOF, PalpEar

Sensor Variant 1a.0 was developed by Sensoptic SA prior to our research. Original application of such sensor was the measurement of ossicular chain mobility\(^2\). Our investigation of the future sensor variants was started by modification of stiffness of this initial sensor Variant 1a.0, by modification of the flexure dimensions. This dimensions are shown in Figure 4.4. Values tested within the scope of this thesis are presented in table 4.1.

![Figure 4.3 – Load cell of Variants V1a and V1b with key dimensions for PRBM](image)
Figure 4.4 – V1a (PalpEar) flexure body, from right to left: complete tool, transducer with tool tip, transducers with its dimensions

<table>
<thead>
<tr>
<th>Variant</th>
<th>DOF(s)</th>
<th>Material</th>
<th>Dimensions</th>
<th>Work done</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>OD - outer diameter [mm]</td>
<td>ID - inner diameter [mm]</td>
</tr>
<tr>
<td>V1a.0</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.00</td>
</tr>
<tr>
<td>V1b</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.00</td>
</tr>
<tr>
<td>V1a.1</td>
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<td>Ti-6Al-4V</td>
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<td>0.75</td>
</tr>
<tr>
<td>V1c.2</td>
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<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.00</td>
</tr>
<tr>
<td>V1a.3</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.00</td>
</tr>
<tr>
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<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.25</td>
</tr>
<tr>
<td>V1a.5</td>
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<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.25</td>
</tr>
<tr>
<td>V1a.6</td>
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<td>4.00</td>
<td>1.25</td>
</tr>
<tr>
<td>V1a.7</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>4.00</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Ale above sensor Variants were tested in force range from 0N to 1N
1. multiple
2. multiple by 3 forces and few by test bench mapping
3. multiple instances and techniques

Table 4.1 – Considered dimensions and work done for Variant 1
Figure 4.5 – Views of Variants a) V1a and b) V1b
4.2 Variant 2 - $\varnothing 4\text{[mm]}$, 3DOF, modified PalpEar

Further modification of sensor Variant 1 led to the development of the sensor Variant 2, with longer, symmetrical flexures, which allowed characterization of forces exerted during ophthalmic microsurgery, such as epiretinal membrane peeling. The characteristic dimensions of sensor Variant 1 are presented in Figure 4.7. Tested configurations are presented in table 4.2.
4.2. Variant 2 - \( \varnothing 4 \text{[mm]}, 3 \text{DOF}, \text{modified PalpEar} \)

Figure 4.7 – Variant V2 modified PalpEar flexure body with dimensions indicated

Figure 4.8 – Variant V2 in FEA of modified PalpEar flexure body deformation under force applied in a) \( X \) direction, b) \( Y \) direction, c) \( Z \) direction
### Chapter 4. Catalogue

#### Table 4.2 – Considered dimensions and work done for Variant 2

<table>
<thead>
<tr>
<th>Variant</th>
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<th>Material</th>
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<th>Work done</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>3</td>
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<tr>
<td>V2.3</td>
<td>3</td>
<td></td>
<td>4.00 - 2.00 0.075 1.50 37.00 0.3</td>
<td>✓</td>
</tr>
</tbody>
</table>

Ale above sensor Variants were tested in force range from 0N to 0.5N

1. multiple prototypes manufactured
2. for multiple prototypes by 3 orthogonal forces and few by test bench mapping
3. multiple instances and techniques

Table 4.2 – Considered dimensions and work done for Variant 2

#### 4.3 Variant 3 - $\phi$2[mm], 3DOF, with safety structures

![Figure 4.9 – Views of sensor Variant V3a](image)

Figure 4.9 – Views of sensor Variant V3a
### 4.3. Variant 3 - Ø2 [mm], 3DOF, with safety structures

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
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<td>Ti-6Al-4V</td>
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<td>-</td>
<td>1.30</td>
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<td>0.12</td>
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<td>0.12</td>
<td>2.6 / 5.8</td>
<td>✓</td>
<td>Type C</td>
<td>✓</td>
</tr>
</tbody>
</table>

Above sensor Variants were tested in force range from 0N to 100mN

Table 4.3 – Considered dimensions and work done for Variant 3

![Figure 4.10 – Views of sensor Variant V3b](image-url)
4.4 Variant 4 - Ø0.5[mm], 3DOF, with safety structures

<table>
<thead>
<tr>
<th>Variant</th>
<th>DOF(s)</th>
<th>Material</th>
<th>Dimensions</th>
<th>Work done</th>
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<tr>
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<td>Ti-6Al-4V</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>V4b</td>
<td>3</td>
<td>Ti-6Al-4V</td>
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<td>-</td>
</tr>
<tr>
<td>V4c</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>0.50</td>
<td>-</td>
</tr>
</tbody>
</table>

Ale above sensor Variants were tested in force range from 0N to 100mN

Table 4.4 – Considered dimensions and work done for Variant 4

Figure 4.11 – Views of sensor Variant V4a
4.4. Variant 4 - \(\varnothing 0.5\text{[mm]}\), 3DOF, with safety structures

Figure 4.12 – Views of sensor Variant V4b

Figure 4.13 – Views of sensor Variant V4c
4.5 Variant 5 - Ø0.9[mm], 3DOF

Figure 4.14 – Load cell Variant V5a with flexure consisting of 4 triangular blades. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure

Figure 4.15 – Load cell Variant V5b with 2-section flexure: pivots for rotation in X-Z and Y-Z planes. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure
4.5. Variant 5 - \( \varnothing 0.9 \text{[mm]}, 3\text{DOF} \)

Figure 4.16 – Load cell Variant V5c with 3 identical flexure sections combining a pivot and 2 blades each. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure

Figure 4.17 – Load cell Variant V5d with 3 arc-shaped blades. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure
Figure 4.18 – Load cell Variant V5e with 3 arc-shaped blades and offset F-P gap locations. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure

Figure 4.19 – Load cell Variant V5f with 3 sine-shaped flexures. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure
Figure 4.20 – Load cell Variant V5g with 3 ‘c’-shaped flexures. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure

Figure 4.21 – Load cell Variant V5h with 3 ‘s’-shaped flexures. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure
Chapter 4. Catalogue

Figure 4.22 – Load cell Variant V5i with 2 double arc-blade flexure. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure

Figure 4.23 – Load cell Variant V5j with 2 double arc-blade flexure and 3 fiber located on front side of the hook. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure
4.5. Variant 5 - 0.9[mm], 3DOF

Figure 4.24 – Load cell Variant V5k with 2 double arc-blade flexure and 1 fiber located on back side of the hook. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure

Figure 4.25 – Load cell Variant V5l with single blade hollow flexure. 1st row: peeling hook - front, side, back; 2nd row: bottom view showing fiber grooves location; 3rd row: cross-section of flexure
## Table 4.5 – Considered dimensions and work done for Variant 5

<table>
<thead>
<tr>
<th>Variant</th>
<th>DOF(-s)</th>
<th>Material</th>
<th>OD - outer diameter [mm]</th>
<th>ID - inner diameter [mm]</th>
<th>z [mm]</th>
<th>flexure topology</th>
<th>Load cell</th>
<th>Work done</th>
<th>Manufacturing</th>
<th>Characterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>V5a</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>4 triangles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5b</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2.0</td>
<td>2 pivots</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5c</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>3 t-shape flexures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5d</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>3 triangles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5e</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>3 blades</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5f</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>3 snake-shape</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5g</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>3 c-shape</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5h</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>3 s-shape</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5i</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>2 levels of 2 blades each</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5j</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>2 levels of 2 blades each</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5k</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>0.45</td>
<td>2</td>
<td>2 levels of 2 blades each</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V5l</td>
<td>3</td>
<td></td>
<td>0.90</td>
<td>-</td>
<td>2</td>
<td>1 blade</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ale above sensor Variants were tested in force range from 0N to 100mN
1. test hooks and demonstrators only
2. by FEA
4.6 Variant 6 - Ø0.6\[mm\], 1DOF, PalpEye

Figure 4.26 – Views of sensor Variant V6a

Figure 4.27 – Views of sensor Variant V6b in scale with used optical fiber
### Table 4.6 – Considered dimensions and work done for Variant 6

<table>
<thead>
<tr>
<th>Variant</th>
<th>DOF(-s)</th>
<th>Material</th>
<th>Dimensions</th>
<th>Work done</th>
</tr>
</thead>
<tbody>
<tr>
<td>V6a.1</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>OD - outer diameter [mm]</td>
<td>FEA</td>
</tr>
<tr>
<td>V6a.2</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>ID - inner diameter [mm]</td>
<td>Manufacturing</td>
</tr>
<tr>
<td>V6a.3</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>Blade length - L [mm]</td>
<td>Characterization</td>
</tr>
<tr>
<td>V6b.1</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>Cut depth [mm]</td>
<td></td>
</tr>
<tr>
<td>V6b.2</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>EDM Slit width [mm]</td>
<td></td>
</tr>
<tr>
<td>V6b.3</td>
<td>3</td>
<td>Ti-6Al-4V</td>
<td>OP [mm]</td>
<td></td>
</tr>
</tbody>
</table>

All above sensor Variants were tested in force range from 0N to 100mN
5 Modeling

5.1 Fundamental conceptual contribution

The main goal of the modeling approach is to replace implicit models, as shown in Figure 5.1 with explicit models, as shown in Figure 5.2. Both figures show a generic 3-DOF, which can be adapted to 2-DOF and 1-DOF, as shown further.

The implicit modeling approach is the one presently used by industry as analyzed in [25]. This approach requires multiple iterations during sensor design stage, either via prototyping and stiffness matrix identification or via FEA. Based on a single design this implicit model does not allow deducing how changing the position of optical fibers influences sensor resolution. This is a strong limitation to the design and optimization of novel force sensors dedicated to specific application.

Figure 5.1 – Implicit model for measured force components decomposition based on read displacements
This thesis introduces explicit models composed of a geometric model and a stiffness model which are separate from one another. The use of explicit models in the design phase of the sensors leads to the following key advantages:

- Tool tip position identification using internal variables \((\Delta O_z, \theta, \phi)\) and known tip geometry,
- Adaptation of sensor resolution based on single experimental characterization, by changing matrix \(M\),
- Straight forward adaptation of force measurement range in X and Y, by changing OP distance.

The proposed conceptual approach is generally applicable to multi-degree-of-freedom force sensors using a parallel kinematic architecture. In the present thesis, the proposed approach is applied to the design of a 3-DOF force sensor based on a single monolithic 3-DOF compliant flexure structure. This approach significantly reduces the duration and cost of the prototyping phase of new sensors, making the design more efficient.

Figure 5.2 – Explicit model for measured force components decomposition based on read displacements, with separate geometric and stiffness calculation sections
5.2 Assumptions

This chapter uses analytical models of flexures (compliant mechanisms) as developed in [16] [15], in particular, regarding the virtual center of rotation of flexures.

Hooke’s law is assumed, so displacement is proportional to force where the constant ratio represents flexure stiffness.

Throughout the model we take into account only first order effects, since translations and rotations are small. Higher order effects are smaller than the approximations of Euler-Bernoulli theory.

For this reason, we systematically use the first order approximations \( \sin x = x, \cos x = 1 \) and \( x^2 = 0 \).

**Load cell force detection from unidirectional Fabry-Pérot displacement**

The small diameter cylindrical shape of our instruments along with the high bending radius of our optical fibers constrain us to have axial Fabry-Pérot displacement sensing (z-direction). It follows that force sensing is done by linear springs in the z-direction with rotational springs in the two orthogonal x and y directions. Table 5.1 shows the possible combinations of the linear spring and the rotational springs.

<table>
<thead>
<tr>
<th>1-DOF</th>
<th>lin(_z)</th>
<th>rot(_x)</th>
<th>rot(_y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-DOF</td>
<td>lin(_z) and rot(_x)</td>
<td>lin(_z) and rot(_y)</td>
<td>rot(_x) and rot(_y)</td>
</tr>
<tr>
<td>3-DOF</td>
<td>lin(_z) and rot(_x) and rot(_y)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1 – Possible load cell configurations

Figure 5.3 shows the conceptual configuration of a single linear spring, essentially equivalent, up to first order, to the 1-DOF rotational spring of Section 3.2.2. Figures 5.6-5.10 show conceptual configurations of linear and rotational springs, with flexure realizations given in Chapter 4.

Figures in this section present load cell configurations where only nominal load is applied. Nominal load takes into account only load forces applied at a specified point \( P \), in the sensing directions of a particular load cell.

5.2.1 1-DOF linear model in \( z \)

The model presented here is relevant for sensors having load cells that are compliant in the axial direction \( Z \), while other rotations and translations are considered infinitely stiff, as shown
in Figure 5.3. The governing stiffness equation is

$$|\mathbf{F}| = F_{z}^{appl} = \Delta A_z K_{z}^{lin}$$

(5.1)
5.2. Assumptions

5.2.2 1-DOF rotational model in $x$

This model is compliant in rotation about $X$, while other directions are considered to be infinitely stiff, as in Figure 5.4 and the generic Figure 5.5. This corresponds to sensor Variant V6.

Figure 5.4 – 1 DOF rotational force sensor (generic)

To find the governing stiffness equation, for rotation as in Figure 5.5, we introduce the rotation
matrix
\[
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\] (5.2)

applied to point \(R = (0, A_z)\)

Therefore
\[
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix} 0 \\ A_z \end{bmatrix} = \begin{bmatrix} -A_z \sin \theta \\ A_z \cos \theta \end{bmatrix} = R'
\] (5.3)

\[A_z(-\sin \theta, \cos \theta) + t(\cos \theta, \sin \theta) = (A_y, A'_z)\] (5.4)

in the \(z\) direction:
\[-A_z \sin \theta + t \cos \theta = A_y\] (5.5)

therefore
\[t = \frac{A_y + A_z \sin \theta}{\cos \theta}\] (5.6)

in the \(y\) direction
\[A'_z = A_z \cos \theta + t \sin \theta = A_z \cos \theta + \frac{A_y + A_z \sin^2 \theta}{\cos \theta}\] (5.7)

taking into account assumptions: \(\cos \theta = 1, \sin \theta = \theta, \theta^2 = 0, \sin^2 \theta = 0\). Note that the error due to these assumptions is easily estimated explicitly.

We get \(A'_z = A_z + A_y \theta\), in other words \(\Delta A_z = \theta A_y\)

Therefore: \(\theta = \frac{\Delta A_z}{A_y}\)

Hooke's law states that
\[|F| = F_{applied} = \theta K_{rot} = \frac{\Delta A_z}{A_y} K_{rot}\] (5.8)
Figure 5.5 – 1-DOF rotational force sensor (generic)
5.2.3 2-DOF: linear in $z$ and rotational in $x$

Using equations derived in Sections 5.2.1 and 5.2.2, we obtain

\[
\Delta A_z = \Delta O_z + \theta A_y, \quad \Delta B_z = \Delta O_z + \theta B_y
\]

This system of linear equations, following our assumptions is used to describe kinematic as in...
5.2. Assumptions

Figure 5.6 and is written in matrix form

\[
\begin{bmatrix}
\Delta A_z \\
\Delta B_z
\end{bmatrix}
= \begin{bmatrix} 1 & A_y \\ 1 & B_y \end{bmatrix}
\begin{bmatrix}
\Delta O_z \\
\theta
\end{bmatrix}
\]  \hspace{1cm} (5.9)

so that

\[
\begin{bmatrix}
\Delta O_z \\
\theta
\end{bmatrix}
= \begin{bmatrix} 1 & A_y \\ 1 & B_y \end{bmatrix}^{-1}
\begin{bmatrix}
\Delta A_z \\
\Delta B_z
\end{bmatrix}
= \frac{1}{B_y - A_y} \begin{bmatrix} B_y & -A_y \\ -1 & 1 \end{bmatrix}
\begin{bmatrix}
\Delta A_z \\
\Delta B_z
\end{bmatrix}
\]  \hspace{1cm} (5.10)

which results in

\[
\Delta O_z = \frac{B_y \Delta A_z - A_y \Delta B_z}{B_y - A_y}, \quad \theta = \frac{-\Delta A_z + \Delta B_z}{B_y - A_y}
\]

So according to Hooke’s law, we can determine the following components of force applied at point \( P \)

\[
F_{y \ k \ rot}^{appl.} = \theta K_{x \ rot} = \frac{-\Delta A_z + \Delta B_z}{B_y - A_y} K_{x \ rot}
\]  \hspace{1cm} (5.11)

and

\[
F_{z \ k \ rot}^{appl.} = \Delta O_z K_{x \ lin} = \frac{B_y \Delta A_z - A_y \Delta B_z}{B_y - A_y} K_{x \ lin}
\]  \hspace{1cm} (5.12)

Introducing the symbols:

\[
M = \begin{bmatrix} 1 & A_y \\ 1 & B_y \end{bmatrix}, \quad \delta_M = \det(M) \quad \text{and} \quad K_{lin,rot} = \begin{bmatrix} K_{lin}^{x \ rot} & 0 \\ 0 & K_x^{rot} \end{bmatrix}
\]

we can write above equations in matrix form, as

\[
\begin{bmatrix}
F_z \\
F_y
\end{bmatrix}
= K_{lin,rot} \begin{bmatrix}
\Delta O_z \\
\theta
\end{bmatrix}
= K_{lin,rot} M^{-1}
\begin{bmatrix}
\Delta A_z \\
\Delta B_z
\end{bmatrix}
\]  \hspace{1cm} (5.13)
These can be extended as:

\[
\begin{align*}
\mathbf{F}^{\text{appl}} &= (0, F_y^{\text{appl}}, F_z^{\text{appl}}) = (0, F_y^{\text{appl}}, 0) + (0, 0, F_z^{\text{appl}}) = \\
&= \left(0, \frac{-\Delta A_z + \Delta B_z}{B_y - A_y} K_{x}^{\text{rot}}, 0\right) + \left(0, \frac{B_y \Delta A_z - A_y \Delta B_z}{B_y - A_y} K_{z}^{\text{lin}}\right) = \\
&= \Delta A_z \left(0, \frac{-K_{x}^{\text{rot}}}{B_y - A_y}, \frac{B_y K_{z}^{\text{lin}}}{B_y - A_y}\right) + \Delta B_z \left(0, \frac{K_{x}^{\text{rot}}}{B_y - A_y}, -\frac{A_y K_{z}^{\text{lin}}}{B_y - A_y}\right)
\end{align*}
\]
5.2. Assumptions

Figure 5.7 – F-P 2-DOF geometry: rotation $\Delta \theta$ and translation $\Delta O_z$
Figure 5.8 – F-P 2-DOF geometry: rotation $\Delta \theta$ and translation $\Delta O_z$
5.2.4 2-DOF model with rotation in $x$ and rotation in $y$

Using the equations derived in Section 5.2.2 with the simplification of excluding rotation around the $z$ axis, one obtains

$$\Delta A_z = \theta A_y - \phi A_x, \quad \Delta B_z = \theta B_y - \phi B_x$$

This is written in matrix form as

$$\begin{bmatrix} \Delta A_z \\ \Delta B_z \end{bmatrix} = \begin{bmatrix} A_y & -A_x \\ B_y & -B_x \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix}$$

so that

$$\begin{bmatrix} \theta \\ \phi \end{bmatrix} = \begin{bmatrix} A_y & -A_x \\ B_y & -B_x \end{bmatrix}^{-1} \begin{bmatrix} \Delta A_z \\ \Delta B_z \end{bmatrix} = \frac{1}{A_y B_x - A_x B_y} \begin{bmatrix} -B_x & A_x \\ -B_y & A_y \end{bmatrix} \begin{bmatrix} \Delta A_z \\ \Delta B_z \end{bmatrix}$$

(5.16)

which results in

$$\theta = \frac{B_x \Delta A_z - A_x \Delta B_z}{A_y B_x - A_x B_y}, \quad \phi = \frac{B_y \Delta A_z - A_y \Delta B_z}{A_y B_x - A_x B_y}$$

So according to Hooke's law we can determine the following components of force applied at point $P$

$$F_{y_{appl}} = \theta K_{rot}^x = \frac{B_x \Delta A_z - A_x \Delta B_z}{A_y B_x - A_x B_y} K_{rot}^x$$

(5.17)

and

$$F_{x_{appl}} = \phi K_{rot}^y = \frac{B_y \Delta A_z - A_y \Delta B_z}{A_y B_x - A_x B_y} K_{rot}^y$$

(5.18)

Introducing the notation

$$M = \begin{bmatrix} A_y & -A_x \\ B_y & -B_x \end{bmatrix}$$

(5.19)
\[ \delta_M = \text{det}(M) \text{ and } K_{rot,rot} = [K_{rrot}^{x} K_{rrot}^{y}], \text{ we can write the above equations in matrix form} \]

\[
\begin{bmatrix}
F_y \\
F_x
\end{bmatrix} = K_{rot,rot}^{x} \begin{bmatrix}
\theta \\
\phi
\end{bmatrix} = K_{rot,rot}^{x} M^{-1} \begin{bmatrix}
\Delta A_z \\
\Delta B_z
\end{bmatrix}
\]

These can be written as

\[
F_{appl} = \begin{bmatrix}
F_x^{appl} \\
F_y^{appl}
\end{bmatrix} = \begin{bmatrix}
F_x^{appl} \\
F_x^{appl}
\end{bmatrix} + \begin{bmatrix}
0, 0 \\
0, 0
\end{bmatrix} = \\
\begin{bmatrix}
B_y \Delta A_z - A_y \Delta B_z \\
A_y B_x - A_x B_y
\end{bmatrix} K_{rrot}^{x} + \begin{bmatrix}
0, 0 \\
0, 0
\end{bmatrix} = \\
\Delta A_z \begin{bmatrix}
B_y K_{rrot}^{x} \\
A_y B_x - A_x B_y
\end{bmatrix} + \Delta B_z \begin{bmatrix}
-A_y K_{rrot}^{x} \\
A_y B_x - A_x B_y
\end{bmatrix}
\]

(5.21)
Figure 5.9 – F-P 2-DOF with rotation $\theta$ and rotation $\phi$
5.2.5 3-DOF model: linear in $z$, rotational in $x$ and in $y$

Using the equations derived in subsections 5.2.1 and 5.2.2 with the further simplification of excluding rotation around the $z$ axis, it can be assumed that:

$\Delta A_z = \theta A_y - \phi A_x + \Delta O_z$

$\Delta B_z = \theta B_y - \phi B_x + \Delta O_z$

$\Delta C_z = \theta C_y - \phi C_x + \Delta O_z$

This is written in matrix form as

$$
\begin{bmatrix}
\Delta A_z \\
\Delta B_z \\
\Delta C_z
\end{bmatrix}
= 
\begin{bmatrix}
A_y & -A_x & 1 \\
B_y & -B_x & 1 \\
C_y & -C_x & 1
\end{bmatrix}
\begin{bmatrix}
\theta \\
\phi \\
\Delta O_z
\end{bmatrix}
$$

so that

$$
\begin{bmatrix}
\theta \\
\phi \\
\Delta O_z
\end{bmatrix}
= 
\begin{bmatrix}
A_y & -A_x & 1 \\
B_y & -B_x & 1 \\
C_y & -C_x & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta A_z \\
\Delta B_z \\
\Delta C_z
\end{bmatrix}
$$

which results in

$$
\theta = \frac{A_x(\Delta C_z - \Delta B_z) - B_x\Delta C_z + C_x\Delta B_z + (B_x - C_x)\Delta A_z}{A_x(C_y - B_y) - B_xC_y + B_yC_x + A_y(B_x - C_x)}
$$

$$
\phi = \frac{A_y(\Delta C_z - \Delta B_z) - B_y\Delta C_z + C_y\Delta B_z + (B_y - C_y)\Delta A_z}{A_x(C_y - B_y) - B_xC_y + B_yC_x + A_y(B_x - C_x)}
$$
5.2. Assumptions

\[
\Delta O_z = \frac{A_x(C_y\Delta B_z - B_y\Delta C_z) + A_y(B_x\Delta C_z - C_x\Delta B_z) + (B_yC_x - B_xC_y)\Delta A_z}{A_x(C_y - B_y) - B_xC_y + B_yC_x + A_y(B_x - C_x)}
\]

So according to Hooke's law, we can determine the following components of force applied at point P

\[
F_{y}^{\text{appl}} = \theta K_x^{\text{rot}} = \frac{A_x(\Delta C_z - \Delta B_z) - B_x\Delta C_z + C_x\Delta B_z + (B_y - C_y)\Delta A_z}{A_x(C_y - B_y) - B_xC_y + B_yC_x + A_y(B_x - C_x)} K_x^{\text{rot}}
\]

\[
F_{x}^{\text{appl}} = \phi K_y^{\text{rot}} = \frac{A_y(\Delta C_z - \Delta B_z) - B_y\Delta C_z + C_y\Delta B_z + (B_x - C_x)\Delta A_z}{A_x(C_y - B_y) - B_xC_y + B_yC_x + A_y(B_x - C_x)} K_y^{\text{rot}}
\]

and

\[
F_{z}^{\text{appl}} = \Delta O_z K_z^{\text{lin}} = \frac{A_x(C_y\Delta B_z - B_y\Delta C_z) + A_y(B_x\Delta C_z - C_x\Delta B_z) + (B_yC_x - B_xC_y)\Delta A_z}{A_x(C_y - B_y) - B_xC_y + B_yC_x + A_y(B_x - C_x)} K_z^{\text{lin}}
\]

Introducing the notation

\[
M = \begin{bmatrix}
-A_x & A_y & 1 \\
-B_x & B_y & 1 \\
-C_x & C_y & 1 \\
\end{bmatrix}
\]

\[
\delta M = \det(M) \text{ and}
\]

\[
K_{\text{lin,rot,rot}} = \begin{bmatrix}
K_x^{\text{rot}} & 0 & 0 \\
0 & K_y^{\text{rot}} & 0 \\
0 & 0 & K_z^{\text{lin}} \\
\end{bmatrix}
\]

we can write above equations in matrix form

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
\end{bmatrix} = K_{\text{lin,rot,rot}} \begin{bmatrix}
\phi \\
\theta \\
\Delta O_z \\
\end{bmatrix} = K_{\text{lin,rot,rot}} M^{-1} \begin{bmatrix}
\Delta A_z \\
\Delta B_z \\
\Delta C_z \\
\end{bmatrix}
\]

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Chapter 5. Modeling

These can be written as

\[
\mathbf{F}^{app} = \left( F_x^{app}, F_y^{app}, F_z^{app} \right) = \left( F_x^{app}, 0, 0 \right) + \left( 0, F_y^{app}, 0 \right) + \left( 0, 0, F_z^{app} \right) = \\
\left( \phi K_y^{rot}, 0, 0 \right) + \left( 0, \theta K_x^{rot}, 0 \right) + \left( 0, 0, \Delta O_z K_z^{lin} \right) = \\
\left( \frac{A_x (\Delta C_z - \Delta B_z) - B_y \Delta C_z + C_y \Delta B_z + (B_y - C_y) \Delta A_z}{A_x (C_y - B_y) - B_x C_y + B_y C_x + A_y (B_x - C_x)} K_y^{rot}, 0, 0 \right) \\
+ \left( 0, \frac{A_x (\Delta C_z - \Delta B_z) - B_x \Delta C_z + C_x \Delta B_z + (B_x - C_x) \Delta A_z}{A_x (C_y - B_y) - B_x C_y + B_y C_x + A_y (B_x - C_x)} K_x^{rot}, 0 \right) \\
+ \left( 0, 0, \frac{\Delta A_z}{A_x (C_y - B_y) - B_x C_y + B_y C_x + A_y (B_x - C_x)} K_z^{lin} \right) \\
(5.30)
\]
5.2.6 3-DOF model: explicit units (International System of Units)

All calculations use the SI units. As an example, the calculation below shows explicitly the unit consistency of formula 5.29.

\[
[M] = \begin{bmatrix}
-A_x[m] & A_y[m] & 1[1] \\
-B_x[m] & B_y[m] & 1[1] \\
-C_x[m] & C_y[m] & 1[1]
\end{bmatrix}
\] (5.31)
Chapter 5. Modeling

implies:

\[
[M^{-1}] = \begin{bmatrix}
w_{11}[m^{-1}] & w_{12}[m^{-1}] & w_{13}[m^{-1}] \\
w_{21}[m^{-1}] & w_{22}[m^{-1}] & w_{23}[m^{-1}] \\
w_{31}[1] & w_{32}[1] & w_{33}[1]
\end{bmatrix}
\]  

(5.32)

Therefore

\[
K_{lin,rot,rot} M^{-1} \begin{bmatrix}
\Delta A \\
\Delta B \\
\Delta C
\end{bmatrix} = \begin{bmatrix}
\Delta A [m] \\
\Delta B [m] \\
\Delta C [m]
\end{bmatrix} = \begin{bmatrix}
K_{y}^{rot}[\text{rad}] & 0 & 0 \\
0 & K_{x}^{rot}[\text{rad}] & 0 \\
0 & 0 & K_{z}^{lin}[\text{rad}]
\end{bmatrix} \begin{bmatrix}
w_{11}[m^{-1}] & w_{12}[m^{-1}] & w_{13}[m^{-1}] \\
w_{21}[m^{-1}] & w_{22}[m^{-1}] & w_{23}[m^{-1}] \\
w_{31}[1] & w_{32}[1] & w_{33}[1]
\end{bmatrix} \begin{bmatrix}
\Delta A [m] \\
\Delta B [m] \\
\Delta C [m]
\end{bmatrix}
\]  

(5.33)

5.2.7 Correction for axial degrees of freedom (Z force component)

The model presented in Section 5.2.6 assumes an ideal design, where force components \(F_x\), \(F_y\) and \(F_z\) are decoupled and proportional respectively to internal variables \(\phi\), \(\theta\) and \(\Delta O_z\). In reality force coupling often has an important impact. This is visible for example in Figure 8.11 where the maximum of \(\Delta O_z\) is translated in X direction, showing coupling with \(F_x\) and thus angle \(\phi\). To compute correctly \(F_z\), as expected from Figure 8.27, a correction procedure is introduced below to provide the required decoupling.

We consider a case, where \(\Delta O_z\) is correlated to the component of \(F_{appl}\) lying on a line \(g\) in the \(Z\)-\(X\) plane, this line being offset from the \(Z\) axis by an angle \(\gamma\), as shown in Figure 5.11.
5.2. Assumptions

This assumption can be formulated as

$$\Delta O_z = \kappa \text{proj}_g(F_{\text{appl}}^{\text{z-meas}}) = \kappa \left( \text{proj}_g(F_{x_{\text{appl}}}^{\text{meas}}) + \text{proj}_g(F_{y_{\text{appl}}}^{\text{meas}}) + \text{proj}_g(F_{z_{\text{appl}}}^{\text{meas}}) \right),$$

where $\kappa$ is the stiffness along $g$. Since $g$ lies in the $Z$-$X$ plane, it is orthogonal to the $Y$ axis, therefore $g \perp F_{y_{\text{appl}}}^{\text{appl}}$ and $\text{proj}_g(F_{y_{\text{appl}}}^{\text{appl}}) = 0$. This simplifies the previous equation to:

$$\text{proj}_g(F_{\text{appl}}^{\text{z-meas}}) = \text{proj}_g(F_{x_{\text{appl}}}^{\text{meas}}) + \text{proj}_g(F_{z_{\text{appl}}}^{\text{meas}}).$$
From figure 5.11 we obtain

\[ \text{proj}_g(F_{\text{appl}}^z) = F_{\text{appl}}^z \cos \gamma - F_{\text{appl}}^x \sin \gamma, \]

which gives

\[ F_{\text{appl}}^z = \frac{\text{proj}_g(F_{\text{appl}}^z) + F_{\text{appl}}^x \sin \gamma}{\cos \gamma} = \frac{\Delta O_z + F_{\text{appl}}^x \sin \gamma}{\cos \gamma}. \]

This term can now be used to provide the correct stiffness matrix from equation 5.29 as follows:

\[
K_{\text{lin}, \text{rot}, \text{rot}}(\gamma) = \begin{bmatrix}
K_y^{\text{rot}} & 0 & 0 \\
0 & K_x^{\text{rot}} & 0 \\
K_y^{\text{rot} \sin \gamma} & 0 & K_x^{\text{rot} \cos \gamma}
\end{bmatrix} = \begin{bmatrix}
K_y^{\text{rot}} & 0 & 0 \\
0 & K_x^{\text{rot}} & 0 \\
K_y^{\text{rot} \tan \gamma} & 0 & K_x^{\text{rot} \cos \gamma}
\end{bmatrix} (5.34)
\]
Manufacturing

The manufacturing organization is illustrated in Figure 6.1. Manufacturing was a collaboration between my EPFL laboratory and outsourcing by the companies listed in Figure 6.1. The Figure applies to sensor Variants 5k and 6a, which were the most promising for Epiretinal Membrane Peeling application.

Manufacturing began after optimization of the designs. Priority was given to the modified 3-DOF PalpEar, Variant 2, in order to see the influence of down scaling of existing force sensors. The manufacturing techniques of EDM, micro milling, laser machining and 3-D printing of metals were used. Subsequently, several manufacturers were consulted for potential fabrication of the submillimetric probe, Variant 5k. For this purpose we prepared multiple CAD models and drawings combined with revisions in consultations with manufacturers. The manufacturing phase focused not only on the force-sensing element of the instrument, but provided a review of all components in terms of biocompatibility, reliability and ergonomics. Modified PalpEar prototypes were manufactured and tested for manufacturing quality.
6.1 Fabrication techniques

6.1.1 Electro-Discharge Machining (EDM)

Force sensing Variant 5k shown in Figure 6.2 required complex consecutive manufacturing operations, which would reduced the success rate of such fabrication process. The cost of this procedure was estimated at 2000CHF making it too expensive for future medical use. It was therefore not manufactured.

We were able to manufacture the 1-DOF instrument, Variant 6a, with diameter 0.6mm (23g). This tool features the most common retinal tool diameter and was identified as the one with most market potential, according to surgeons with whom we collaborate. The prototypes were manufactured with monolithic structures having grooves adapted for optical fiber placement, taking into account that the section closest to the Fabry-Pérot gap has a diameter 125µm, while a further section with cladding has diameter 250µm. This resulted in a fragile prototype made from two parts glued together but good enough to pass initial tests and crosscheck force sensing capabilities. This problem was later solved, resulting in a robust final product.

6.1.2 Rapid prototyping by 3D printing

Rapid prototyping was used extensively in our research, especially for upscaled demonstrators in order to explain our force measuring principles to industrial partners. We also used rapid prototyping to manufacture our dedicated eye simulator for surgeon training shown in Figure 71.
Chapter 6. Manufacturing

9.2.

Ophthalmic instruments can have various handle shapes, so a number of shapes were proposed. Initial handle shape designs are shown in Figure 6.4. They were quickly prototyped by 3D printing and tested by end users in clinical trials. With fast prototype production we could produce several new designs with ongoing consultation with surgeons, leading to an optimal shape.

Figure 6.4 – Handle designs

6.2 Manufactured instruments

Figures 6.6, 6.7, 6.8, 6.9, show the various fabricated instruments.
6.2. Manufactured instruments

Figure 6.6 – Manufactured instruments with sensors V1a, V1b, V2 and customized handle

Figure 6.7 – Tool tips for instruments with sensors V1a, V1b, V2
6.3 Quality control

The quality control was done using visual inspection of microscope data to validate manufacturing dimensions and achieved tolerances. It was performed in two stages. Firstly after manufacturing the monolithic titanium part comprising the flexure. Secondly to control the optical fiber placement. In addition to validation between CAD model and manufactured load cell dimensions, this quality control process gives also precise geometrical data, allowing to match force range and accuracy between FEA and real sensor. Qualitative visual control allows to exclude the presence of defects such as nicks in the machined edges, grooves resulting from an inappropriate electrodischarge process, glue overflows, etc.

The flexure dimension control was done using a Marcel-Aubert microscope, Figure 6.10. For each load cell design keeping the individual manufacturing tolerances of flexures was
6.3. Quality control

important, as they directly influence stiffness and therefore also measurement range and accuracy of the sensor.

Figure 6.10 – Variant 2 sensor flexure-body measured with a Marcel-Aubert microscope and with a CAD drawing overlay

Scanning electron microscope (SEM) imaging was performed to check for potential surface cracks due to thermal stress during manufacturing (Figure 6.11). None of these issues were reported during our quality control. This ensured the absence of influence of the manufacturing process on the stiffness of individual sensor prototypes of the same Variant.

Figure 6.11 – Variant 2 sensor flexure-body tested with a scanning electron microscope

The quality of Fabry-Pérot gap was validated using a Marcel-Aubert microscope, as in Figure
6.12 in terms of reflective surfaces alignment and distance $d_{Az}$ between them, when no load was applied on the tip.

Measured distance $d_{Az}$ to pass the quality control, had to fulfill the following condition, on the example of measurement line A: $A_{z \text{limit}^- + s_{am} * |\Delta A_{z \text{min}}| < d_{Az} < A_{z \text{limit}^+ - s_{am} * |\Delta A_{z \text{max}}|}$,

where: $d_{Az}[\mu m]$ distance in axial direction between two parts of the optical fiber, constituting the FP gap,

$\Delta A_{z \text{min}}[\mu m]$ - minimum value of displacement reading preview for load in nominal range of characterized load cell,

$\Delta A_{z \text{max}}[\mu m]$ - maximum value of displacement reading preview for load in nominal range of characterized load cell,

$A_{z \text{limit}^-}[\mu m]$ - minimum of sensor measurement range - in case of interferometers used within the thesis it was 10 $\mu$m,

$A_{z \text{limit}^+}[\mu m]$ - maximum of sensor measurement range - in case of interferometers used within the thesis it was 20 $\mu$m,

$s_{am}$ - security factor for displacement measurement range.

Where possible also relative off axis displacement between two optical fiber parts was evaluated as $d_{Ay}$. To pass the quality control ensuring necessary optical signal the following condition had to be fulfilled:

$d_{Ay} < 20[\mu m]$
6.3. Quality control

Figure 6.12 – Fabry-Pérot gap in manufactured instrument from first batch of Variant 6a

The controls performed with optical and SEM microscopes allowed to validate that the EDM manufactured load cells remain within the designed geometric tolerances surface quality (no cracks, flat surfaces) required to provide real measurement data fitting with the data obtained by FEM simulations.
A test bench was developed in order to characterize the manufactured sensors and future load cells, see Figure 7.1. This allows testing in different positions covering all orientations with respect to gravity. It also covers most of the sensor workspace using a reference force sensor. Depending on requirements, the test can be performed without load or reference mass or without any applied external forces.

The software controlling this structure was written using the LabView environment, which allowed direct integration with the software controlling the interferometer units, since it also used the LabView platform.

### 7.1 Measurement setup hardware

The initial configuration of our test bench was introduced in [26]. We extended this with three groups of significant elements of system hardware: motorized stages, reference sensors, calibration masses.
7.1.1 Motorized stages

To ensure precise and repeatable force application direction throughout the test, we chose Standa actuators, shown in Figure 7.2. These control position with 0.01° angular resolution for rotational stages and 58nm resolution for linear stage. Moreover rotary stages are hollow in the center, simplifying the mounting of our characterized sensors. Position control is done by Standa control units shown in Figure 7.3.
7.1. Measurement setup hardware

(a) 8MR190-2 - Rotary stage for angle $\beta$
(b) 8MR151 - Rotary stage for angle $\alpha$
(c) 8MT173-DCE2 - Linear stage for force application

Figure 7.2 – Actuators by Standa used in the experimental setup

(a) 8SMC5-USB-B8-1
(b) 8SMC5-USB-B9-2

Figure 7.3 – Control equipment by Standa used in the experimental setup: a) linear axis, b) rotational axes
Chapter 7. Experimental Setup and Procedures

7.1.2 Reference force sensors

The nominal force range proposed in our catalogue ranged from 50mN to 1N so required the use of two different sensors for characterization purposes. We choose a Kistler 9207 for the high force range, that is, for sensor Variant 1, and a Futek LPM200 for sensor Variants 2 and 6.

Kistler 9702

The uniaxial Kistler 9702 force sensor shown in Figure 7.5 measures forces ranging from -50N to 50N. Moreover, it has two additional calibrated measuring ranges: -5N to 5N and -0.5N to 0.5N and a sensitivity of 0.5mN.

Futek LPM200/FSH03395

The Futek LPM200 force sensor has a unidirectional force measurement range of -100mN to 100mN, which perfectly fits with our requirements for epiretinal membrane peeling applications. This allowed us to characterize sensors Variants 2 and 6.
7.1.3 Reference masses for direct gravity load

Calibration and characterization of force sensors requires comparable load conditions for various methods: analytical, FEM and experimental. To use the coordinate system and load conditions introduced by Sensoptic [16], we needed to introduce a system of calibration masses for loading medical probes on our test bench. As all calculations assume force application at the origin of the coordinate system, see Figure 8.15, the use of load masses was investigated. We therefore adjoined a load mass with known center of gravity to the flexure. After fixing it to the probe, an internal blade is pre-stressed in order to keep the mass in the same position relative to the probe. Moreover, the center of gravity (COG) of the mass is chosen to be the origin of the coordinate system of the probe, introduced in Chapter 5, as point P. This is consistent with the load conditions used in our analytical and FEA models. Note that this method does not require glue to attach mass to the probe, which would complicate load exchange and does not introduce any moving parts such as wires which would change the force application point during testing. Prototypes load masses were prepared for all our experimentally tested probes: Variant 1 (Figure. 7.6 top), Variant 2 (Figure. 7.6 middle) and catheter (Figure. 7.6 bottom).
Chapter 7. Experimental Setup and Procedures

Figure 7.6 – Reference masses manufactured by rapid prototyping with center of gravity marked for probes: a) Variant V1a.0, b) Variant V2.2, c) endosense catheter

7.2 Characterization procedures and software

To control the active stages (2 rotational, 1 linear) and acquire data from sensors (interferometers and reference force sensors), we developed software in the LabView environment. The front panel allowing us to choose one of the measurement procedures is shown in Figure 7.8. Details of this software were introduced in[27]. We were able to characterize our sensors using our reference masses as shown in Figure 7.7 a) or by applying forces measured by a reference sensor, as shown in Figure 7.7 b). Moreover, subsequent procedures combined with FEA results allowed us to test the maximal load for our load cells.
7.2. Characterization procedures and software

Figure 7.7 – Test bench measurement modes: a) direct, with rotational positioners 1 and 2, b) indirect, with rotational positioners 1, 2 and translation stage 3

Figure 7.8 – Front panel of our software for choosing the measurement mode and other parameters, e.g., max load or angular resolution
Chapter 7. Experimental Setup and Procedures

7.2.1 Force vs displacement characterization using gravity load

The first procedure was direct - using a reference mass attached to the tip of the sensor with two rotational stages to position the measured sensor with 0.01° angular resolution. The test is a static measurement, so axes 1 and 2, as shown in Figure 7.7 a), is set in measurement position and the system waits until the reading from interferometers to stabilize, then writes it into a file and continues with the automatic procedure by moving on to another position.

This test can be also performed without any reference masses in order to test the influence of sensor tip weight on the interferometer reading.

7.2.2 Force vs displacement characterization using a reference force sensor

The second measurement method was indirect, where non-measurable friction forces may arise between reference sensor and measured sensor. As with the first procedure, the tested structure is positioned by rotational axes 1 and 2, but load is applied along a third linear axis with a reference sensor mounted at the top, see Figure 7.7 b). Since the measured structures typically deform by less than 100µm, an additional flexure was introduced in order to make the system more compliant and apply more precise external force values.

Moreover, using this method, we were able to apply multiple reference loads. For example, in case of manufactured sensor Variant 6, we measured interferometer values without load, and with loads of 2.5mN, 5mN, 7.5mN and 10mN, as shown in Figure 7.9 b). To realize these measurements for each direction, the first interferometer reading was done without any applied load. A linear stage then translates a reference sensor until it indicates a force of 2.5mN, the sensor reading is stored for this value and a linear stage then continues until a reference force sensor indicates 5mN, etc. These measurements are shown in Figure 7.9 a). The top chart shows interferometer readings from sensor Variant 6, while the bottom chart shows force measured by the reference sensor decomposed in the coordinate system of sensor Variant 6.
7.2. Characterization procedures and software

7.2.3 Max Stress testing

The FEA data shown in Figure 7.10 allows us to identify directions having the highest stress of a given sensor Variant. In order to ensure that our sensor remains in the elastic domain and preserve Hooke's Law during measurement, we apply a nominal force multiplied by a sensor safety factor in these directions on our test bench. If the difference indicated by the interferometers before the load was applied and after it was removed remains within 0.2% of the total stroke during this procedure, we consider that the sensor remains in the elastic domain for this direction.

Figure 7.9 – Measurement procedure with multiple reference force values applied in each characterized direction
Figure 7.10 – Stress values for sensor Variant 6 analyzed by FEA with 10mN applied force in multiple directions.
Figure 7.10 – Stress values for sensor Variant 6 analyzed by FEA with 10mN applied force in multiple directions (cont.)
8 Finite Element Analysis and Experimental Results

In this chapter, we study sensor Variant 1 of Section 4.1 by comparing the FEA and experimental data with the analytical model of Section 5.1.5. Our analysis is based on five approaches. The first three are briefly mentioned in Section 8.2 and rely on the scheme of Figure 8.1. Our model takes interferometer displacements and directly computes force components, as opposed to Approaches 4 and 5 described in Sections 8.3.1 and 8.4, which first transform interferometer displacements into internal variables \((\phi, \theta, \Delta O_z)\) based on the sensor configuration matrix \(M\) and then into forces using the proposed stiffness matrix \(K\).

The initial versions of Approaches 4 and 5, schematically shown in Figure 8.2 were later corrected as shown in Figure 8.21, details of the proposed corrections are described in Section 8.5.

Approaches 4 and 5 combined with the corrections are the core of this section. Our work shows that the FEA and experimental data correspond to the theoretical model, up to two rotational shifts due to the placement of the sensor. We deal with this shift by proposing additional stiffness model correction which resolves the difference between theory and measurement, as shown in Sections 8.5.4 and 8.5.4. The relative error between the theoretical model and the FEA data was thus reduced from 50% to 4% and reduced to 7% in the experimental case. Moreover, this approach should prove being useful in handling asymmetric sensor topologies that could arise in future research – the design was chosen due to the asymmetric behavior of the \(\Delta O_z\) parameter, thereby making it more generic for future designs.

Another reason for this design is that we had previously manufactured a number of prototypes with this architecture making it possible to obtain experimental data for more than one unit.

Our analysis decomposes the force to be measured into three components, and uses interferometer data and Hooke’s Law to compute them. Moreover, our data allow us to retrieve more information, e.g., the maximum force that our structure can handle and the range of measurable force, as described in Section 3.2.

Remark. We use the term residual force to mean the measurement error of our device. We
have diverged from the usual sign convention by defining

\[
\text{residual force} = \text{applied force} - \text{measured force}
\]

Figure 8.1 – Block diagram showing how the analytical model was validated based on FEA and experimental measurements in Approaches 1, 2 and 3
Applied force (constant amplitude and variable direction) $F_{\text{appl.}}^{\text{applied}} (1N, \alpha, \beta)$

FEA or Measurement

Optical displacements $\Delta A_z, \Delta B_z, \Delta C_z$

Sensor configuration matrix $M$

Geometric model

Deformation of the flexure joints $\varphi, \theta, \Delta O_z$

Stiffness model

Measured force (three components) $F_{\text{meas.}}^x, F_{\text{meas.}}^y, F_{\text{meas.}}^z$

Residual error (three components) $F_{\text{res.}}^x, F_{\text{res.}}^y, F_{\text{res.}}^z$

- Stiffness matrix $K$
  - Offsets $A_z, B_z, C_z$

Analytical pseudo rigid body model

Figure 8.2 – Block diagram showing how the analytical model was validated based on FEA and experimental measurements of Approaches 4 and 5
Chapter 8. Finite Element Analysis and Experimental Results

8.1 Approaches

We list five different approaches to model the major differences in measured force and the identification of their parameters (calibration)

- **Approach 1.** Linear model, where 12 parameters \( k_{11}, \ldots, k_{33} \) and \( A_z, B_z, C_z \) were identified based on 4 sensor positions, without load, based on sensor tip weight only. The force is decomposed as:

\[
\begin{bmatrix}
F_x^{\text{measured}} \\
F_y^{\text{measured}} \\
F_z^{\text{measured}}
\end{bmatrix} =
\begin{bmatrix}
k_{11} & k_{12} & k_{13} \\
k_{21} & k_{22} & k_{23} \\
k_{31} & k_{32} & k_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta A_z - A_z \\
\Delta B_z - B_z \\
\Delta C_z - C_z
\end{bmatrix}
\]

(8.1)

- **Approach 2.** Linear model, using the same equation as in Approach 1, where the parameters were identified by best fit in terms of root mean square, using the complete data set of 1387 sensor positions measured by the experimental setup.

- **Approach 3.** Second order model, where 21 parameters were identified using the same experimental dataset. This decomposes force as follows

\[
\begin{bmatrix}
F_x^{\text{measured}} \\
F_y^{\text{measured}} \\
F_z^{\text{measured}}
\end{bmatrix} =
\begin{bmatrix}
k_{11} & k_{12} & k_{13} \\
k_{21} & k_{22} & k_{23} \\
k_{31} & k_{32} & k_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta A_z - A_z \\
\Delta B_z - B_z \\
\Delta C_z - C_z
\end{bmatrix} +
\begin{bmatrix}
l_{11} & l_{12} & l_{13} \\
l_{21} & l_{22} & l_{23} \\
l_{31} & l_{32} & l_{33}
\end{bmatrix}
\begin{bmatrix}
(\Delta A_z - A_z)^2 \\
(\Delta B_z - B_z)^2 \\
(\Delta C_z - C_z)^2
\end{bmatrix}
\]

(8.2)

- **Approach 4.** The proposed linear analytical pseudo rigid body model is applied to the FEA dataset, where parameters are identified using 7 sensor positions. The force decomposes as follows

\[
\begin{bmatrix}
F_x^{\text{measured}} \\
F_y^{\text{measured}} \\
F_z^{\text{measured}}
\end{bmatrix} =
\begin{bmatrix}
N_{\phi_{\text{range}}} & 0 & 0 \\
0 & -2N_{\theta_{\text{range}}} & 0 \\
N_{\phi_{\text{range}}} \tan \gamma & 0 & N_{\theta_{\text{range}}} \cos \gamma
\end{bmatrix}
\begin{bmatrix}
-A_x & A_y & 1 \\
-B_x & B_y & 1 \\
-C_x & C_y & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta A_z - A_z \\
\Delta B_z - B_z \\
\Delta C_z - C_z
\end{bmatrix}
\]

(8.3)

- **Approach 5.** The linear analytical pseudo rigid body model is applied to the experimental data set using the equation of Approach 4, where parameters are identified by a limited number of applied force directions. The minimum number is 7 but can increase depending on the applied corrections, see Section 8.3.
8.2 Approaches 1 to 3: Root mean square identification of sensor parameters

Approach 2 showed a degree of magnitude improved calibration of Variant 1 as compared to Approach 1. Details of these first three approaches are given in [25]. Summary of these results is shown in Figure 8.3.

The results shown here indicate a minimal achieved relative error of 3.21% for the linear model of Approach 2, where 12 parameters $k_{11}, ..., k_{33}$ and $offs_A, offs_B, offs_C$ where fit using all 1387 points of the acquired dataset to define the measured force relative error in % shown in Figure 8.3. This result is to be compared with the final error values of Approach 5 in Section 8.7. It is important to mention that our final error, of similar magnitude, was found in Approach 5 using stiffness matrices generated with just 7 points of the data set, so much easier to generate.

**Approach 1:**
Original linear calibration, 4 points fit

**Approach 2:**
Optimized linear, 1387 points fit

**Approach 3**
Optimized 2nd order, 1387 points fit

| Approach | Description | Points Fit | Error (%)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Original linear calibration</td>
<td>4</td>
<td>max=12.75% rms=4.51%</td>
</tr>
<tr>
<td>2</td>
<td>Optimized linear</td>
<td>1387</td>
<td>max=3.21% rms=1.14%</td>
</tr>
<tr>
<td>3</td>
<td>Optimized 2nd order</td>
<td>1387</td>
<td>max=2.49% rms=0.69%</td>
</tr>
</tbody>
</table>

Figure 8.3 – Results showing possible calibration improvement without using the pseudo rigid body model, and using all dataset points to generate best linear fit of model shown in the equation below

The error values shown in Figure 8.3 were calculated using

$$\text{error} = \frac{||F_{measured}|| - ||F_{applied}||}{||F_{applied}||} * 100$$

8.3 Approach 4: Finite Element Analysis of Variant 1 for a 1 Newton load

The analysis of sensor Variant 1 shown in this chapter was done using Comsol.
Chapter 8. Finite Element Analysis and Experimental Results

As shown in Figure 8.4, the meshing was required to be very fine in order for the FEA data to correspond to the experimental results. We used a multi-layer mesh on the compliant elements and a coarse mesh on the rigid elements.

Figure 8.4 – Example of load cell flexure body mesh, showing manually adapted layer structure on most compliant elements

8.3.1 Measurement range and raw data from FEA

The FEA results took approximately 24 hours on a dedicated PC for a single sensor configuration for the 1387 measurement points illustrated in Figure 8.6, using Comsol software.

Figure 8.5 shows the direction of force as applied on the tool tip, of magnitude 1N in the directions given by $\alpha$ and $\beta$. 96
8.3. Approach 4: Finite Element Analysis of Variant 1 for a 1 Newton load

Figure 8.5 – \( \alpha \) and \( \beta \) angles defining direction of applied force \( F_{appl} \)

Figure 8.6 – FEA simulation points: the unitary (1N) force was applied in 1387 directions onto the tool tip for the FEA validation. This polar graph shows the spherical coordinates \( \alpha, \beta \), of each point (crosses). Raster: \( \alpha \) ranges from 0° to 360° by 5° steps; \( \beta \) ranges from 0° to 90° by 5° steps.
Figure 8.7 – Components of applied force for \( \| F_{appl} \| = \sqrt{(F_{appl}^x)^2 + (F_{appl}^y)^2 + (F_{appl}^z)^2} = 1 \text{N} \)
8.3. Approach 4: Finite Element Analysis of Variant 1 for a 1 Newton load

Figure 8.8 – Example of FEA of displacements corresponding to force components of 0.1N magnitude applied in a) X direction, b) Y direction, c) Z direction. The 0.1N magnitude is shown because this was the only rendered force value.
Figure 8.9 – Optical displacements seen by interferometers A, B, C, respectively
8.3.2 Geometric model (part of PRBM) applied to FEA data

This subsection presents results of comparing the FEA data with the analytical model of Chapter 5. The FEA returns displacements for given applied forces, and we then use these displacements as input for the analytical model and compare the computed forces with the original input forces of the FEA computation.

Figure 8.10 – Deformation of flexures ($\phi, \theta, \Delta O_z$) based on the optical displacement calculated by FEA and the pseudo-rigid body model
8.3.3 Identification of the stiffness matrix $K$

Hooke’s Law gives the correspondence between force and displacement, with the linear relation given by the stiffness matrix $K$. This matrix was computed analytically by the theoretical model of Section 5.1.5. The matrix given there was diagonal, therefore, every force component was directly proportional to a displacement derived from the geometrical model.

In order to compute these diagonal elements experimentally, we applied the maximum forces of $-1N$ and $+1N$ and measured the corresponding $\phi$, $\theta$ angles, and applied $0N$ and $1N$ force and compared them to the extremal measured $\Delta O_z$ values.

From the analysis shown in figures 8.10 and 8.11, we can deduce the following extreme values

$\phi_{\text{min}} = -0.0985^\circ$, $\phi_{\text{max}} = 0.0985^\circ$,

$\theta_{\text{min}} = -0.1870^\circ$, $\theta_{\text{max}} = 0.1870^\circ$,

$\Delta O_{z\text{min}} = -1051.5\text{nm}$, $\Delta O_{z\text{max}} = 441.3905\text{nm}$

which gives

$\phi_{\text{range}} = \phi_{\text{max}} - \phi_{\text{min}} = 0.1969^\circ$, $0.1969^\circ \frac{\pi}{180^\circ} = 0.00344 \text{ rad}$

$\theta_{\text{range}} = \theta_{\text{max}} - \theta_{\text{min}} = 0.3740^\circ$, $0.3740^\circ \frac{\pi}{180^\circ} = 0.00653 \text{ rad}$

$\Delta O_{z\text{range}} = \Delta O_{z\text{max}} - \Delta O_{z\text{min}} = 1492.9\text{nm}$
8.3. Approach 4: Finite Element Analysis of Variant 1 for a 1 Newton load

Therefore, up to first order, we get the following stiffness matrix:

\[
\begin{bmatrix}
K_{Y}^{\text{rot}} & 0 & 0 \\
0 & K_{x}^{\text{rot}} & 0 \\
0 & 0 & K_{\text{lin}}^{z}
\end{bmatrix}
= \begin{bmatrix}
\frac{2N}{\phi_{\text{range}}} & 0 & 0 \\
0 & \frac{-2N}{\theta_{\text{range}}} & 0 \\
0 & 0 & \frac{1N}{\Delta O_{z}}
\end{bmatrix}
\]

(8.4)

This results in the following equation developed for a generic pseudo-rigid-body model

\[
F_{\text{appl}} = F_{\text{meas}} + F_{\text{residual}} = K_{\text{lin,rot,rot}}^{\text{FEA}} \begin{bmatrix}
\phi \\
\theta \\
\Delta O_{z}
\end{bmatrix} + F_{\text{residual}}
\]

(8.5)

Figures 8.12, 8.13, 8.14 illustrate the analytically derived force then the relative error as compared to the FEA model. Figure 8.12 shows that the relative error is of order 1% in \(X\) and similarly, Figure 8.13 shows that the relative error is of order 1% in \(Y\). However, Figure 8.14 shows a relative error of 50%.

This means that the model appears to hold in \(X\) and \(Y\), but further analysis is required to handle the \(Z\) component. This will be done below.
Figure 8.12 – Estimated force ($X$ component) and residual error (FEA): the upper graph presents the force estimated on the deformation of the flexures ($\phi, \theta, \Delta O_z$) based on the optical displacement calculated by FEA and the pseudo rigid body model, and on the stiffness matrix. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all simulated directions) and the estimated force.
8.3. Approach 4: Finite Element Analysis of Variant 1 for a 1 Newton load

Figure 8.13 – Estimated force (Y component) and residual error (FEA): the upper graph presents the force estimated on the deformation of the flexures ($\phi, \theta, \Delta O_z$) based on the optical displacement calculated by FEA and the pseudo rigid body model, and on the stiffness matrix. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all simulated directions) and the estimated force.
Figure 8.14 – Estimated force (Z component) and residual error (FEA): the upper graph presents the force estimated on the deformation of the flexures ($\phi, \theta, \Delta O_2$) based on the optical displacement calculated by FEA and the pseudo rigid body model, and on the stiffness matrix. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all simulated directions) and the estimated force.
8.4 Approach 5: Experimental measurements on Variant 1 with a 1N weight

In this section, we repeat the procedure of the previous section but where the applied force was generated by a 100 gram hanging mass on the tool tip generating a 1N force due to gravity. The major difference with the previous section is that the force now acts in the opposite $Z$ direction.

8.4.1 Measurement range and data

The acquisition of the measurement data set of 1387 applied force directions shown in Figure 8.16 took about 55 minutes. This relatively short period of time was made possible thanks to our automatized calibration setup and proposed calibration masses. Although it was much faster than the FEA analysis which took on the order of 24 hours, we note that it was performed on the manufactured sensor Variant 1. The advantage of the FEA analysis presented in the previous section is not having used any manufacturing time.

Figure 8.15 shows the direction of force as applied on the tool tip, of magnitude 1N in the directions given by $\alpha$ and $\beta$.

![Diagram showing $\alpha$ and $\beta$ angles](image)

Figure 8.15 – $\alpha$ and $\beta$ angles
Figure 8.16 – Measurement points: the unitary (1N) force was applied in 1387 directions onto the tool tip for the FEA validation. This polar graph shows the spherical coordinates $\alpha$, $\beta$, of each point (crosses). Raster: $\alpha$ ranges from $-180^\circ$ to $180^\circ$ by $5^\circ$ steps; $\beta$ ranges from $0^\circ$ to $-90^\circ$ by $5^\circ$ steps. Orange arrows represent the order of sample collecting.
8.4. Approach 5: Experimental measurements on Variant 1 with a 1N weight

Figure 8.17 – Components of applied force $\|F_{\text{appl}}\| = \sqrt{F_{x}^{\text{appl}})^2 + (F_{y}^{\text{appl}})^2 + (F_{z}^{\text{appl}})^2} = 1$N
Figure 8.18 – Optical displacements measured by interferometers A, B, C, respectively
8.4. Approach 5: Experimental measurements on Variant 1 with a 1N weight

8.4.2 Pseudo rigid body model

In this section, we take the interferometer displacement data and using the model of Section 5.1.5, we calculate the $\phi$, $\theta$, $\Delta O_z$ displacements of the sensor tip. These calculations match the results obtained by FEA.

Figure 8.19 – Deformation of the flexures ($\phi, \theta$) based on the optical displacements measured by the interferometers and the pseudo rigid body model
Chapter 8. Finite Element Analysis and Experimental Results

8.4.3 Identification of the stiffness matrix $K$

Identification of the stiffness matrix $K_{\text{linear, rotational, rotational}}^{\text{experimental}}$ was included in the Appendix A in section A.1. Graphs of resulting forces were also included there, as this is only an intermediary step, which required corrections, that are included further in this chapter.

Result of identification:

$$K_{\text{linear, rotational, rotational}}^{\text{experimental}} = \begin{bmatrix} 514 \text{ N rad} & 0 & 0 \\ 0 & -291.3 \text{ N rad} & 0 \\ 0 & 0 & 500 \times 10^3 \text{ N m} \end{bmatrix}$$  \hspace{1cm} (8.6)

8.5 Stiffness matrix correction

The previous Sections 8.3.1 and 8.4 demonstrated a good match between the analytical model and the FEA and experimental data, except for a significant difference in $\Delta O_z$ and a rotational offset in the experimental data. Below in this section corrections for this behavior is introduced for Approaches 4 and 5, as modification of initial diagonal matrices. These modifications are shown schematically in Figure 8.21, which is the difference if compared to Figure 8.2.
8.5. Stiffness matrix correction

**Applied force**
*constant amplitude and variable direction*

\[ F_{\text{appl.}} (1N, \alpha, \beta) \]

FEA or Measurement

**Optical displacements**
\[ \Delta A_z, \Delta B_z, \Delta C_z \]

Sensor configuration matrix \( M \)

**Geometric model**

Deformation of the flexure joints
\[ \varphi, \theta, \Delta O_z \]

Stiffness matrix \( K \)

- Offsets \( A_z, B_z, C_z \)

Stiffness model

**Measured force**
*three components*
\[ F_{x, \text{meas}}, F_{y, \text{meas}}, F_{z, \text{meas}} \]

Stiffness matrix corrections for \( \gamma \)

Stiffness matrix correction for \( \alpha_{\text{offs}} \)
method1 / method2

**Residual error**
*three components*
\[ F_{x, \text{res}}, F_{y, \text{res}}, F_{z, \text{res}} \]

Analytical pseudo rigid body model

Figure 8.21 – Block diagram showing how the analytical model was validated based on FEA and experimental measurements in Approaches 4 and 5 with proposed corrections
8.5.1 Z rotation offset identification

Offset angle was identified as:
\[ \alpha_{offset} = -7.725^\circ, \]
which corresponds to -0.13483 rad.

Procedure to obtain this value is described in details in Appendix A, subsection A.2.

8.5.2 Solution for rotational degrees of freedom - method 1: based on residual forces

Observation of \( F_{\text{residual}1} \) shows a clear correlation between angle \( \phi \) and \( F_{\text{residual}1}^x \), as well as between angle \( \theta \) and \( F_{\text{residual}1}^y \). This allows to further reduce sensor error by extending the stiffness matrix by the following coefficients:

\[
K_{\text{experimental}}^2 = \begin{bmatrix}
0 & -F_{\text{residual}1}^y \cdot \frac{\phi}{\phi_{\text{range}}} & 0 \\
-F_{\text{residual}1}^x \cdot \frac{\theta}{\theta_{\text{range}}} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
0 & -0.3018N & 0 \\
0 & -0.2873N & 0 \\
0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
0 & -44N \text{ rad} & 0 \\
0 & -73.8N \text{ rad} & 0 \\
0 & 0 & 0
\end{bmatrix} (8.7)
\]

\[
F_{\text{appl.}} = F_{\text{meas.}} + F_{\text{residual}2} = \left( K_{\text{experimental}}^{\text{lin,rot,rot}} + K_{\text{experimental}}^2 \right) \begin{bmatrix}
\phi \\
\theta \\
\Delta O_z
\end{bmatrix} + F_{\text{residual}2} (8.8)
\]

Therefore, in conclusion, we can apply the stiffness matrix

\[
K_{\text{experimental}} = K_{\text{experimental}}^{\text{lin,rot,rot}} + K_{\text{experimental}}^2
\]
for sensor Variant 1, where

\[
K_{\text{exp,res}} = K_{1}^{\text{experimental}}_{\text{lin,rot,rot}} + K_{2}^{\text{experimental}} = \begin{bmatrix}
2N_{\phi} & 0 & 0 \\
0 & -2N_{\theta} & 0 \\
0 & 0 & K_{l}^{\text{lin}}
\end{bmatrix} + \begin{bmatrix}
0 & -\frac{F_{\text{res,1}}}{\phi_{\text{range}}} & 0 \\
\frac{F_{\text{res,1}}}{\theta_{\text{range}}} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
\frac{2N}{\phi_{\text{range}}} & -0.3018N/\text{rad} & \frac{2N}{\theta_{\text{range}}} \\
0.2873N/\phi_{\text{range}} & 0 & -2N/\theta_{\text{range}} \\
0 & 0 & \frac{1N}{\Delta \theta_{\text{range}}}
\end{bmatrix}
\begin{bmatrix}
514 N/\text{rad} & -44 N/\text{rad} & 0 \\
-73.8 N/\text{rad} & -291.3 N/\text{rad} & 0 \\
0 & 0 & 500 \times 10^{3} N/m
\end{bmatrix}
\] (8.9)
Figure 8.22 – Measured force (X component) and residual error: the upper graph presents the force estimated on the deformation of the flexures $\phi$ based on measured optical displacement and the pseudo rigid body model, taking into account correction for $\alpha_{offset}$ of the stiffness matrix by method 1. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
8.5. Stiffness matrix correction

Figure 8.23 – Measured force (Y component) and residual error: the upper graph presents the force estimated on the deformation of the flexures $\theta$ based on measured optical displacement and the pseudo rigid body model, taking into account correction for $\alpha_{off set}$ of the stiffness matrix by method 1. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
8.5.3 Solution for rotational degrees of freedom - method 2: based on rotation of the stiffness matrix $K$

An alternative method to compensate for the $\alpha_{\text{offset}}$ is to apply a rotation matrix to the stiffness matrix $K_{\text{lin,rot,rot}}$.

This matrix is defined by:

$$\text{Rot}_z(\alpha_{\text{corr}}) = \begin{bmatrix} \cos(\alpha_{\text{corr}}) & -\sin(\alpha_{\text{corr}}) & 0 \\ \sin(\alpha_{\text{corr}}) & \cos(\alpha_{\text{corr}}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8.10)$$

therefore

$$K_{\text{exp,Rot}} = \text{Rz}(\alpha_{\text{corr}}) K_{\text{lin,rot,rot}} = \begin{bmatrix} \frac{509.3}{\text{N rad}} & \frac{-39.2}{\text{N rad}} & 0 \\ \frac{-69.1}{\text{N rad}} & \frac{-288.6}{\text{N rad}} & 0 \\ 0 & 0 & 500 \times 10^3 \text{N m} \end{bmatrix} \quad (8.11)$$

This allows us to calculate the measured and residual force components as follows:

$$F_{\text{appl}} = F_{\text{meas,Rot}} + F_{\text{res,R}} = K_{\text{exp,Rot}} K_{\text{exp,Rot}} \begin{bmatrix} \phi \\ \theta \\ \Delta O_z \end{bmatrix} + F_{\text{res,R}} \quad (8.12)$$

The resulting components of these forces along the $X$ and $Y$ directions are shown in the figures below:
Figure 8.24 – Measured force (X component) and residual error: the upper graph presents the force estimated on the deformation of the flexures φ based on measured optical displacement and the pseudo rigid body model, taking into account correction for \( \alpha_{off} \) of the stiffness matrix by method 2. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
Figure 8.25 – Measured force (Y component) and residual error: the upper graph presents the force estimated on the deformation of the flexures $\theta$ based on measured optical displacement and the pseudo rigid body model, taking into account correction for $\alpha_{offset}$ of the stiffness matrix by method 1. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
Note that following this correction to the stiffness matrix, the $X$ and $Y$ relative error has decreased from 15%, as computed in Section 8.4.3, to 4.3%.

8.5.4 Correction for axial degrees of freedom ($Z$ force component) for FEA and experimental measurements

As noted in Section 8.3.3 the $\Delta O_z$ values deduced by the pseudo rigid body model as applied to the FEA and experimental data do not directly follow Hooke's Law in the $Z$ direction.

However, we note that $\Delta O_z$ seems to be correlated to the component of $F^{appl}$ lying on a line $g$ in the $Z$-$X$ plane, this line being offset from the $Z$ axis by an angle $\gamma$, as shown in Figure 5.11. This coupling was further resolved according to procedure described in Section 5.2.7

The magnitude of $\gamma$ angle was computed to be $31.8^\circ$ (0.5550 rad) using FEA and $19.74^\circ$ (0.34453 rad) using the experimental data.

Correction for FEA

The corrected stiffness matrix from the above computation gives the updated figure with $Z$ maximal error now reduced to $+4\%$ and $-3\%$ as opposed to the previous $50\%$ value.
Figure 8.26 – Estimated force (Z component) and residual error (FEA): the upper graph presents the force estimated on the deformation of the flexures (\(\phi, \Delta O_Z\)) based on measured optical displacement and the pseudo rigid body model, and on the stiffness matrix corrected for asymmetrical \(\Delta O_z\) behavior by angle \(\gamma\). The lower graph shows the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
Correction for experimental measurements

This section provides the similar computation for experimental measurements, with errors of +6.5% and −7.6%.

Figure 8.27 – Measured force (Z component) and residual error: the upper graph presents the force estimated on the deformation of the flexures (φ, ΔOZ) based on measured optical displacement and the pseudo rigid body model, and on the stiffness matrix corrected for asymmetrical ΔOZ behavior by angle γ. The lower graph shows the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
Chapter 8. Finite Element Analysis and Experimental Results

8.5.5 Stiffness comparison of FEA and experimental data

The final updated stiffness matrix $K^{FEA}$ for the FEA data of Sensor Variant 1, taking into account the $\Delta O_z$ correction by angle $\gamma$, is

$$
K^{FEA} = \begin{bmatrix}
581.9 \frac{N}{rad} & 0 & 0 \\
0 & -306.4 \frac{N}{rad} & 0 \\
-360.8 \frac{N}{rad} & 0 & 1200 \times 10^3 \frac{N}{m}
\end{bmatrix}.
$$

Including the $\Delta O_z$ offset, this gives the following force-displacement equation

$$
F_{appl} = F_{meas} + F_{res} = K^{FEA} \begin{bmatrix}
\phi \\
\theta \\
\Delta O_z
\end{bmatrix} - \begin{bmatrix}
0 \\
0 \\
\Delta O_{z_a, \phi = 0}
\end{bmatrix} + F_{res} = \begin{bmatrix}
\frac{2N}{\theta_{range}} & 0 & 0 \\
0 & -\frac{2N}{\theta_{range}} & 0 \\
\frac{2N}{\theta_{range}} \tan \gamma & 0 & \frac{1N}{(\Delta O_{z_{max}} - \Delta O_{z_{a, \phi = 0}}) \cos \gamma}
\end{bmatrix} \begin{bmatrix}
\phi \\
\theta \\
\Delta O_z
\end{bmatrix} - \begin{bmatrix}
0 \\
0 \\
\Delta O_{z_a, \phi = 0}
\end{bmatrix} + F_{res} = \begin{bmatrix}
\frac{2N}{\phi_{range}} & 0 & 0 \\
0 & -\frac{2N}{\phi_{range}} & 0 \\
\frac{2N}{\phi_{range}} \tan \gamma & 0 & \frac{1N}{(\Delta O_{z_{max}} - \Delta O_{z_{a, \phi = 0}}) \cos \gamma}
\end{bmatrix} \begin{bmatrix}
\phi \\
\theta \\
\Delta O_z
\end{bmatrix} - \begin{bmatrix}
0 \\
0 \\
\Delta O_{z_{a, \phi = 0}} - 70 \times 10^{-9} m
\end{bmatrix} + F_{res}
$$

where

$$
\begin{bmatrix}
\phi \\
\theta \\
\Delta O_z
\end{bmatrix} = M_{\text{Variant1}}^{-1} \begin{bmatrix}
\Delta A_z \\
\Delta B_z \\
\Delta C_z
\end{bmatrix} = \begin{bmatrix}
1.2021 \times 10^{-3} [m] & -1.2021 \times 10^{-3} [m] & 1 \\
1.2021 \times 10^{-3} [m] & 1.2021 \times 10^{-3} [m] & 1 \\
1.7000 \times 10^{-3} [m] & 0 [m] & 1
\end{bmatrix}^{-1} \begin{bmatrix}
\Delta A_z [m] \\
\Delta B_z [m] \\
\Delta C_z [m]
\end{bmatrix}
$$

Similarly, for the experimental data of Sensor Variant 1, taking into account the $\Delta O_z$ correction by angle $\gamma$, and handling the $\alpha$ correction, we get the final updated stiffness matrices $K^{\text{Experimental1}}$ and $K^{\text{Experimental2}}$

$$
K^{\text{Experimental1}} = \begin{bmatrix}
514 \frac{N}{rad} & -44 \frac{N}{rad} & 0 \\
-73.8 \frac{N}{rad} & 291.3 \frac{N}{rad} & 0 \\
-184.4 \frac{N}{rad} & 0 & 700 \times 10^3 \frac{N}{m}
\end{bmatrix}
$$

(8.15)
8.6. The discussion of the discontinuous nature of some of the experimental data

\[
K^{\text{Experimental2}} = \begin{bmatrix}
509.3 \frac{N}{\text{rad}} & -39.2 \frac{N}{\text{rad}} & 0 \\
-69.1 \frac{N}{\text{rad}} & -288.6 \frac{N}{\text{rad}} & 0 \\
-184.4 \frac{N}{\text{rad}} & 0 & 700 \times 10^3 \frac{N}{m}
\end{bmatrix}
\] (8.16)

where \(K^{\text{Experimental1}}\) corresponds to the correction of Section 8.5.2 and \(K^{\text{Experimental2}}\) corresponds to the correction of Section 8.5.3

8.6 The discussion of the discontinuous nature of some of the experimental data

As compared to FEA results the experimental measurements have two sources of discontinuities. The first one is the O-ring friction effects. The second one is the drift of interferometer bias.

To understand it better it is important to consider the sequence of sample collection, as shown in Figure 8.16, with an orange arrow. Measurement started for \(\alpha = 180^\circ\) and \(\beta = 0^\circ\). Subsequently value of \(\beta\) angle was reduced to \(-90^\circ\) in 18 steps. While in \(\beta = -90^\circ\) the gravity load was always the same, independent from \(\alpha\) angle. After each decrease of \(\alpha\) angle by 5\(^\circ\) the direction of changing \(\beta\) angle was inverted.

The stripes on neighboring \(\alpha\) values (ex. \(\alpha = -120^\circ\) and \(\alpha = -125^\circ\)), visible in Figures from 8.22 to 8.25 reflect the fact of inverting \(\beta\) angle direction. Such hysteresis can be explained by the fact of using the O-ring preventing dust and liquid from penetrating the load cell. It introduces an additional stiffness between the handle and tool tip, and results in error lower than 1\% of sensor measurement range, as reflected in Figure 8.28 b). Such error could be further reduced by changing the sealing solution, which was not required for sensor Variant 1 application.

Another source of discontinuity is the bias value of interferometer indication, which was drifting with time. Hence the stepwise change between measurements taken for \(\alpha = 180^\circ\), where the test started and \(\alpha = -180^\circ\), where it finished on hour later. This behavior was observed mainly on residual force component in X direction, as in Figures 8.22 and 8.24. This resulted in overall difference at the level of 1\% of the sensor force range between beginning and the end of measurement, as seen in Figure 8.28 b). Influence of such interferometer value drift can be reduced by offsetting the bias each time, when no load is applied on the tool tip. In practice ear inspections performed with sensor Variant 1 rarely take longer than 5 minutes. Even without offsetting the interferometer bias the error caused by this phenomena can be estimated as lower than 0.1\% of sensor force range and is acceptable by clinicians.
8.7 Conclusion of the FEA and experimental results

Variant 1 of our sensor catalogue was simulated by FEA and thoroughly tested experimentally.

The residual force of the model using the FEA results, Approach 4, is within a range of -25 mN to 3 mN for a 1 N applied force, in all tested directions, which corresponds to an error range of -2.5% to 0.3% (Figure 8.28a). We consider this to validate our analytical model, including the geometric model and stiffness models.

Figure 8.28 – Absolute force magnitude error for a) FEA and b) experimental data, taking into account all proposed corrections for offsets ($\alpha_{offset}, \gamma$) is shown as $||F_{residual}|| = ||F_{applied}|| - ||F_{measured}|| = 1N \cdot \sqrt{(F_{x,measured})^2 + (F_{y,measured})^2 + (F_{z,measured})^2}$.
Our experimental results show that a diagonal stiffness matrix is not sufficient to precisely model the sensor. Two correction methods were proposed to identify the full (non-diagonal) stiffness matrix: the corrections of the rotational degrees of freedom (X and Y force components) were based either on measurement of the residual force, or on rotating the stiffness matrix; the correction for the axial degrees of freedom (Z force component) was based on correcting the direction of force.

Applying the proposed corrections to the stiffness matrix leads to a significant reduction of sensor measurement error: for a 1N applied force, the residual force error decreases from 450 mN (45% error) to 76 mN (7.6% error), see Figure 8.29.
Figure 8.29 – Residual force reduction by our proposed methods
The residual force in all directions tested by our model using measurements in 7 directions, Approach 5, is within -43 mN to 34 mN for a 1N applied force. This corresponds to an error of -4.3% to 3.4%, see Figure 8.28 b).

The original approach based on measurements in 4 directions, Approach 1, has an error lying between -5% and 12.75%, see Figure 8.30.

Increasing the number of measurement points from 4, Approach 1, to 1387, Approaches 2 and 3, reduces the error to lie between -3% and 3.2% and between -2% and 2.5%, respectively, see Figure 8.30.

Figure 8.30 – Maximal error values comparing Approaches 4 and 5 using our proposed pseudo rigid body model with three previous methods of measured force calculation based on interferometer displacement

In conclusion Approach 5 using only 7 measurement directions for the identification of the model parameters gives results which are significantly better than those of the original Approach 1 which used 4 measurement directions. Moreover, Approach 5 is almost as good as Approaches 2 and 3, each requiring a time-consuming calibration procedure to test 1387 measurement directions.
Chapter 8. Finite Element Analysis and Experimental Results

These results demonstrate the added value of our analytical method based on the pseudo rigid body model. It reduces the calibration procedure from over a thousand measurements to only seven, while keeping an accuracy which is sufficient for the application.
9 Application: VivoForce instrument for retinal microsurgery

This chapter highlights the primary application of this thesis, an innovative 0.6mm diameter force sensing instrument allowing for safer epiretinal membrane peeling surgery, described wider in [23]. The flexure mechanism corresponds to Sensor Variant 6 of Chapter 4.6.

9.1 Introduction

Our VivoForce instrument applies to retinal microsurgery. Epiretinal membranes severely degrade human vision and must be surgically peeled from the retina, a delicate procedure since the retina must not be damaged. The principal difficulty is the limitation of human performance at the required millinewton force range [28]. Current surgery relies on classical passive tools such as a membrane pick or forceps. This results in significant risk of retinal damage and long surgery time (up to 40 minutes) so the procedure is highly dependent on surgeon skill and experience. Our proposed force sensing instrument minimizes the possibility of irreversible retinal damage, thus simplifying the procedure and making it accessible to a wider range of surgeons. Other force sensing instruments are given in [3] [29] [30].

9.2 Tool development

The instrument was developed according to procedure described in Section 3.2, where it is shown in Figures 3.3 and 3.5.

9.2.1 Design and calibration

The tool shaft entering the eye has diameter 0.6mm, compatible with commonly used 23 gauge retinal instruments. This shaft is made of medical grade titanium due to biocompatibility requirements and its desirable properties for the flexible section. Calibration of the instrument was conducted on the setup as in [24], where the only modification was a more sensitive reference sensor (Futek FSH03395).
Chapter 9. Application: VivoForce instrument for retinal microsurgery

9.3 Experimental setup

Due to the lack of a widely accessible model for performing epiretinal membrane peeling, we propose the novel simulator shown in Figure 9.2a), which we call ArtEye. It consists of a sphere having the size of a human eye. Our instrument is inserted through a flexible membrane mimicking the sclera and there is a disposable membrane whose peeling is similar to an epiretinal membrane, see Figure 9.2b). As compared to pig eyes or fertilized chicken eyes previously used for instrument testing, these membranes are low cost and can be manufactured in high volume. Our simulator coupled with our force sensing instrument facilitates surgeon training.

Figure 9.1 – Frames from surgical microscope recordings of test membrane peelings carried out at Jules-Gonin hospital in Lausanne

(a) Dry environment

(b) Wet environment
9.4 Experimental results

Tests of the VivoForce instrument performed at the Jules Gonin Hospital, Lausanne, indicate that surgeons quickly take into account force sensing. Professor Thomas Wolfensberger, a participant in the tool development, performed three consecutive membrane peelings on ArtiEye, with threshold sound feedback set to 30mN, 20mN, 15mN, respectively. This resulted in a lowering of both peak and average measured force, showing the potential for decreased damage during real surgeries. Forces exerted during these tests are shown in Figure 9.3.
9.5 Application summary

We showed that the VivoForce instrument is ready for surgical testing. Together with packaging safe for transportation it is shown in Figure 9.4. We also validated the Fabry-Pérot force sensing transducer at the submillimeter scale. Our current research applies this technique to other medical instruments at the submillimeter scale.
Conclusion

10.1 Contributions

Our thesis makes the following contributions:

• Elaboration of a catalogue of novel flexure-based mecano-optical transducers: one variant with 1-DOF; five variants with 3-DOF

• Manufacturing of two variants: one with 1-DOF (Variant 6) and one with 3-DOF (Variant 2)

• Integration of optical components (three fiber optic Fabry-Perot interferometers) into mecano-optical transducers

• Development of a dedicated generic analytical pseudo rigid body model for 1, 2 and 3-DOF flexure-based transducers, including a geometric model and a stiffness model

• Application of the analytical model of Variant 1

• Validation of the analytical model by Finite Element Analysis of Variant 1

• Design and assembly of a dedicated motorized test bench for the automatic scan of the applied force direction on the various transducers, and implementation of a dedicated control and acquisition system

• Validation of the analytical model by experimental measurement (force applied in 1387 directions) on transducer Variant 1

• Demonstration of the reduction of sensor force measurement error thanks to a new analytical model and rapid calibration approach (only seven measurement direction needed), as compared to the original approach

• Design of five functional surgical instrument prototypes based on Variant 6 with 1-DOF (three prototypes) and Variant 2 with 3-DOF (two prototypes)
• Ex-vivo tests of the designed prototypes by a surgeon on pig eyes, and on a dedicated eye simulator.

10.2 Perspectives

The new modeling approach proposed in this thesis significantly improves the sensor development cycle. As compared to models currently used in industry, see Figure 8.1, our analytical pseudo rigid body model, see Figure 8.2, introduces new geometric variables $\phi, \theta, \Delta O_z$. The advantage is that they allow maximal Fabry-Pérot interferometer mirror inclination at the early design phase, thus reducing sensor development cycle duration.

Moreover, having separate stiffness and geometric models facilitates utilisation of the catalogue since stiffness characterization is based only on mechanical properties independent of interferometer position. Our analytical stiffness model and Finite Element Analysis allow us to estimate stiffness matrices for each design satisfying the force requirements and having the correct physical dimensions. However, the FEA computation is very time consuming. Our approach also allows us to more quickly estimate the stiffness matrix using design of experiment methods. Knowledge of the stiffness matrix potentially allows for automatic dimensioning of sensor variants for future applications. Moreover, together with the geometric model and the sensor parameter matrix, we will be able to refine sensor resolution. The stiffness matrix can be used in the design phase, depending on manufacturing quality.

Subsequent research will continue with a new stiffness model approach covering all sensor variants, in particular, sensor Variant 5.
Bibliography


Bibliography


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Being a **research oriented mechatronics engineer** gives me detailed understanding of physical phenomena and allows me to propose innovative solutions fulfilling required objectives. I do this by combining versatile skills in mechanical design, software development and electronics. Achieving this goal brings me joy!

**Professional experience**

2014 1.01-2019 31.08
**Doctoral assistant at École Polytechnique Fédérale de Lausanne (EPFL)**
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**Supervisor:** Professor Simon Henein, Instant-Lab
**Principal activities:**
- Research on the systematic design of force sensing medical instruments
- Writing a Ph.D. thesis on this research
- Teaching assistant, directing student projects

2013 1.02- 2013 30.09
**Research assistant at EPFL**
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**Instant-Lab (Professor Simon Henein)**
**Principal activities:**
- Developing innovative med-tech projects
- Teaching assistant, directing student projects

2011 1.08-2013 31.01
**Research assistant at EPFL**
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**Laboratory of Robotic Systems (Professor Reymond Clavel)**
**Principal activities:**
- Software and Hardware integration of haptic interfaces and sensors for medical robots
- Teaching assistant, directing student projects

2011 2.05-29.07
**Junior Engineer at Emerson Process Management Power and Water Solutions**
Warsaw, Poland
**Principal activities:**
- Development and installation of a reporting system at the Żarnowiec power plant
- DCS for the Konin power plant

2009 1.07-30.09
**Internship at KOLETON**
Orawka, Poland
**Principal activities:**
- Design of 3D parts. Software: Autocad, Inventor

2009 1.06-26.06
**Undergraduate degree, ABB**
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**Principal activities:**
- Programming and operation of industrial robots.
- Motion programming based on 3D CAD models using Inventor.
2008 14.07-8.08  Internship at “Mechanical workshop Jabłoński”
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Principal activities:
Implementing PLC’s and HMI panels for machine control systems.
Documentation of electric circuits.
Realization of control box for self-developed project.

Education

2014-2020  Ph.D. candidate in Engineering
École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
Thesis venue: Instant-Lab, Microcity, Neuchatel, Switzerland
Supervisors: Professor Simon Henein and Dr Charles Baur

2005-2010  M.Sc. in Automatic Control and Robotics with major in Robotics
Warsaw University of Technology, Poland
Thesis title: Diagnosis of leakages in pneumatic systems with single actuator.
Thesis venue: Faculty of Mechatronics, WUT, Poland
Supervisor: Professor Mariusz Olszewski

Languages

Polish  Mother tongue
English  Very good (C1)
French  Basics (B1)
Russian  Basic (A1/A2)
German  Basic (A1)

Skills

Computer Aided Design Software: Autocad, Inventor, SolidWorks
Programming languages: Pascal, C++, G code, Rapid, VBA, Asembler, Python, Latex
Simulation software: Matlab, Simulink, LabView + data acquisition hardware
Finite element analysis software: Comsol, ANSYS
3D animation software: Blender.org
Manufacturing: CNC, Electro Discharge Machining, Laser cutting, Conventional machining, Rapid prototyping

Training

2019/2020  Academia Industry Training – AIT Switzerland-Colombia
2016  Management of Innovation and Technology Transfer (by TTO, EPFL)
2012/2013  Venture challenge (entrepreneurial training at EPFL)
2012  Object-oriented design and programming in LabView
2012  Managing Software Engineering in LabView
2012  LabView FPGA
2012  LabView Real-Time
2012  LabView Core III
2009  SEP certificate of competence in operation up to 1kV
2008  Programming SAIA, Festo and Siemens PLC’s (within studies)
2008  Basics Fatek PLC programming
2002  First Certificate in English
Publications

Publications in Peer-reviewed Scientific Journals

Conference proceedings

International Conferences – Posters

Invited talks
1. S. Fifanski, T. Fussinger, *Instruments for intraocular microsurgery*, Hamlyn Symposium in Medical Robotics, Imperial College, London, United Kingdom, June 24, 2018
2. S. Fifanski, *Swiss model of start-up incubation on examples*, Conference on model of doctoral studies, National Representation of Doctorants, Warsaw, Poland, November 2015

Awards

2013 EPFL - Special Award of the Dean for services of exceptional value
2012 "Best Business Idea' Award" as a part of VRAI research group, Laboratory of Robotic Systems (KB Medical startup)
2011 Distinction for Masters’ Thesis in Polish nationwide competition of Young Innovators.

Societies

2015 2019 Euspen, European society for precision engineering and nanotechnology
2012 2015 NCCR Co-Me, Computer Aided and Image Guided Medical Interventions
2008 2010 Students interests club in Biomedical Devices, WUT, Warsaw, Poland

Hobbies

Crossfit, sailing, 3rd class glider pilot, home automation
Appendices
A Treatment of experimental data

This appendix contains intermediary calculation steps and charts of corresponding results, which were performed to obtain the results shown in Chapter 8.

A.1 Identification of the stiffness matrix $K$ for experimental data

We repeat the procedure of Section 8.3.3 to compute the stiffness matrix $K$. However, there is a rotational $\alpha_{offset}$ offset around the $Z$ axis, calculated below to be $-7.725^\circ$. We believe that this is due either to an assembly error or to the sensor setup during the measurement procedure. This offset could be eliminated by introducing a rotational reference on the flexure body during the manufacturing process. As it stands, this issue can be addressed by introducing a supplementary constant rotational term, as done in the discussion below.
Figure A.1 – Resulting displacements calculated using the Pseudo Rigid Body Model
Figure A.1 – Resulting displacements calculated using the Pseudo Rigid Body Model (cont.)
Appendix A. Treatment of experimental data

From the analysis shown in Figure A.1, we deduce the following extreme values

$$\phi_{\text{min}} = -0.1133^\circ, \phi_{\text{max}} = 0.1096^\circ$$

and

$$\theta_{\text{min}} = -0.1938^\circ, \theta_{\text{max}} = 0.1996^\circ$$

which gives

$$\phi_{\text{range}} = \phi_{\text{max}} - \phi_{\text{min}} = 0.2230^\circ, \quad 0.2230 \frac{\pi}{180^\circ} = 0.00389 \text{rad}$$

$$\theta_{\text{range}} = \theta_{\text{max}} - \theta_{\text{min}} = 0.3934^\circ, \quad 0.3934 \frac{\pi}{180^\circ} = 0.00687 \text{rad}$$

Therefore, to first order, we get the following stiffness matrix:

$$K_{1\text{experimental}}^{\text{lin,rot,rot,rot}} = \begin{bmatrix} K_{\text{rot}}^y & 0 & 0 \\ 0 & K_{\text{rot}}^x & 0 \\ 0 & 0 & K_{\text{lin}}^z \end{bmatrix} = \begin{bmatrix} -2N \phi_{\text{range}} & 0 & 0 \\ 0 & 2N \theta_{\text{range}} & 0 \\ 0 & 0 & 1N \Delta O_z \text{range} \end{bmatrix}$$  \quad (A.1)

This results in the following equation developed for the generic pseudo rigid body model:

$$F^{\text{appl.}} = F^{\text{meas.1}} + F^{\text{residual1}} = K_{1\text{experimental}}^{\text{lin,rot,rot,rot}} \begin{bmatrix} \phi \\ \theta \\ \Delta O_z \end{bmatrix} + F^{\text{residual1}}$$  \quad (A.2)
A.1. Identification of the stiffness matrix $K$ for experimental data

Figure A.2 – Measured force ($X$ component) and residual error: the upper graph presents the force estimated on the deformation of the flexures ($\phi, \theta, \Delta O_z$) based on measured optical displacement and the pseudo rigid body model, and on the stiffness matrix. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
Figure A.3 – Measured force (Y component) and residual error: the upper graph presents the force estimated on the deformation of the flexures ($\phi, \theta, \Delta O_z$) based on measured optical displacement and the pseudo rigid body model, and on the stiffness matrix. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
A.1. Identification of the stiffness matrix $K$ for experimental data

Figure A.4 – Measured force ($Z$ component) and residual error: the upper graph presents the force estimated on the deformation of the flexures ($\phi, \theta, \Delta O_z$) based on measured optical displacement and the pseudo rigid body model, and on the stiffness matrix. The lower graph presents the residual error, i.e., the difference between the magnitude of the applied force (1N in all measured directions) and the measured force.
Appendix A. Treatment of experimental data

A.2 Z rotation offset identification

We now address the rotational offset described in Section 8.5 and only observed in the experimental data. We can identify the rotational $\alpha_{offset}$ using the inherent symmetry of the sensor displacement data and the extremal measured sensor displacement values, as computed here.

Figure A.5 – Resulting displacements calculated using the pseudo rigid body model
Using a polynomial fit we obtain the following extreme values:

\( \alpha(\min(\Delta A_z)) = 47.32^\circ \),

\( \alpha(\max(\Delta A_z)) = -131.99^\circ \),

\( \alpha(\min(\Delta B_z)) = -62.12^\circ \),
Appendix A. Treatment of experimental data

\[ \alpha(\text{max}(\Delta B_z)) = 116.40^\circ, \]
\[ \alpha(\text{min}(\Delta C_z)) = -7.38^\circ, \]
\[ \alpha(\text{max}(\Delta C_z)) = 171.42^\circ. \]

The symmetric sensor design in the ideal case allows us to expect the following values

\[ \alpha(\text{min}(\Delta A_z)) + \alpha(\text{min}(\Delta B_z)) = 0^\circ, \]
\[ \alpha(\text{max}(\Delta A_z)) + \alpha(\text{max}(\Delta B_z)) = 0^\circ, \]
\[ \alpha(\text{min}(\Delta C_z)) + \alpha(\text{max}(\Delta C_z)) = 180^\circ. \]

This condition is not fulfilled due to manufacturing errors, mainly the misalignment of the sensor hook in relation to the flexure body. We can correct this value by \( \alpha_{corr} \) assuming the following:

\[ \alpha(\text{min}(\Delta A_z)) + \alpha(\text{min}(\Delta B_z)) = 2\alpha_{corr1}, \]
\[ \alpha(\text{max}(\Delta A_z)) + \alpha(\text{max}(\Delta B_z)) = 2\alpha_{corr2}, \]
\[ \alpha(\text{min}(\Delta C_z)) + \alpha(\text{max}(\Delta C_z)) = 180^\circ + 2\alpha_{corr3} \]

so

\[ \alpha_{corr1} = \frac{47.32^\circ - 62.12^\circ}{2} = -7.40^\circ, \]
\[ \alpha_{corr2} = \frac{-131.99^\circ + 116.40^\circ}{2} = -7.795^\circ, \]
\[ \alpha_{corr3} = \frac{-7.38^\circ + 171.42^\circ - 180^\circ}{2} = -7.980^\circ. \]

we therefore conclude:

\[ \alpha_{offset} = \frac{\alpha_{corr1} + \alpha_{corr2} + \alpha_{corr3}}{3} = -7.725^\circ, \quad \frac{-7.725^\circ \pi}{180^\circ} = -0.13483 rad \]
B Generic 3-DOF pseudo rigid body model

The model described in this chapter is an alternative to the 3-DOF model described in Section 5.2.6. The former has the following advantages compared to latter one:

- Geometric part of the model is valid for long range angular displacement, making it therefore suitable not only to model flexures like described in Chapter 4. It is capable of modeling long range angular flexures, as well as identify linear and angular displacements of other mechanisms with surge-pitch-yaw kinematic (i.e. telescope mirrors, or spherical oscillators) using 3 linear displacements along $Z$ axis.

- Position of reflective surfaces in $Z$ direction is taken into account and might be different for each measurement line as parameters $A_z, B_z$ and $C_z$.

- This model has two complementary stiffness models covering all sensors shown in Chapter 4.

- Model downscaling indications are shown, allowing to model sensors from 1-DOF to 3-DOF with the same set of equations, as presented in the Table B.1.

- Discussion of geometric model limitations was performed, indicating forbidden configurations of interferometer locations and a set of conditions, which should be fulfilled in desired sensor workspace.

B.1 3-DOF geometry model with derivation of the formulas

Considering the kinematic of 3DOF sensor as $lin_z - rot_x - rot_y$ and following previously introduced naming convention without applied forces, we have a general situation as in Figure B.1. The X,Y,Z axes of carthesian coordinate system are linked to the base of the transducer. Points A,B,C show initial position of mirror surfaces being measured by 3 interferometers. Point $A_0, B_0, C_0$ are their orthogonal projections on XY plane. Therefore sections $\overrightarrow{A_0A}, \overrightarrow{B_0A}, \overrightarrow{B_0A}$ mark the measurement directions. To avoid ambiguity and singularity these three sections
Appendix B. Generic 3-DOF pseudo rigid body model

cannot be coplanar. Moreover in position where no external load is applied on a sensor we consider that mirror surfaces a, b, c, for all interferometric measurements lie in planes parallel to XY plane and containing respectively points A, B, C. As these planes are linked with sensor tip, which is considered as a rigid body, distances between them will remain constant, while tip will change its position. Therefore distance between planes a and b will be always equal to $|A_z - B_z|$, while distance between planes a and c will be equal to $|A_z - C_z|$.

![Figure B.1 – 3 DOF force sensor without load(generic)](image)

Applied force will result in change of reading from interferometric measurements. New measured points will be respectively marked as $A'$, $B'$, $C'$. Therefore interferometer readings are: $\Delta A_z = A'_z - A_z$, $\Delta B_z = B'_z - B_z$, $\Delta C_z = C'_z - C_z$.

Mirror surfaces will now be $a'$, $b'$, $c'$, each of them containing one of measured points. As said before distances between these mirror surfaces will remain unchanged. Knowing that we will focus on identification of plane $a'$.

Knowing the above we can imagine a point $C'' \in a'$, such that its distance to point $C'$ would be equal to distance between planes $a'$ and $c'$. It would mean that vector $\overrightarrow{C''C'}$ is perpendicular to plane $a'$ and therefore parallel to it’s normal vector $\overrightarrow{n}$, which together with point $A'$ is sufficient to identify plane $a'$.
B.1. 3-DOF geometry model with derivation of the formulas

By analogy we will introduce point $B'' \in a'$, such that its distance to point $B'$ would be equal to distance between planes $a'$ and $b'$. It would mean that vector $\overrightarrow{B'B''}$ is perpendicular to plane $a'$ and therefore parallel to it's normal vector $\overrightarrow{n}$.

Mentioned points are shown in Figure B.2.

![Figure B.2 – 3 DOF force sensor measurement points](image)

To formulate conditions required for $\overrightarrow{n}$ identification we will need dependencies from plane defined by points $A', C', C''$. They are better visible in Figure B.3.

As points $A'$ and $C'$ are fully defined by measurement and distance $|C'C''|$ is known, from Pitagorean theorem we have:

$$|A'C''|^2 + |C'C''|^2 = |A'C'|^2,$$

therefore

$$|A'C''| = \sqrt{|A'C|^2 - |C'C''|^2}.$$

It is also necessary to note that

$$\sin(\phi_{C''A'C}) = \frac{|C'C''|}{|A'C'|} = \frac{A_z - C_z}{|A'C'|}$$

and
Appendix B. Generic 3-DOF pseudo rigid body model

Figure B.3 – 3 DOF force sensor - view on $A', C', C''$ plane
B.1. 3-DOF geometry model with derivation of the formulas

\[ \phi_{\overrightarrow{A'C'}} = 90^\circ + \phi_{C''A'C'}, \]

Therefore \( \cos(\phi_{\overrightarrow{A'C'}}) = -\sin \phi_{C''A'C'} \).

By analogy, looking at plane \( A', B', B'' \), which is shown in Figure B.4 following equations are formulated:

\[ |A'B''| = \sqrt{|A'B'|^2 - |B'B''|^2}, \]

\[ \sin(\phi_{B''A'B'}) = \frac{|B'B''|}{|A'B'|} = \frac{A_z - B_z}{|A'B'|}, \]

\[ \cos(\phi_{\overrightarrow{nA'B'}}) = -\sin \phi_{B''A'B'}. \]

Figure B.4 – 3 DOF force sensor - view on \( A', B', B'' \) plane

Having above information unit vector \( \overrightarrow{n} \) is defined by the following relations:

\[
\begin{cases}
|\overrightarrow{n}| = \sqrt{n_x^2 + n_y^2 + n_z^2} = 1 \\
\angle(\overrightarrow{n}, A'B') = 90^\circ + \phi_{B''A'B'} \\
\angle(\overrightarrow{n}, A'C') = 90^\circ + \phi_{C''A'C'}
\end{cases}
\]

They can be solved if we further develop expressions for angles between vectors as:
Appendix B. Generic 3-DOF pseudo rigid body model

\[
\left\{ \begin{array}{l}
| \vec{n} | = \sqrt{n_x^2 + n_y^2 + n_z^2} = 1 \\
\frac{\vec{n} \cdot \vec{A}'B'}{|\vec{n}||\vec{A}'B'|} = \cos(90^\circ + \phi_{B'\vec{A}'B'}) = -\sin(\phi_{B'\vec{A}'B'}) = -\frac{A_z - B_z}{|\vec{A}'B'|} \\
\frac{\vec{n} \cdot \vec{A}'C'}{|\vec{n}||\vec{A}'C'|} = \cos(90^\circ + \phi_{C'\vec{A}'C'}) = -\sin(\phi_{C'\vec{A}'C'}) = -\frac{A_z - C_z}{|\vec{A}'C'|}
\end{array} \right.
\]

which simplifies to:

\[
\left\{ \begin{array}{l}
n_x^2 + n_y^2 + n_z^2 = 1 \\
n_x(B_x - A_x) + n_y(B_y - A_y) + n_z(B_z' - A_z') = B_z - A_z \\
n_x(C_x - A_x) + n_y(C_y - A_y) + n_z(C_z' - A_z') = C_z - A_z
\end{array} \right.
\]

Going further it gives:

\[
\left\{ \begin{array}{l}
n_x^2 + n_y^2 + n_z^2 = 1 \\
n_x(B_x - A_x) + n_y(B_y - A_y) + n_z(B_z' - A_z') = B_z - A_z \\
n_x(B_x - A_x) + n_y(B_y - A_y) + n_z(B_z' - A_z') = B_z - A_z
\end{array} \right.
\]

To simplify solving of above equation system vectors \(\vec{s}\) and \(\vec{t}\) are introduced as follows:

\[
\vec{s} = [s_x, s_y, s_z] = \vec{A}'B' = |B_x - A_x, B_y - A_y, B_z - A_z|,
\]

\[
\vec{t} = [t_x, t_y, t_z] = \vec{A}'B' = |C_x - A_x, C_y - A_y, C_z - A_z|.
\]

Only z coordinates of these vectors change while force is applied on the sensor tip. Direct influence of these displacements \(\Delta A_z, \Delta B_z, \Delta C_z\) is shown by following relations:

\[
s_z = B_z' - A_z' = \Delta B_z + B_z - \Delta A_z - A_z,
\]

\[
t_z = C_z' - A_z' = \Delta C_z + C_z - \Delta A_z - A_z.
\]

Using components of vectors \(\vec{s}\) and \(\vec{t}\) above system of equations can be written as:

\[
\left\{ \begin{array}{l}
n_x^2 + n_y^2 + n_z^2 = 1 \\
n_x s_x + n_y s_y + n_z s_z = B_z - A_z \\
n_x t_x + n_y t_y + n_z t_z = C_z - A_z
\end{array} \right.
\]

Above system of equations has two solutions \(\vec{n}_1 = [n_{x1}, n_{y1}, n_{z1}]\) and \(\vec{n}_2 = [n_{x2}, n_{y2}, n_{z2}]\), out
B.1. 3-DOF geometry model with derivation of the formulas

of which first fulfills our conditions and will be further referred to as \( \vec{n} \). Example of such solutions is shown in Figure B.5.

![Figure B.5 – 3 DOF force sensor - \( a' \) plane normal vector identification](image)

Explicit form of both solutions are:

\[
\begin{align*}
  n_{x1} &= n_z = \frac{-l_2 + \sqrt{l_2^2 - 4l_1l_5}}{2l_1} \\
  n_{y1} &= n_y = n_{x1}m_1 + m_2 \\
  n_{z1} &= n_x = \frac{C_y - A_y}{t_x} - n_{y1} \frac{t_x}{t_z} - n_{z1} \frac{t_z}{t_x} \\
  n_{x2} &= \frac{-l_2 - \sqrt{l_2^2 - 4l_1l_5}}{2l_1} \\
  n_{y2} &= n_{x2}m_1 + m_2 \\
  n_{z2} &= n_x = \frac{C_y - A_y}{t_x} - n_{y2} \frac{t_x}{t_z} - n_{z2} \frac{t_z}{t_x}
\end{align*}
\]

where:

\[
\begin{align*}
  l_1 &= 2m_1k_0 + m_2^2k_2 + k_4 \\
  l_2 &= m_1k_1 + 2k_0m_2 + 2m_1m_2k_2 + k_3 \\
  l_3 &= m_2k_1 + m_2^2k_2 + k_3 \\
  k_0 &= \frac{t_1t_2}{t_1^2} \\
  k_1 &= \frac{2t_2(A_y - C_y)}{t_2^2}
\end{align*}
\]
Appendix B. Generic 3-DOF pseudo rigid body model

\[ k_2 = 1 + \left( \frac{t_x}{l_x} \right)^2 \]
\[ k_3 = \frac{2t_z(A_z-C_z)}{l_z^2} \]
\[ k_4 = 1 + \left( \frac{t_z}{l_z} \right)^2 \]
\[ k_5 = 1 - \left( \frac{(C_z-A_z)}{l_z} \right)^2 \]
\[ m_1 = \frac{s_x t_z - s_z t_x}{s_x s_y - s_z s_y} \]
\[ m_2 = \frac{l_z(B_z-A_z) - s_z(C_z-A_z)}{l_z s_y - s_z s_y} \]

B.1.1 Discussion of model limitations

Definition of this geometric model and its particular kinematic already constraints some setups of measurement lines. Further constraints result directly from the form of introduced solution. They are presented below:

1. Denominator of parameter \( m_1 \) cannot be equal 0. Therefore:

\[ m_1 \neq 0 \Rightarrow t_x s_y - s_x t_y \neq 0 \Rightarrow s_x t_y \neq s_y t_x \Rightarrow \frac{t_x}{t_y} \neq \frac{s_x}{s_y} \]

Geometric interpretation of last implication is that vectors \( \overrightarrow{A_0B_0} \) and \( \overrightarrow{A_0C_0} \) cannot be parallel. In such situation all three measurement lines of interferometers A, B, C would be coplanar. This possibility was already excluded by definition of our geometric model.

Three coplanar measurement lines might be used in further sensors with more than 3DOF. Such situation may also appear in redundant force sensors, especially important in medical field on which we are currently working.

2. Denominator in few of above parameters is equal to \( t_x = C_x - A_x \). This means it cannot be equal to 0. In practice it prevents interferometers A and C to have the same x coordinate. If such configurations occurs it is advised to change symbols between interferometers B and C.

\[ t_x \neq 0 \Rightarrow C_x \neq A_x \]

3. Furthermore looking at coordinate \( n_z \) we have 2 additional conditions:

3A. Denominator of \( n_z \neq 0 \Rightarrow l_1 \neq 0 \).

3B. \( l_2^2 - 4l_1 l_3 > 0 \)

We did not find geometric interpretation of these two conditions, therefore it is advised to check if they are not equal to zero for expected measurement range (i.e. for all possible values of \( \Delta A_z, \Delta B_z, \Delta C_z \) for particular sensor and measurement method). For sensors developed in Instant-Lab we didn't find any, where conditions 3A or 3B were broken.
B.1. 3-DOF geometry model with derivation of the formulas

Figure B.6 – Forbidden positions according to model limitation 1

Figure B.7 – Forbidden positions according to model limitation 2
Appendix B. Generic 3-DOF pseudo rigid body model

Figure B.8 – Allowed relative positions, with suggested clockwise naming convention of interferometers

Figure B.9 – Allowed positions with two interferometers with the same X coordinate and suggested clockwise naming convention - example 1
### B.1. 3-DOF geometry model with derivation of the formulas

Figure B.10 – Allowed positions with two interferometers with the same X coordinate and suggested clockwise naming convention - example 2

#### B.1.2 Tooltip position identification

Next step is to identify position and orientation of sensor tip, defined by points $P'$ and $O'$. For this purpose we will introduce point $R' = [R'_x, R'_y, R'_z]$ lying on the intersection of direction designated by points $P', O'$ and plane $a'$.

Knowing $\mathbf{n}$ and coordinates of point $A'$ plane $a'$ is defined by equation:

$$a': n_x(x - A'_x) + n_y(y - A'_y) + n_z(z - A'_z) = 0$$

As point $O'$ lies on z axis, vector $\overrightarrow{O'R'}$ is parallel to $\mathbf{n}$ and $|O'R'| = |OR| = |AZ|$ true are the following equations:

$R'_x = n_x|OR| = n_x A_z$,  

and

$R'_y = n_y|OR| = n_y A_z$.

Knowing $R'_x$ and $R'_y$ we can find $R'_z$ from $a'$ plane equation:

$$\frac{n_x(R'_x - A_z) + n_y(R'_y - A_z)}{-n_z} + A'_z = R'_z.$$
Appendix B. Generic 3-DOF pseudo rigid body model

Knowing that we can find point $O'$, as:

$O' = [0, 0, R'_{z} - n_{z}A_{z}]$.

This finally allows to identify point $P'$, as:

$P' = [n_{x}P_{z}, n_{y}P_{z}, O'_{z} + n_{z}P_{z}]$

At current stage all the points shown in Figure B.11 have identified positions.

![Figure B.11 – identified geometry of 3 DOF force sensor](image)

Direction of vector $\overrightarrow{O'P'}$ can be identified by above coordinates or using Euler angles defined as in Figure B.11. They can be deduced from below equations:

$$P'_{z} = \sqrt{P'^{2}_{x} + P'^{2}_{y} + P'^{2}_{z}} \cos \beta$$

$$
\begin{align*}
P'_{x} &= \sqrt{P'^{2}_{x} + P'^{2}_{y} + P'^{2}_{z}} \cos \alpha \\
P'_{y} &= \sqrt{P'^{2}_{x} + P'^{2}_{y} + P'^{2}_{z}} \sin \alpha
\end{align*}
$$

Model introduced above allows for identification of tool tip position and orientation required.
Figure B.12 – Direction of vector $O'P'$ defined by Euler angles $\alpha$ and $\beta$
for force identification using reading from 3 interferometric measurements performed in parallel directions in any position. The only condition is that measurements lines representing optical fibers cannot be coplanar.

Moreover said model can be scaled down to any of introduced before 2DOF or 1DOF geometries.

### B.2 3 DOF stiffness models

Two further models are introduced to link calculated geometry with force \( F = [F_x, F_y, F_z] \) applied on point \( P \), both for small displacements.

Having a sensor design where we can decouple stiffness for rotations around axes \( x \) and \( y \), we have:

\[
F_xP_z = K_{Rot}^x P_x'
\]

\[
F_yP_z = K_{Rot}^y P_y'
\]

\[
F_z = K_{Lin}^z O_z'
\]

Where we cannot decouple \( F_x \) and \( F_y \) and stiffness can be expressed as function of \( \alpha \) angle \( K_{Rot}(\alpha) \), we have:

\[
F_xP_z = K_{Rot}(\alpha)P_x' = K_{Rot}(\alpha)\sqrt{P_x'^2 + P_y'^2 \cos \alpha}
\]

\[
F_yP_z = K_{Rot}(\alpha)P_y' = K_{Rot}(\alpha)\sqrt{P_x'^2 + P_y'^2 \sin \alpha}
\]

\[
F_z = K_{Lin}^z O_z'
\]

The first model can be used with second set of equations if we introduce function \( K_{Rot}(\alpha) \) according to equation below: \( K_{Rot}(\alpha) = K_{Rot}^x \cos \alpha + K_{Rot}^y \sin \alpha \).

### B.3 Model downscaling indications

The introduced model can be applied to 1-DOF or 2-DOFs sensors. This versatility is specially useful in research conditions, where development tools will be able to base on single model no matter what sensor we are dealing with.

Realization of this downscaling can be achieved by modification of model inputs. Modifications will apply to both measurement line coordinates defined by points A, B, C and interferometer readings \( \Delta A, \Delta B, \Delta C \). Operations below show how to adapt our 3DOF model to other configurations listed in Table B.1.
B.3. Model downscaling indications

<table>
<thead>
<tr>
<th>load cell kinematic</th>
<th>measured variables</th>
<th>simulated variables</th>
<th>fiber location parameters</th>
</tr>
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<tbody>
<tr>
<td>1DOF</td>
<td>lin</td>
<td>ΔA</td>
<td>∆B = ∆A, ∆C = ∆A</td>
</tr>
<tr>
<td></td>
<td>rots</td>
<td>ΔA</td>
<td>∆B = 0, ∆C = 0</td>
</tr>
<tr>
<td></td>
<td>rot</td>
<td>ΔA</td>
<td>∆B = 0, ∆C = 0</td>
</tr>
<tr>
<td>2DOF</td>
<td>lin, rot</td>
<td>ΔA</td>
<td>∆B = 0, ∆C = ∆B</td>
</tr>
<tr>
<td></td>
<td>lin, lin-rot</td>
<td>ΔA</td>
<td>∆B = 0, ∆C = ∆B</td>
</tr>
<tr>
<td></td>
<td>rot, rot</td>
<td>ΔA</td>
<td>∆B = 0, ∆C = 0</td>
</tr>
<tr>
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<td></td>
<td>lin-rot, lin-rot, rot</td>
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<tr>
<td>3DOF</td>
<td>lin-rot, rot</td>
<td>ΔA</td>
<td>-</td>
</tr>
</tbody>
</table>

Table B.1 – Model adaptation for different cell configurations