

A Betz-Inspired Principle for Kite-Power Generation Using Tethered Wings

Sean Costello^{a,*}, Colm Costello^b, Grégory François^a, Dominique Bonvin^a

^a*Laboratoire d'Automatique, École Polytechnique Fédérale de Lausanne, Switzerland*

^b*Department of Mechanical Engineering, Institute of Technology Tallaght, Ireland*

Abstract

This paper's main contribution is a theoretical result that can be used to evaluate the maximum power-generating potential of any kite-power system. An upper bound is derived for the power a wing can generate in a given wind speed. It is proven that the angle of the restraining forces on the system modulates this upper bound. In order to derive practically useful results, this is linked to the strength-to-weight ratio of the different system components through an efficiency factor. The result is a simple analytic expression that can be used to calculate the maximum power-producing potential for any system of lifting surfaces, dynamic or static, supported by a tether. As an example, the analysis is applied to two systems currently under development, namely, pumping-cycle generators and jet-stream wind power.

Keywords: Wind Power, Kite Power, Efficiency, Optimization.

1. Introduction

The concept of using kites, or airplanes on tethers, to harness energy from the wind has gained increasing attention in recent years. A number of start-ups are working to commercialize kite power, and the body of scientific literature on the subject is rapidly expanding. The main motivation driving this development is to access winds at high altitudes without constructing a large, costly tower. Due to friction between the ground and the air, a wind gradient is present in the first several hundred meters of the atmosphere [18, 20], with the result that, on average, the wind is stronger and less turbulent once an altitude of several hundred meters is reached. Some ambitious projects even aim to harness winds at altitudes of several kilometers, where the wind is predicted to be almost an order of magnitude stronger than near the surface [15]. It is unlikely the size of conventional wind turbines can increase indefinitely; increased material costs (the square-cubed law), site-access limitations and engineering challenges will probably prevent turbines from continuing to grow at the rate of roughly 10 m per-year that occurred during the last decade [19]. Hence, alternative strategies must be developed to access higher winds. Kites are wings (just like the blades of a turbine, the wings of an airplane or the sails of a yacht) that are connected to the ground using a system of cables. The force the wind generates on the kite can be transmitted to the ground via these cables in order to do work. As the cables are relatively inexpensive, the kite can operate several hundreds of meters (or even several kilometers) from the ground. Another advantage is that the cable (or system of cables) transmits the force on the kite directly to the ground, so supporting structures such as the towers used by wind turbines or the masts on ships are unnecessary.

A range of diverse concepts have emerged for harnessing high-altitude wind, from hovering quad-copters to giant floating balloon generators. However, the majority of the teams in this field are working on kite/airplane based designs. These designs use one or more wings, connected in different manners using cables. The cables may be used to drive a variety of ground-based generator setups, generally simple winch generators, yet some wish to employ more imaginative schemes such as rotating merry-go-rounds [7]. In addition, the wing

*Corresponding author

Email addresses: sean.costello@epfl.ch (Sean Costello), colm.costello@ittdublin.ie (Colm Costello), gregory.francois@epfl.ch (Grégory François), dominique.bonvin@epfl.ch (Dominique Bonvin)

may be loaded with wind turbines in order to generate electricity directly. Indeed, many of the proposed kite systems bear little or no resemblance to the traditional kite, yet in this article they will be referred to as kite-power systems because fundamentally they are all based on wings supported by cables.

As an emerging technology, kite power has yet to prove its economic merit. Practitioners must surmount numerous practical obstacles related to structures, generator systems, sensing, automatic control and take-off and landing. The aim of this article is to develop a concept of aerodynamic efficiency that can be used to analyze and design kite-power systems. Notions of efficiency are extremely important in energy generation; Carnot efficiency is fundamental to the study of heat engines, while wind turbines cannot exceed Betz's limit [4]. An understanding of efficiency is based on an upper bound. The efficiency of a wind turbine is defined as the ratio of the derived mechanical power to the maximum power that can be removed from the wind passing through the disc occupied by the turbine. This is a very useful measure because the diameter of the turbine and its cost are strongly related. Hence, the Betz efficiency of a turbine is one of the quantities that can be used to predict the return on investment. Unfortunately, the Betz limit cannot meaningfully be applied to kites as the area a kite moves in is generally very large and the kite will only remove a small fraction of the wind energy passing through that area; thus a kite system would have a very low Betz efficiency.

Previous work on the efficiency of kite-power systems has focused on maximizing the power output for a particular setup [13, 11, 2, 3]. In kite-power's seminal paper, Loyd [14] derived expressions for the surprisingly high power that a tethered kite can produce flying in a crosswind direction. Much of the contemporary development is based on Loyd's analysis, and there is a misconception in the kite-power community that Loyd derived an upper bound for the power a wing can produce. In fact, Loyd derived upper bounds for the power production for three particular configurations, the so-called 'lift power', 'drag power' and 'simple kite' configurations. One could imagine flying a kite in many more configurations. Moreover, Loyd's upper bound applies only to a very idealized system; for a more realistic kite-power system, it has yet to be determined to what extent this bound is affected by parameters such as weight. Diehl [10] extended Loyd's analysis to derive a general upper bound for the power any kite can produce, pointing out that a misalignment between the wind direction and the aerodynamic force exerted on the kite will result in losses (which he calls "cosine losses"). The main result in our paper is to quantify these "cosine losses". In doing so we are able to derive a much more restrictive (and hence informative) upper bound that can be applied to all kite power systems.

Our analysis is based on two bounds. First, in Section 3, we derive an upper bound, referred to as P_{\max} , for the power any given wing may generate with a given wind speed (this is the bound derived by Diehl [10]). However, this bound has a limited use in analyzing a system as, just like Betz's limit for turbines, practical systems are unlikely to approach it. For this reason, we derive in Section 4 the more restrictive bound \tilde{P} ; its application requires more information about the system and in return it is more accurate. Although deriving this second bound necessitates an abstract mathematical formulation, its potential becomes apparent in Section 5. This bound is used to investigate the effect of the main parameters of a generic high-altitude wind-power system on its overall power coefficient. This yields a simple analytic expression for the maximum derivable power, applicable to almost any airborne system of wings. To illustrate the results, the analysis is applied to two systems currently under development, namely, pumping-cycle generators [1, 17] and jet-stream wind power [15].

2. Problem Formulation

2.1. Aerodynamic power

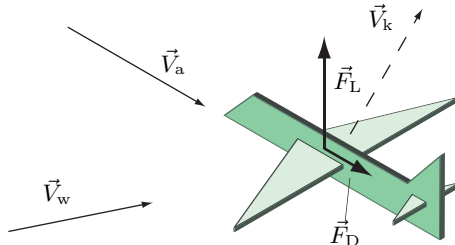


Figure 1: A foil (in this case a plane) moving in 3-D space.

Consider a wing moving in a 3-D space with velocity \vec{V}_k as shown in Figure 1. The flow of the apparent wind over the wing, \vec{V}_a , results in a net force on the foil. The velocity of the apparent wind is given by $\vec{V}_a = \vec{V}_w - \vec{V}_k$, where \vec{V}_w is the wind velocity. This aerodynamic force can be separated into two components: the drag \vec{F}_D , which acts in the direction of \vec{V}_a , and the lift \vec{F}_L , which acts perpendicular to \vec{V}_a :

$$|\vec{F}_D| = \frac{1}{2} \rho A C_D |\vec{V}_a|^2, \quad (2.1)$$

$$|\vec{F}_L| = \frac{1}{2} \rho A C_L |\vec{V}_a|^2, \quad (2.2)$$

where ρ is the air density, A is the area of the wing and C_L and C_D are the wing-specific lift and drag coefficients. These coefficients depend on the apparent wind speed and on the orientation of the wing. A more detailed discussion on the orientation of the wing is given in [Appendix B](#). We define *aerodynamic power* as

$$P_{\text{aero}} = \vec{V}_k \cdot (\vec{F}_L + \vec{F}_D). \quad (2.3)$$

This is the instantaneous power being transferred to the kite by the aerodynamic force. This power can be used to raise/lower the potential or kinetic energy of the kite, or it can be used to do work on an external object, for example pull a boat, drive a generator or even drag a turbine through the air. It is logical that, for a given wing and wind conditions, there will be an upper bound for P_{aero} ; we will call this P_{max} . Regardless of the manner in which the aerodynamic power is harnessed, the average power produced (taking into account the net change in the system's potential/kinetic energy) cannot exceed P_{max} .

We will derive a simple expression for P_{max} . What is more, we will derive the more restrictive bound \tilde{P} , which varies depending on the angle the restraining forces on the kite make with the wind. This shows that most kite systems will derive considerably less power than P_{max} .

2.2. Loyd's analysis

Maximizing (2.3) will give an upper bound for the power a wing can generate. It is also possible to include constraints in the formulation. Loyd solved this optimization problem with 3 different constraints:

1. 'Simple kite' configuration: the velocity of the wing is parallel with the total aerodynamic force, $\vec{V}_k \parallel (\vec{F}_D + \vec{F}_L)$.
2. 'Lift power' configuration: the total aerodynamic force is parallel with the wind, $\vec{V}_w \parallel (\vec{F}_D + \vec{F}_L)$.
3. 'Drag power' configuration: the velocity of the wing is perpendicular to the wind, $\vec{V}_k \perp \vec{V}_w$.

The reason Loyd addressed these three particular cases is because, from a practical viewpoint, they are the most intuitive configurations in which a kite would be used to generate power. His main result was that almost equally high powers can be generated in the ‘lift’ and ‘drag’ power configurations. These are certainly the most popular configurations under investigation today.

Loyd only considered the three most likely kite-power configurations; his analysis cannot be applied to alternative schemes. For example, what is the maximum power output for the ‘lift power’ and the ‘drag power’ modes combined? In order to answer this question, we will maximize (2.3) *without constraints*.

3. A General Upper Bound

We now re-derive the upper bound from Diehl [10]. In doing so, we show that the maximum power a wing can generate occurs when it is operated in Loyd’s ‘lift power’ mode. We use the fact that P_{aero} can be shown to only depend on two parameters: the angle between the aerodynamic force vector and the wind ζ , and the velocity of the kite in the direction of the aerodynamic force v_1 .

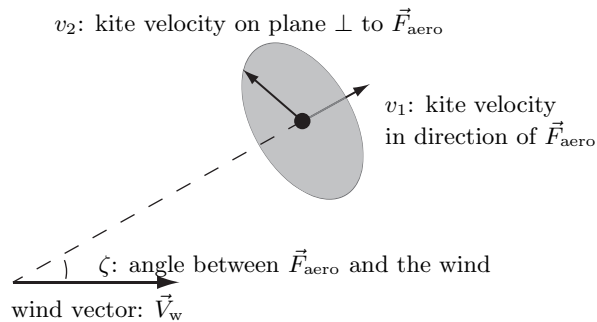


Figure 2: Decomposition of the kite’s velocity into the two perpendicular components v_1 and v_2 .

Consider the kite shown in Figure 2. The apparent wind in the direction of \vec{F}_{aero} is

$$v_{\parallel} = |\vec{V}_w| \cos \zeta - v_1. \quad (3.1)$$

The key observation made by Argatov et al. [2] and Dadd et al. [8] is that the apparent wind in the plane perpendicular to \vec{F}_{aero} is given by

$$v_{\perp} = \frac{C_L}{C_D} v_{\parallel}. \quad (3.2)$$

Note that, while Argatov and Dadd derived this in the context of a ‘zero-mass’ kite model, we make no assumption regarding the mass of the system here. Hence, the magnitude of the apparent wind is given by

$$|\vec{V}_a|^2 = v_{\perp}^2 + v_{\parallel}^2 \quad (3.3)$$

$$= v_{\parallel}^2 \left(1 + \left(\frac{C_L}{C_D} \right)^2 \right). \quad (3.4)$$

As the aerodynamic force is proportional to the square of the apparent wind speed, we have

$$|\vec{F}_{\text{aero}}| = \frac{1}{2} \rho A |\vec{V}_a|^2 \sqrt{C_L^2 + C_D^2} \quad (3.5)$$

$$= \frac{1}{2} \rho A N v_{\parallel}^2, \quad \text{with } N = C_D \left(1 + \left(\frac{C_L}{C_D} \right)^2 \right)^{\frac{3}{2}} \simeq C_L \left(\frac{C_L}{C_D} \right)^2, \quad (3.6)$$

as $\left(\frac{C_L}{C_D}\right)^2 \gg 1$ in general. The maximal value of N occurs close to the angle of attack which maximizes $\frac{C_L}{C_D}$; this is discussed further in [Appendix B](#). Substituting in the expression (3.1) for v_{\parallel} , we obtain

$$|\vec{F}_{\text{aero}}| = \frac{1}{2}\rho AN(|\vec{V}_w| \cos \zeta - v_1)^2. \quad (3.7)$$

We can now write the aerodynamic power in terms of v_1 :

$$P_{\text{aero}} = |\vec{F}_{\text{aero}}| v_1 \quad (3.8)$$

$$= \frac{1}{2}\rho AN(|\vec{V}_w| \cos \zeta - v_1)^2 v_1. \quad (3.9)$$

It is straightforward to maximize this expression with respect to v_1 . For a given ζ , the speed $v_1^* = \frac{1}{3}|\vec{V}_w| \cos \zeta$ yields the maximum power

$$P_{\text{aero}}^*(\zeta) = \left(\frac{1}{2}\rho AN\right) \frac{4}{27} |\vec{V}_w|^3 \cos^3 \zeta. \quad (3.10)$$

In turn, this power is maximized when $\zeta = 0$, yielding

$$P_{\text{max}} = \left(\frac{1}{2}\rho A |\vec{V}_w|^3\right) C_p, \quad \text{with } C_p = \frac{4N}{27}. \quad (3.11)$$

Note that the term in brackets is the power (kinetic energy per unit time) of the wind passing through an area A . C_p is the maximum power coefficient (comparable to the power coefficient for wind turbines, except that the area being considered is the wing area, not the swept area). $C_p = 5$ is a typical value for a standard surf kite. We can conclude that the energy that can be produced is several times the kinetic energy of the wind passing through a cross-section of area A . This is the attraction of kites; just as the narrow blades of a wind turbine cover a large area, a relatively small kite can derive power from a large area by flying fast across the wind. If a more efficient wing is used (meaning it has a greater $\frac{C_L}{C_D}$), for example a Boeing 747 with $C_p = 34$, the kinetic energy equivalent to an area 34 times that of the wing can be removed!

It is worth emphasizing that P_{max} is the *absolute limit* for the amount of power that can be derived by a given wing at a given wind speed, regardless of the configuration of tethers or generators being used. Loyd also obtained this bound for the ‘lift power’ configuration (in fact Houska [12] derived this exact expression, Loyd made some slight simplifications). So the ‘lift power’ configuration is in fact optimal, which means that, in this configuration, a wing generates its largest aerodynamic power. The difference is that here we derive the bound independently of the configuration.

4. An Upper Bound for Average Force Angles

To harness the difference in speed between the ground and the air (wind), we must use the ground to push against the air through the intermediary of the wing. Conceptually, the wing creates resistance between the ground and the air, which slows down the air and some of the loss in kinetic energy is converted into aerodynamic power. Of primal importance is the force the ground exerts on the wing; this must be transmitted through a system of flexible tethers. If we could design a configuration such that this force is aligned with the wind, it would be easy to operate the wing optimally at all times. The only way to exert a force in line with the wind would be if the wing was flying just above the ground. But the whole point of kite power is to harness winds at altitudes of at least several hundred meters. This imposes an angle between the restraining force and the wind.

When considering a single weightless kite on a weightless tether, the aerodynamic force can reasonably be assumed to be aligned with the tether. In this case, it is apparent that the angle between the tether and the wind will decide the maximum aerodynamic power, thus equation (3.10) can be applied to establish an upper bound on P_{aero} . However, the tether is no longer necessarily aligned with the aerodynamic force once factors such as tether drag, the mass of the kite, or the mass of the tether are taken into account. In this

case, the angle of the instantaneous aerodynamic force may vary arbitrarily, depending on the maneuvers performed. The analysis becomes even more complex if several kites are attached together, as has been proposed by Houska [12]. Argatov et al. [2] and Argatov and Silvennoinen [3] established that for the case of a single kite attached to a generator, both the mean and the maximum achievable mechanical power depend on the cosine of the average tether angle *cubed*. We now derive a very simple upper bound for P_{aero} that can be applied to any system of kites, regardless of configuration or mass. As suggested by Argatov et al. [2], it transpires that the cosine of the angle of the *average* aerodynamic force *cubed* determines the maximum power that can be derived from the wind.

Theorem 1. (Maximum Aerodynamic Power for a Force Angle)

Consider the period of time T , and let ζ_0 be the angle that the average aerodynamic force $\frac{1}{T} \int_0^T \vec{F}_{\text{aero}} dt$ makes with the wind. The average aerodynamic power is upper bounded as follows:

$$\frac{1}{T} \int_0^T P_{\text{aero}} dt \leq P_{\text{max}} \cos^3 \zeta_0 \quad \forall \zeta_0 \in]-\frac{\pi}{2}, \frac{\pi}{2}[. \quad (4.1)$$

Proof. The idea is to show that, for a given ζ_0 , the average power is largest when both the direction of the aerodynamic force and the kite's velocity in this direction are constant. To do so, we solve a path optimization problem. We use the spherical co-ordinate system shown in Figure 3; the zenith is aligned with the wind, ζ is the polar angle, and θ is the azimuth angle.

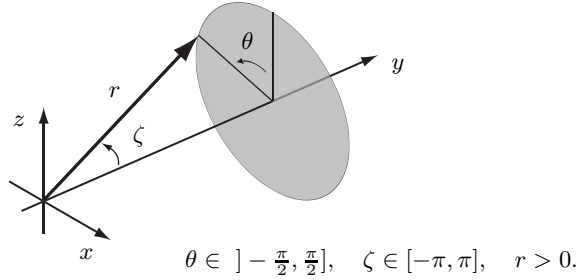


Figure 3: Spherical co-ordinate system with the wind vector as the zenith. The wind is in the y direction.

In these co-ordinates, the aerodynamic force is

$$\vec{F}_{\text{aero}} = -r \sin(\zeta) \sin(\theta) \hat{x} + r \cos(\zeta) \hat{y} + r \sin(\zeta) \cos(\theta) \hat{z}, \quad \text{with } r = |\vec{F}_{\text{aero}}|. \quad (4.2)$$

Let the average aerodynamic force point in the direction

$$\hat{n}_1 = \cos(\zeta_0) \hat{y} + \sin(\zeta_0) \hat{z}, \quad (4.3)$$

which forms an orthonormal basis along with

$$\hat{n}_2 = -\sin(\zeta_0) \hat{y} + \cos(\zeta_0) \hat{z} \quad \text{and} \quad \hat{n}_3 = \hat{x}. \quad (4.4)$$

The path optimization problem to be solved is

$$\begin{aligned} & \max_{v_1(t), \zeta(t), \theta(t)} \int_0^T P_{\text{aero}}(v_1(t), \zeta(t)) dt \\ & \text{s.t. } \exists c \in \mathbb{R} \text{ satisfying } \int_0^T \vec{F}_{\text{aero}}(v_1(t), \zeta(t), \theta(t)) dt = c \hat{n}_1. \end{aligned} \quad (4.5)$$

In the following, the argument t will generally be omitted, for example $\zeta(t)$ will be referred to as ζ . The expressions in this optimization problem can be simplified without modifying the problem itself. Normalizing

v_1 w.r.t. $|\vec{V}_w|$ and removing the constant terms in (3.7) and (3.8) that do not affect the optimization problem, we can write the scaled aerodynamic power and force as:

$$P(v, \zeta) = (\cos \zeta - v)^2 v, \quad |\vec{F}(v, \zeta, \theta)| = (\cos \zeta - v)^2, \quad \text{with } v = \frac{v_1}{|\vec{V}_w|}. \quad (4.6)$$

Note that the constraint in Problem 4.5 can be reformulated as

$$\dot{x} = f(u) = \begin{bmatrix} \vec{F} \hat{n}_2 \\ \vec{F} \hat{n}_3 \end{bmatrix} = (\cos \zeta - v)^2 \begin{bmatrix} \cos \theta \sin \zeta \cos \zeta_0 - \cos \zeta \sin \zeta_0 \\ -\sin \zeta \sin \theta \end{bmatrix}, \quad x(0) = 0. \quad (4.7)$$

Hence, f is the instantaneous force perpendicular to \hat{n}_1 , expressed in the \hat{n}_2, \hat{n}_3 basis. Using the notation $u = [v, \zeta, \theta]^T$, the optimization problem can be written as:

$$\max_{u(t)} \int_0^T P(u) dt \quad (4.8)$$

$$\text{s.t.} \quad \dot{x} = f(u), \quad x(0) = 0 \\ \psi(x(T)) = x(T) = 0. \quad (4.9)$$

This is an optimal control problem that can be tackled using Pontryagin's Maximum Principle (PMP) [6]. We form the Hamiltonian:

$$H = -P + \lambda^T f \quad \text{with} \quad \dot{\lambda}^T = -\frac{\partial H}{\partial x}; \quad \lambda^T(T) = \nu^T \frac{\partial \psi}{\partial x(T)}, \quad \nu \in \mathbb{R}^2. \quad (4.10)$$

PMP states that a necessary condition for $u^*(t)$ to be a maximizer is that $\exists \nu$ s.t. H is maximized by $u^*(t) \forall t$. This implies that $\frac{\partial H}{\partial u} = \left[\frac{\partial H}{\partial v} \quad \frac{\partial H}{\partial \zeta} \quad \frac{\partial H}{\partial \theta} \right] = [0 \ 0 \ 0]$, $\forall t$. By solving this equation we can find all possible maximizers. Notice that H does not depend on x , and thus $\dot{\lambda} = 0$; also $\frac{\partial \psi}{\partial x} = I_2$ and so $\lambda(t) = \nu$, i.e. $\lambda \in \mathbb{R}^2$ is constant. The full expression for the Hamiltonian is

$$H = (\cos \zeta - v)^2 \left(\lambda_1 (\cos \theta \sin \zeta \cos \zeta_0 - \cos \zeta \sin \zeta_0) - \lambda_2 \sin \zeta \sin \theta - v \right). \quad (4.11)$$

Let us first consider the equation $\frac{\partial H}{\partial \theta} = 0$:

$$\frac{\partial H}{\partial \theta} = -(\cos \zeta - v)^2 \sin \zeta \left(\lambda_2 \cos \theta + \lambda_1 \cos \zeta_0 \sin \theta \right) = 0. \quad (4.12)$$

We discard the solution $v = \cos \zeta$ as this yields $P = 0$ and $F = 0$. This is clearly not part of a maximizing solution because other values of u , e.g. $u = [\frac{1}{3} \cos \zeta_0, \zeta_0, 0]$ yield $P > 0$ and $F = 0$. Therefore,

$$\text{either } \sin \zeta = 0 \quad \text{or } \tan \theta = -\frac{\lambda_2}{\lambda_1 \cos \zeta_0}. \quad (4.13)$$

These two conditions combined mean \vec{F} must lie in one plane for all t (θ is constant except at the singular point $\zeta = 0$). As \hat{n}_1 , the required direction of the average \vec{F} , must lie in this plane in order to satisfy the constraint in Problem 4.5, we can conclude that this plane is characterized by $\theta = 0$, $\forall t$. Consequently $f_2 = 0$, $\forall t$:

$$f(u) = (\cos \zeta - v)^2 \begin{bmatrix} \sin(\zeta - \zeta_0) \\ 0 \end{bmatrix}, \quad (4.14)$$

and the Hamiltonian becomes:

$$H = (\cos \zeta - v)^2 (\lambda_1 \sin(\zeta - \zeta_0) - v). \quad (4.15)$$

The condition $\frac{\partial H}{\partial v} = 0$ gives

$$v = \frac{1}{3} (\cos \zeta + 2\lambda_1 \sin(\zeta - \zeta_0)). \quad (4.16)$$

Combining this with $\frac{\partial H}{\partial \zeta} = 0$ and solving for λ_1 gives

$$\lambda_1 = \frac{\cos \zeta}{\sin(\zeta - \zeta_0)} \quad \text{or} \quad \lambda_1 = -\frac{\sin \zeta}{\cos(\zeta - \zeta_0)}. \quad (4.17)$$

Note that the first solution implies that $v = \cos \zeta$ so, once again, we discard this possibility. Therefore, the second possibility must hold $\forall t$. At this point we must consider the values of ζ that can occur during an optimal profile. If a profile is optimal, for a given value f_1 , (4.14) gives:

$$v(\zeta, f_1) = \cos \zeta - \sqrt{\frac{f_1}{\sin(\zeta - \zeta_0)}}, \quad (4.18)$$

$$\text{and so } P(\zeta, f_1) = \frac{f_1}{\sin(\zeta - \zeta_0)} \left(\cos \zeta - \sqrt{\frac{f_1}{\sin(\zeta - \zeta_0)}} \right). \quad (4.19)$$

We need the fact that, for $\zeta_0 \in]-\frac{\pi}{2}, \frac{\pi}{2}[$, it is straightforward to show that

$$\cos(\zeta_0 \pm \frac{\pi}{2} \pm \delta) \leq \cos(\zeta_0 \pm \frac{\pi}{2} \mp \delta) \quad \forall \delta \in]0, \frac{\pi}{2}[. \quad (4.20)$$

Using this, and noting that $\sin(\zeta - \zeta_0)$ is symmetric about $\zeta_0 \pm \frac{\pi}{2}$, we can conclude that $\forall \zeta_1 \in]\zeta_0 + \frac{\pi}{2}, \zeta_0 - \frac{\pi}{2}[$, there exists a $\zeta_2 \in]\zeta_0 - \frac{\pi}{2}, \zeta_0 + \frac{\pi}{2}[$ s.t. $P^*(\zeta_2, f_1) > P^*(\zeta_1, f_1)$. Hence, in the search for a maximum, we can restrict our attention to the interval

$$\zeta \in [\zeta_0 - \frac{\pi}{2}, \zeta_0 + \frac{\pi}{2}]. \quad (4.21)$$

Now, returning to the main argument, from (4.17), λ_1 is given by:

$$\lambda_1 = -\frac{\sin \zeta}{\cos(\zeta - \zeta_0)} \quad \text{and} \quad \dot{\lambda}_1 = -\dot{\zeta} \frac{\cos \zeta_0}{\cos^2(\zeta - \zeta_0)}. \quad (4.22)$$

We already determined that $\dot{\lambda}_1 = 0$, thus either $\dot{\zeta} = 0$, or $\zeta_0 = \pm \frac{\pi}{2}$ (which we exclude as it lies outside the range for ζ_0 that we consider in this proof). The only constant ζ satisfying the constraint in 4.5 is $\zeta = \zeta_0$ (if the angle the aerodynamic force makes is constant, and the angle of the average aerodynamic force is ζ_0 , the aerodynamic force must always have angle ζ_0). This produces the optimal aerodynamic power

$$\tilde{P}(\zeta_0) = P_{\max} \cos^3 \zeta_0. \quad (4.23)$$

Hence, any other trajectory with the same direction of average aerodynamic force ζ_0 will produce less power. \square

A few remarks are in order:

- N may be varied, for example by modifying the angle of attack, or reducing the surface area of the kite. However, as it affects \bar{F}_{aero} and P_{aero} proportionally, this does not affect the theorem. The largest possible value of N should be used in the bound (4.1) though; this will correspond to a particular angle of attack.
- Any deviation of the aerodynamic force from its average direction decreases the average aerodynamic power.
- The result can easily be extended to a system of kites by considering the sum of their power-producing potentials as expressed in the following corollary.

Corollary 1. (Maximum Power for Multiple Wings) Consider a system of n_w wings, each with area A_i and maximum power coefficient $C_{p,i}$, $i = 1, \dots, n_w$. Let ζ_0 be the angle the average total aerodynamic force makes with the wind. Then, over the period of time T , the average aerodynamic power generated by the system is upper bounded as follows:

$$\frac{1}{T} \int_0^T P_{\text{aero}} dt \leq \tilde{P}(\zeta_0), \quad (4.24)$$

where \tilde{P} is calculated, as for a single wing, from (3.11) and (4.23) using the overall wing parameters:

$$A = \sum_{i=1}^{n_w} A_i, \quad (4.25)$$

$$C_p = \frac{\sum_{i=1}^{n_w} A_i C_{p,i}}{A}. \quad (4.26)$$

Proof. A proof by contradiction can be constructed using Theorem 1. Let the aerodynamic force angle for each kite be $\zeta_i(t)$ and its velocity in this direction $v_i(t)$. Assume that, for a unit of time, the total aerodynamic power violates the bound, that is,

$$\frac{1}{T} \sum_{i=1}^{n_w} \int_0^T P_{\text{aero},i} dt > \left(\frac{1}{2} \rho |\vec{v}_w|^3 \right) AC_p \cos^3 \zeta_0, \quad (4.27)$$

where ζ_0 is the direction of the sum of the average aerodynamic forces $\frac{1}{T} \sum_{i=1}^{n_w} \int_0^T F_{\text{aero},i} dt$. Then, the same average power and the same average aerodynamic force can be produced using one wing with power coefficient C_p and area A by operating with $\zeta_i(t \frac{T}{T_i})$ and $v_i(t \frac{T}{T_i})$ for amounts of time $T_i = \frac{A_i C_{p,i}}{AC_p}$ for the total time $\sum_{i=1}^{n_w} T_i = 1$. The average power generated is then;

$$\int_0^1 P_{\text{aero}} dt = \sum_{i=1}^{n_w} T_i \frac{AC_p}{A_i C_{p,i}} \left(\frac{1}{T} \int_0^T P_{\text{aero},i} dt \right), \quad (4.28)$$

which by (4.27) is greater than

$$\left(\frac{1}{2} \rho |\vec{v}_w|^3 \right) AC_p \cos^3 \zeta_0. \quad (4.29)$$

Since by Theorem 1 this is impossible, statement (4.27) is false. \square

5. A Practical Upper Bound for a Generic Kite System

5.1. Effect of parameters on the overall power coefficient

The general upper bound P_{max} (3.11) gives the maximum power a wing with given properties can generate at a given wind speed. We will now show how the bound \tilde{P} implies that, for a practical kite-power system, the power produced by the wing(s) is likely to be much lower. This is because, in general, the direction of the restraining forces (transmitted using tethers) acting on the wing(s) are not aligned with the wind. An intuitive explanation is that the kinetic energy of an object (in this case the air) can only be fully removed by exerting a force directly opposing its motion. If the force is misaligned with the direction of motion, only a fraction of this energy can be removed. This can be quantified by a decrease in the upper bound. Indeed, the idea is to squeeze the upper bound P_{max} by taking into account the key parameters which occur in any kite system. Linking these parameters to the angle of the average aerodynamic force ζ_0 allows us to apply Theorem 1. As we are dealing with the *average* aerodynamic force, the result can be applied to both static and dynamic systems.

The generic airborne wind-power system we will consider is shown in Figure 4. The system is composed of two parts: a static tether and a ‘kite system’. The part designed to remove useful energy from the wind, the ‘kite system’, may be any combination of wings, tethers, weights or balloons. They may be static or in motion. Note that the static tether can be removed if not present in the system being analyzed. The parameters required for the following analysis are: the aerodynamic characteristics of the wings and the tether, the magnitude of the time-averaged aerodynamic force produced by the kite system $|\vec{F}_{\text{aero}}|$, the angle of the time-averaged force the tether exerts on the ground ϕ_{TG} , the total mass of all airborne components M , and the static-tether drag \vec{F}_{drag} . In the following the static-tether drag is assumed to be horizontal. If the vertical component of static-tether drag is significant (which is unlikely), it can be added to the total weight.

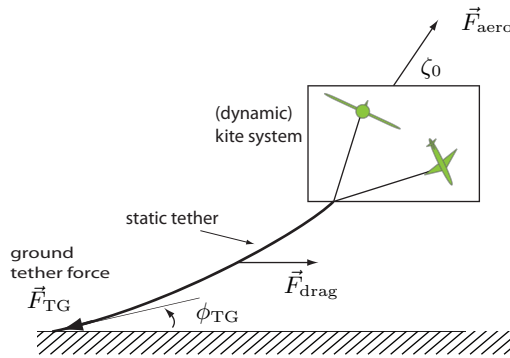


Figure 4: Generic airborne wind-power system.

These basic parameters will generally be known for a projected kite-power system. As the examples will show, even if they are not exactly known, good approximate values should be available. For example, ϕ_{TG} can be approximated by the average ground tether-angle in most cases. Based on the parameters, we derive an efficiency factor e that, when multiplied by P_{max} , gives the more accurate \tilde{P} upper bound for the system. The derivation is given in [Appendix A](#), here we simply state the result

$$e := \frac{\tilde{P}(\zeta_0)}{P_{\text{max}}} = \cos^3 \left(\phi_{\text{TG}} + \sin^{-1} \left(\frac{|\vec{F}_{\text{drag}}|}{|\vec{F}_{\text{aero}}|} \sin \phi_{\text{TG}} + \frac{Mg}{|\vec{F}_{\text{aero}}|} \cos \phi_{\text{TG}} \right) \right). \quad (5.1)$$

The maximum power the generic kite system can generate is therefore

$$\tilde{P} = eAC_p \left(\frac{1}{2} \rho |\vec{V}_w|^3 \right), \quad (5.2)$$

where A and C_p can be calculated from the wings’ characteristics using (4.25) and (4.26). Note that effects such as dynamic-tether drag should already be accounted for when calculating C_p (Houska and Diehl [13] derive a simple formula for incorporating dynamic tether drag into the kite’s glide ratio, yielding an ‘effective’ glide ratio). In the next section, this efficiency factor will be applied to several practical systems, but we first make some observations about the expression for e :

- $|\vec{F}_{\text{aero}}|$ will increase with wind speed, while M remains constant. Hence, the efficiency factor will improve as the wind speed increases. However, the system’s maximum load will be attained at the rated wind speed, and $|\vec{F}_{\text{aero}}|$ can increase no further. Hence, the minimum (best, in terms of efficiency) value of $\frac{Mg}{|\vec{F}_{\text{aero}}|}$ is determined by the system’s strength-to-weight ratio.
- Decreasing ϕ_{TG} will improve efficiency, if all other terms remain unchanged. However, decreasing ϕ_{TG} means a longer tether is required to access winds at a given height. A longer tether weighs more, and may generate more drag.

- The influence of static-tether drag is modulated by $\sin \phi_{\text{TG}}$. As ϕ_{TG} decreases, so does the influence of static-tether drag on the efficiency factor.

Hence, to maximize efficiency, the system's strength-to-weight ratio should be maximized and a compromise must be found between having a low ground tether angle, and minimizing tether weight and tether drag. Figures 5, 6 and 7 plot the efficiency factor vs. $|\vec{F}_{\text{drag}}|/|\vec{F}_{\text{aero}}|$ and $Mg/|\vec{F}_{\text{aero}}|$ for three different values of ϕ_{TG} . Examples of wing weight-to-strength ratios are 0.01 for a surf kite [8, 21], 0.06 for a Boeing 747 with no payload [5], and 0.33 for a DG-808C glider [9]. It can be seen that ϕ_{TG} strongly influences the efficiency, an angle of 45° resulting in a maximum efficiency of about 30%!

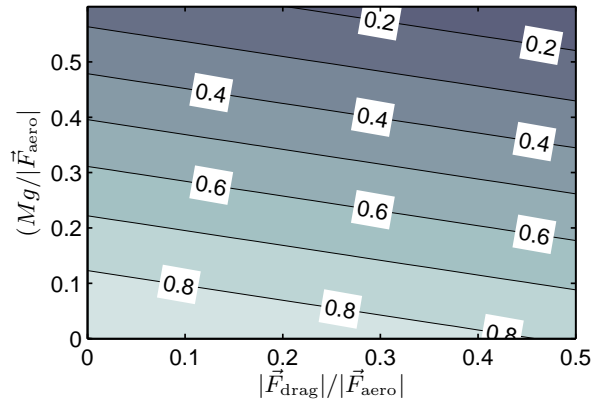


Figure 5: The efficiency factor e for $\phi_{\text{TG}} = 15^\circ$

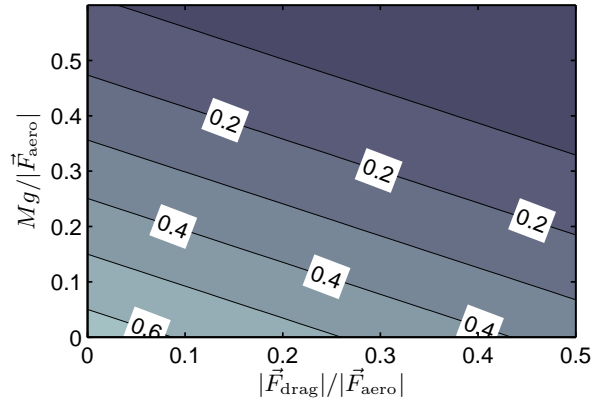


Figure 6: The efficiency factor e for $\phi_{\text{TG}} = 30^\circ$

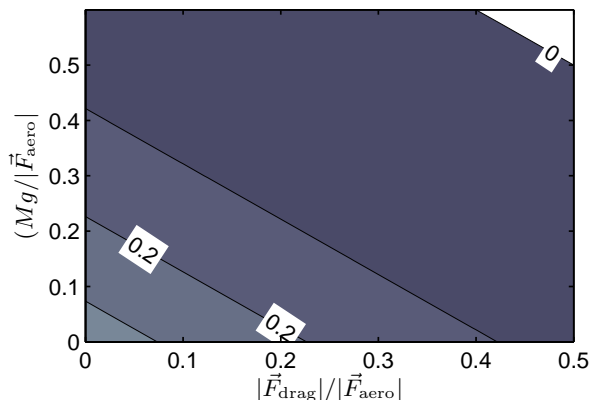


Figure 7: The efficiency factor e for $\phi_{TG} = 45^\circ$

5.2. Example: Computing bounds for kite-power systems

We now apply the preceding analysis to two systems currently being developed by companies. The parameters for the Skysails and Ampyx systems are based on data provided by the companies for their currently functioning systems/prototypes (both companies forecast future improvements).

1. Rigid Wings vs. Flexible Wings for Pumping-Cycle Generators.

The pumping cycle was first suggested by Loyd [14]. Several companies are currently developing this concept; here we study the systems proposed by Ampyx [1] and Skysails [17]. Both companies envisage flying a single wing, at heights of several hundred meters. A generator on the ground unwinds the tether to generate electricity. The main difference between the two systems lies in the wing: Skysails uses a flexible wing, similar to a surf kite; while Ampyx uses a rigid wing, similar to a glider. Skysails' flexible wing does not have a high glide ratio, but it has a very high strength-to-weight ratio. Ampyx's wing has a high glide ratio, but a far lower strength-to-weight ratio, and is undoubtedly more costly to manufacture per square meter of wing. Descriptions of both systems can be found on the company websites [1], [17]. The parameters are given in Table 1. Data provided by both companies are shaded and were used to make the following comparison as accurate as possible. With the exception of ϕ_{TG} , non-shaded parameters were calculated using the shaded ones. The average angle of the ground tether force was assumed to be $\phi_{TG} = 30^\circ$ for both systems, in order to ensure a fair comparison between them (this means they would be accessing wind at the same altitude).

In order to express the average aerodynamic force as a function of the maximum force the system can tolerate (given in Table 1), we introduce the load factor

$$\text{LF} = \frac{|\vec{F}_{\text{aero}}|}{|\vec{F}_{\text{aero}}|_{\text{max}}}. \quad (5.3)$$

Figure 8 shows how the systems' maximum capacities C_p are modulated by e for different values of LF. The result is that at full load the Ampyx system has the potential to be more 'efficient', due to its higher effective C_L/C_D . However, as the load factor decreases, Ampyx's efficiency decreases, whereas the Skysails system can maintain its efficiency even at very low load factors. As aerodynamic force is proportional to effective wind-speed squared, 50% of the full-load wind speed will result in a load factor of approximately 25%. Which wing is a better choice should thus depend not only on the cost per square meter of wing but also on the overall efficiency factor that can be expected, given the wind statistics at a particular site.

Table 1: Characteristics of the Ampyx and Skysails planned systems

Description	Parameter	Skysails	Ampyx
weight/strength system	$\left(\frac{Mg}{ \vec{F}_{\text{aero}} }\right)_{\min}$.02	.08
average ground tether angle	ϕ_{TG}	30°	30°
effective glide ratio (including dynamic tether drag)	$(C_L/C_D)_{\text{eff}}$	5	8.2
area of wing	A	400 m ²	3 m ²
mass of wing		320 kg	28 kg
maximum aerodynamic force	$ \vec{F}_{\text{aero}} _{\max}$	320 kN	3.5 kN
average tether (Dyneema) length	l_t	420 m	425 m
average wing height	h	250 m	200 m
tether diameter	d_t	30 mm	2.2 mm
average tether mass		380 kg	1.6 kg
average airborne mass	M	700 kg	29.6 kg

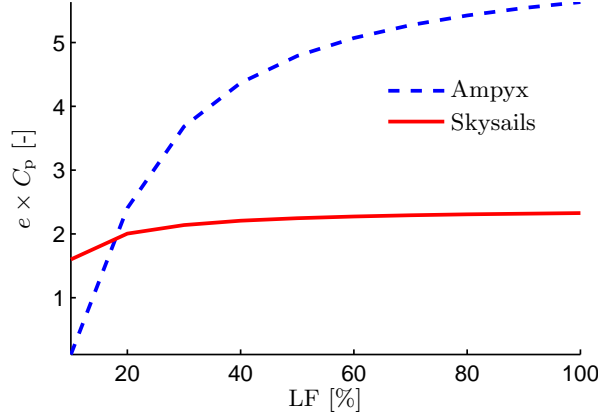


Figure 8: The number of times the wind's power density that can be produced per square meter of wing, for the Ampyx and Skysails systems.

2. Jet-Stream Wind Power.

The idea of accessing winds at altitudes of approximately 10 km is also currently under investigation [15]. To get a rough idea of the efficiency factor such a system could hope to have, we begin by assessing the weight-to-strength ratio of the tether:

$$\left(\frac{M_t g}{|\vec{F}_{\text{aero}}|}\right)_{\min} = \text{SF} \times \frac{g \rho_t l_t}{\sigma_t} = 0.29, \quad (5.4)$$

where M_t is the approximate tether mass, l_t is the approximate tether length of 15 km, $\rho_t = 970 \text{ kg/m}^3$ and $\sigma_t = 9 \times 10^9 \text{ N/m}^2$ are the density and tensile strength of ultra-high-molecular-weight polyethylene (UHMWPE) fiber such as Dyneema[®] (the strongest tether material currently available). SF is the tether safety factor of 6 (to give some perspective the Royal Netherlands Navy uses a safety factor of 7 for mooring vessels [16]). Assuming that the system of wings (for example a turbine) has a weight-to-strength ratio of about 0.05 (this is in between that of Skysail's and Ampyx's systems), we obtain

$$\left(\frac{Mg}{|\vec{F}_{\text{aero}}|}\right)_{\min} \simeq 0.35. \quad (5.5)$$

So, even without taking tether-drag into account, we can see from Figure 6 that at a tether angle of 30° the *maximum* efficiency factor is 0.3. Additionally, this will deteriorate rapidly if the system is not operating at full load. For example, at half of full load (or 70 % of the full-load wind speed) the maximum efficiency factor is 0.1. The system proposed by Roberts et al. [15] relies on having a generator in the air. This can be expected to significantly increase the system's weight, further reducing the efficiency. Based on this analysis, it is questionable whether such a system would be commercially viable.

6. Conclusions

This paper has derived the general upper bound P_{\max} for the power a wing can extract from the wind. Fundamentally, this is based on the fact that the only real source of energy in a kite-power system is the wind. Hence, while variations in potential or kinetic energy of the overall system may cause energy storage/release, net power must come solely from the movement of the wings under the influence of aerodynamic forces. A second upper bound, \tilde{P} , was derived. This bound is in general much more restrictive (and hence accurate) than P_{\max} . Its calculation requires the angle of the average restraining force on the kite system to be known. The use of the *average* restraining force allows the bound to be applied to dynamic kite systems, something that was previously not possible.

It was also shown how to use basic system parameters to compute this bound for a generic high-altitude wind-power system. In particular, the load factor the system operates at and the system weight were shown to strongly affect efficiency. As an example, the analysis was applied to two concrete scenarios, namely, pumping-cycle generators and jet-stream power. Parameters from existing industrial prototypes were used. The results show that, while rigid wings are generally more efficient at full load due to their superior glide ratio, flexible wings are much lighter and conserve their efficiency as the load factor decreases. The feasibility of jet-stream wind power was also examined. In this case, our conclusion is rather sobering: efficiency factors lower than 0.3 are likely.

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Appendix A. Derivation of the Efficiency Factor

The average aerodynamic force must balance the weight of all airborne components, Mg , and the tension in the tether at the ground (assuming on average the system is not accelerating):

$$|\vec{F}_{\text{aero}}| \begin{bmatrix} \cos \zeta_0 \\ \sin \zeta_0 \end{bmatrix} - |\vec{F}_{\text{TG}}| \begin{bmatrix} \cos \phi_{\text{TG}} \\ \sin \phi_{\text{TG}} \end{bmatrix} + \begin{bmatrix} |\vec{F}_{\text{drag}}| \\ Mg \end{bmatrix} = 0. \quad (\text{A.1})$$

Multiplying this equation by $[\sin \phi_{\text{TG}} \quad -\cos \phi_{\text{TG}}]$ gives

$$|\vec{F}_{\text{aero}}| (\cos \zeta_0 \sin \phi_{\text{TG}} - \sin \zeta_0 \cos \phi_{\text{TG}}) + |\vec{F}_{\text{drag}}| \sin \phi_{\text{TG}} + Mg \cos \phi_{\text{TG}} = 0 \quad (\text{A.2})$$

$$\implies \sin(\zeta_0 - \phi_{\text{TG}}) = \frac{|\vec{F}_{\text{drag}}|}{|\vec{F}_{\text{aero}}|} \sin \phi_{\text{TG}} - \frac{Mg}{|\vec{F}_{\text{aero}}|} \cos \phi_{\text{TG}}. \quad (\text{A.3})$$

As $\tilde{P}(\zeta_0) = \cos^3(\zeta_0)P_{\text{max}}$, (5.1) follows directly.

Appendix B. The Orientation of the Wing that Maximizes Power

To define the orientation of the wing, we will need to use a body reference frame (X, Y, Z) that is fixed to the wing. More than one convention exist in aerodynamics; the reference frame we employ is shown in Figure B.9. We then define the wing’s orientation relative to a reference orientation. The reference orientation we will use is with the X axis pointed into the apparent wind. The position of the Y/Z axes is arbitrary. The orientation is then given by three Euler angles, (ψ, β, α) , in the XZY convention:

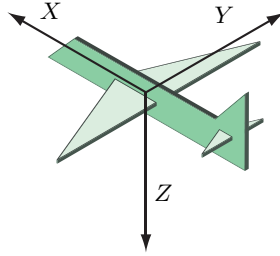


Figure B.9: Body reference frame for the wing. The X axis is aligned with the wing chord.

- (a) **Rolling:** Rotate the body frame around X by ψ . This rotates the lift force in the ZY plane; steering in airplanes and many kites is performed principally in this manner.
- (b) **Yawing:** Rotate the body frame around Z by β , this is termed sideslip.
- (c) **Pitching:** Rotate the body frame around Y by α , this is the angle of attack.

The three rotations are shown in Figure B.10.

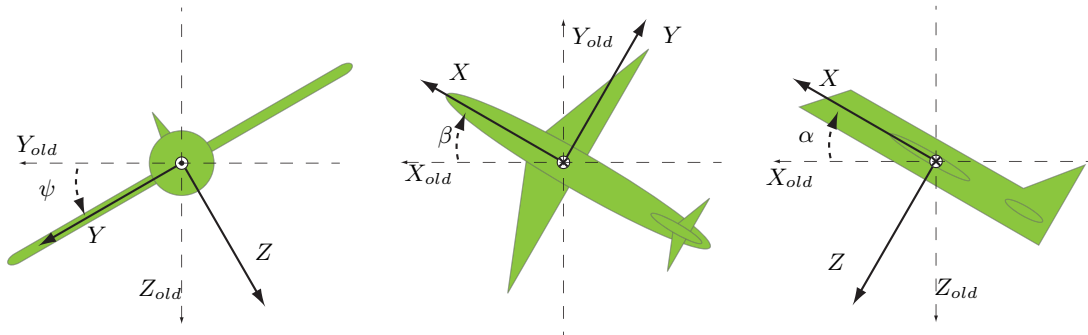


Figure B.10: Rotations specifying the orientation of a wing.

Wings are generally designed to operate most efficiently (achieve the greatest lift to drag ratio) for zero sideslip, i.e. $\beta = 0$. The roll angle as defined here does not affect the magnitude of aerodynamic forces. The question is then: what angle of attack maximizes the power coefficient C_p ? Figure B.11 shows how C_p varies w.r.t. α for a standard wing. From analyzing a number of wings, we can conclude that C_p is maximized for an angle of attack slightly greater than the maximum C_L/C_D point, for $C_L \simeq 1$.

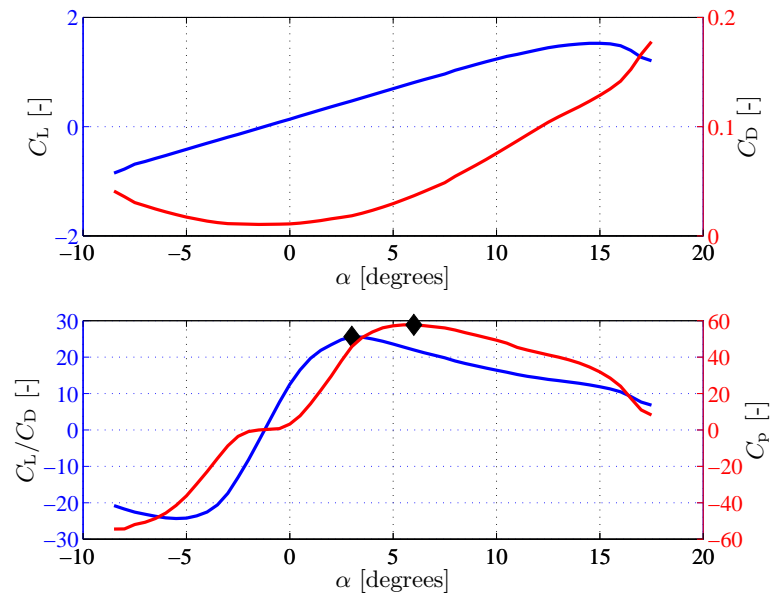


Figure B.11: Data generated using Xfoil (a widely used package for simulating wind-tunnel tests) for the Boeing 737 root airfoil with an aspect ratio of 8. The maximum $\frac{C_L}{C_D}$ and C_p are marked.