NON-LINEAR MODELING OF A BROADBAND SLIC FOR ADSL-LITE-OVER-POTS USING HARMONIC ANALYSIS

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ABSTRACT

A new frequency domain based identification algorithm for the discrete-time Wiener model is presented. The Wiener model belongs to the class of Volterra models with factorizable kernels. Opposed to the general Volterra model, it is shown that the Wiener model can be uniquely identified, using harmonic analysis only. Therefore the given identification scheme is very suitable for real world applications, where only measurements from the spectrum analyzer are available. The derived method is applied to model large scale analog circuits. A broadband SLIC (subscriber line interface circuit) for an ADSL-Lite-over-POTS (asymmetric digital subscriber line, plain old telephone service) central office application is considered. Data acquisition is done by an analog network simulator. Simulation results for an IIR (infinite impulse response) filter representation of the Wiener model are compared to results obtained from the network simulator and to measurements from the laboratory. Emphasis is put on the accuracy of the intermodulation products, occurring when a multi-tone signal is applied. To improve the model performance, a non-linear optimization procedure in the time domain is considered.

1. INTRODUCTION

The investigation is concerned with identification of real world linear systems showing small non-linear distortions. These systems occur frequently in analog integrated circuit designs. Attempts to model these systems by Volterra-like series expansions often leads to many significant terms in the expansion i.e., it leads to high order of non-linearity N. For the considered circuit, the broadband SLIC, the observed significant order N is nine. For such cases Volterra models are not applicable because the number of model parameters exponentially increases with N. Therefore simplified models like the Wiener model have to be considered.

When applying a DMT (discrete multi tone) modulated signal to an ADSL transceiver with non-linear distortions, intermodulations of carriers occur, which do limit the data performance of the ADSL system. The number of bits being allocated to each single carrier determines the overall bit-rate of the system. A proper quantity to describe the possible bit-allocation for each carrier is the MTPR value (multi-tone power ratio), a generalized SNR (signal to noise ratio). The MTPR is defined as

$$MTPR_i = 10\log\left(\frac{S_i}{N_i + \sum_j D_{ij}}\right) \quad \text{in dB}, \qquad (1)$$

where the index i denotes the i-th carrier. S_i stands for the transmitted power of the i-th carrier, which has to be related to the sum of the noise N_i and all the inter-modulations D_{ij}, produced from the other j carriers of the DMT signal

$$u(t) = \sum_{k=n_1}^{n_2} A_k \cos(k\omega_0 t + \varphi_k). \qquad (2)$$

DMT signals, as given in (2), have different peak values, depending on their phase distribution $\{\varphi_k\}$. These peak values are described by the crest factor, which is defined as the ratio of the l_∞-norm to the l_2-norm of the signal $u(t)$.

The motivation for that specific identification problem is to estimate the performance of the overall ADSL system during the design phase of the circuit. Therefore a non-linear discrete-time model of the transceiver, that is accurate in the MTPR value, is required.

Section 2 determines the Wiener model, section 3 outlines the new identification algorithm. The presented algorithms are then applied to identify the broadband SLIC in section 4. Finally, conclusions are drawn in section 5.

2. MODEL STRUCTURE

In the following, the Wiener model is derived and its relation to Volterra series is pointed out. For the sake of generality the investigations are mainly done in the continuous time domain.

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Every stable, causal and non-linear operator with fading memory [1] $\mathbf{K}$, governing the I/O relation

$$y(t) = \mathbf{K}u(t),$$

can be represented by a Volterra series

$$y(t) = h_0 + \sum_{n=1}^{\infty} \int_{\mathbb{R}^n} h_n(\sigma_1, \dots, \sigma_n) \prod_{i=1}^{n} u(t - \sigma_i) d^n\sigma \quad . \tag{3}$$

Assuming that there is no additional DC-offset in the output $y(t)$, the constant h_0 is set to zero in the following. If the kernels $h_n(\sigma_1, \dots, \sigma_n)$ in (3) are full factorizable through

$$h_n(\sigma_1, \dots, \sigma_n) = \prod_{i=1}^{n} h_n(\sigma_i),$$

the Volterra series (3) reduces to the Wiener model

$$y(t) = \sum_{n=1}^{\infty} \left[\int_{\mathbb{R}} h_n(\sigma) u(t - \sigma) d\sigma \right]^n . \tag{4}$$

Realizations of the linear part of the Wiener model through FIR (finite impulse response) and IIR filters are given by

$$y(l) = \sum_{n=1}^{N} \left[\sum_{k=0}^{M_n} h_n(k) u(l - k) \right]^n \tag{5}$$

and

$$y(l) = \sum_{n=1}^{N} \left[\sum_{k=0}^{M_n} b_n(k) u(l - k) - \sum_{k=1}^{\bar{M}_n} a_n(k) y_n(l - k) \right]^n , \tag{6}$$

respectively. With $\bar{M}_n$ and M_n denoting the filter orders, N the order of the non-linearities and y_n the output of the n-th linear dynamic part given by the convolution in (4).

3. HARMONIC ANALYSIS

The steady state response of the Wiener model to a single-tone input is investigated. It will be shown that the Fourier-transform $H_n(\omega)$ of the kernel $h_n(\sigma)$ in (4) can be identified through harmonic analysis. Subsequently an approximate method to find $H_n(\omega)$ is given. This method utilizes only absolute values from harmonic analysis and is considered especially for practical use, when absolute values from a spectrum analyzer are available.

For an input

$$u(t) = 2A\cos(\omega_0 t) = Ae^{i\omega_0 t} + Ae^{-i\omega_0 t}, \tag{7}$$

with the definition

$$H_n(\omega) \equiv \int_{-\infty}^{\infty} h_n(\sigma) e^{-i\omega\sigma} d\sigma, \tag{8}$$

the steady state response of (4) can be written as

$$y(t) = \sum_{n=1}^{\infty} A^n \sum_{k=0}^{n} \binom{n}{k} H_n(\omega_0)^k H_n^*(\omega_0)^{(n-k)} e^{i(2k-n)\omega_0 t}, \tag{9}$$

where $H_n^*(\omega_0)$ denotes the complex conjugate of $H_n(\omega_0)$. With $\varphi_n = \arg(H_n(\omega_0))$ and ordering terms with the same exponent $2k - n$, equation (9) can be expressed as a Fourier-series

$$y(t) = \sum_{k=-\infty}^{\infty} f_k(A, \omega_0) e^{ik\omega_0 t}, \tag{10}$$

with the Fourier-coefficients

$$f_k(A, \omega_0) = \sum_{n=k}^{\infty} \frac{n!}{\left(\frac{n-k}{2}\right)! \left(\frac{n+k}{2}\right)!} A^n |H_n(\omega_0)|^n e^{ik\varphi_n}, \tag{11}$$

for $k \geq 0$ and $f_{-k}(A, \omega_0) = f_k^*(A, \omega_0)$. The summation in (11) has to be performed for indices n with the same parity as k. Equation (10) with (11) determines the response of the Wiener model (4) to a single-tone input (7).

3.1. Parameter estimation

To extract $H_n(\omega_0)$ from the Fourier-coefficients given in (11), the Fourier-transform, as defined in (8), of the system response (10) is regarded. Each harmonic k in the output spectrum gives the contribution

$$Y(k\omega_0) = 2\pi f_k(A, \omega_0). \tag{12}$$

For finite order N of non-linearity, equations (10) and (11) are written as

$$y(t) = \sum_{k=-N}^{N} f_k(A, \omega_0) e^{ik\omega_0 t}, \tag{13}$$

$$f_k(A, \omega_0) = \sum_{n=k}^{\bar{N}} \frac{n!}{\left(\frac{n-k}{2}\right)! \left(\frac{n+k}{2}\right)!} A^n |H_n(\omega_0)|^n e^{ik\varphi_n}, \tag{14}$$

with $\bar{N}$ denoting the highest order of non-linearity, having the same parity as k i.e., $\bar{N}$ is either N or $N - 1$. Each $f_k(A, \omega_0)$ is now constructed by a finite sum. From equation (12) one gets for $k = N$

$$Y(N\omega_0) = 2\pi \left[AH_N(\omega_0) \right]^N . \tag{15}$$

Obviously, $|H_N(\omega_0)|$ and φ_N are determined by $Y(N\omega_0)$. Setting $k = N - 2$, equation (12) results in

$$Y((N-2)\omega_0) = 2\pi \left(\left[AH_{N-2}(\omega_0) \right]^{N-2} + N|AH_N(\omega_0)|^N e^{i(N-2)\varphi_N} \right) .$$

From (16), the terms $|H_{N-2}(\omega_0)|$ and φ_{N-2} can be extracted, making use of (15). This scheme can be applied recursively for all $H_n(\omega)$, over the relevant frequency range i.e., to determine the responses $H_n(\omega)$ a frequency sweep at a given amplitude A has to be performed. The recursive procedure represents an exact identification method for the Wiener model. The accuracy of the procedure depends on the assumption that there are no higher harmonics in the spectrum than the model order N i.e., no undermodeling is given. In real world problems, as non-linear analog circuits, this assumption is not valid in general. The impact of higher order harmonics, which are not modeled, to the accuracy of the proposed algorithm will be subject of further analysis.

The following approximate method to determine the Wiener model parameters can be used if only absolute values of $Y(k\omega_0)$ are available. This includes the assumption, that the phase shifts of all $H_n(\omega_0)$ are approximately the same i.e., $\varphi = \varphi_n$, $n = 1, \ldots, N$. Separating the common factor $e^{ik\varphi_n}$ from (14) and denoting the rest of (14) as $\bar{f}_k(A, \omega_0)$ (13) gives

$$y(t) = \sum_{k=-N}^{N} \bar{f}_k(A, \omega_0) e^{ik(\omega_0 t + \varphi)}, \qquad (16)$$

where $\bar{f}_k(A, \omega_0)$ is real and (12) simplifies to

$$|Y(N\omega_0)| = 2\pi \bar{f}_k(A, \omega_0). \qquad (17)$$

Denoting

$$\mathbf{Y} = [|Y(\omega_0)|, \quad |Y(2\omega_0)|, \quad \cdots \quad , |Y(N\omega_0)|]^T$$

and

$$\mathbf{H} = \left[|H_1(\omega_0)|, \quad |H_2(\omega_0)|^2, \quad \cdots \quad , |H_N(\omega_0)|^N\right]^T,$$

the relation between the $|H_n(\omega_0)|$ and $|Y(k\omega_0)|$ can be written as

$$\mathbf{Y} = \mathbf{A}\mathbf{H},$$

with the upper-triangular amplitude matrix $\mathbf{A}$

$$2\pi \begin{pmatrix} A & 0 & \frac{3!}{1!2!}A^3 & 0 & \frac{5!}{2!3!}A^5 & 0 & \cdots \\ 0 & A^2 & 0 & \frac{4!}{1!3!}A^4 & 0 & \frac{6!}{2!4!}A^6 & \\ 0 & 0 & A^3 & 0 & \frac{5!}{1!4!}A^5 & 0 & \\ 0 & 0 & 0 & A^4 & 0 & \frac{6!}{1!5!}A^6 & \\ 0 & 0 & 0 & 0 & A^5 & 0 & \\ 0 & 0 & 0 & 0 & 0 & A^6 & \\ \vdots & & & & & & \ddots \end{pmatrix}.$$

Matrix inversion yields the vector with powers of the transfer-functions $|H_n(\omega_0)|$ of the Wiener model

$$\mathbf{H} = \mathbf{A}^{-1}\mathbf{Y}.$$

In general, the obtained $|H_n(\omega)|$ can be approximated by FIR (5) or by IIR (6) filters, using standard methods.

4. APPLICATION

The section deals with the application of the derived identification algorithms to real world systems which, in general, do not exhibit a Wiener model structure but can be approximated by a Wiener model. In section 3, it has been shown that a Wiener model can be uniquely identified when excited with single tones at different frequencies. For general non-linear system, this excitation is not sufficient to garantee a unique identification of the system [2, 3]. Therefore, the derived Wiener model approximation for a general non-linear system through the method presented in section 3 is only optimal for single tone inputs. For other inputs the approximation will in general be suboptimal.

To optimize the model parameters for a specific signal form, time domain input/output data for this signal form has to be available. For the considered ADSL application this is outlined below.

4.1. Time domain optimization

The Wiener model parameters can be made optimal for a specific input signal form by minimizing the MSE (mean square error) between the unkown reference system output and the Wiener model output to the given specific input signal. For the broadband SLIC these signals are broadband DMT signals of (2), with $n_1 = 33$, $n_2 = 127$, $A_k = A$ $\forall k$. The crest factor is chosen to be the mean crest factor ($CF_{mean} = 3.3$) produced with a uniform random distribution of the φ_k in (2). The objective function J for the optimization is given by

$$J = \sum_n [y_R(n) - y(n)]^2, \qquad (18)$$

where $y(n)$ comes from (5) or (6). $y_R(n)$ denotes the output signal of the unkown reference system i.e., the broadband SLIC. In general a non-linear optimization yields reasonable results only with a good initialization of the parameters. Thus, the parameters derived in section 3 can serve as initialization of (18). In addition to the objective function J, one can also provide the gradient and the Hessian matrix of (18) in an analytical form for the FIR case (5), which enables one to apply Quasi-Newton methods [4]. With these methods a better convergence concerning speed and accuracy can be obtained.

4.2. Broadband SLIC

A detailed explanation of the broadband SLIC is given in [5]. In short, it serves as a transceiver for an ADSL-Lite-over-POTS central office application. Its non-linear model consists of the transmit path (K_B), the receive path (K_S) and the echo compensation path (K_{BF}). Fig. 1 shows the overall model of the broadband SLIC, constituted by the non-linear operators K_S, K_{BF}, K_B, and a linear line model, given by the impuls-responses h_1, h_2, h_3 and h_4. This model is used for further investigations, concerning the performance of the ADSL system.

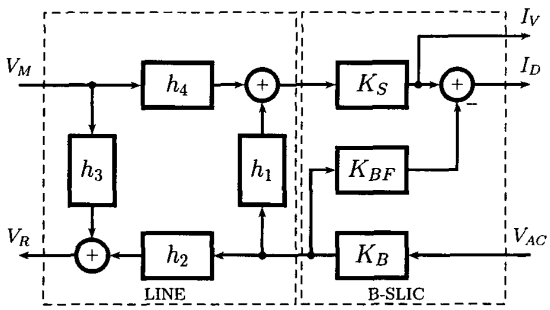

Fig. 1. Linear line model and non-linear model of the broadband SLIC.

4.3. Results

Identification results are given for the most complex part of the broadband SLIC, the buffer K_B, and they are compared to measurements from the laboratory. The operator K_B is approximated by a Wiener model, which has been implemented through an IIR filter structure.

For the harmonic analysis of section 3, single-tone simulations in the range from 4312.5 Hz to 552 kHz are performed. The amplitude of the single-tones is chosen to match the mean crest factor of a DMT signal (2), with $n_1 = 33$ and $n_2 = 127$, used for data transmission. The harmonics $Y(k\omega_0)$ up to the order of nine are extracted from the spectrum and the discussed algorithm of section 3 is applied. In addition to this, a simulation with a DMT signal, $n_1 = 33$ and $n_2 = 127$, and mean crest factor is performed for the application of the time domain optimization described in section 4. All simulations are done for a sampling frequency of 17.664 MHz.

MTPR evaluations corresponding to (1) are performed with the implementation on silicon and with the Wiener model. In Tab. 1, a comparison between measurements, done in the laboratory, and simulations, done with the obtained models, is given. MTPR values averaged over all carriers are presented, where the lab measurements are serving as a frame of reference. The following abbreviations are used: "LAB" - laboratory, "NS" - network simulator, "HA"- harmonic analysis, "TDO" - time domain optimization. From Tab. 1 it can be seen, that the results obtained from the Wiener model, together with the presented identification methods, are in good agreement with those obtained from measurements done with the implementation on silicon.

	LAB	NS	HA	TDO
MSE	-	0	4.5E-5	2.4E-5
MTPR [dB]	64.7	63.3	65.8	64.3
MTPR error [dB]	0	1.4	1.1	0.4

Table 1. Simulation results for the transmit path K_B versus measurements. Average values.

5. CONCLUSION

Non-linear black-box modeling is applied successfully to a large scale analog circuit, namely a broadband SLIC used in an ADSL-Lite-over-POTS central office application.

A new identification method for the Wiener model is presented, which in the minimum case requires only absolute values of the harmonics, occurring in the spectrum of the system response. It is shown, that the Wiener model can be uniquely identified using only single-tones. To find the optimal Wiener model approximation to a general nonlinear system a time domain optimization of the model parameters can be applied.

The models obtained with these methods, satisfy the required accuracy for the MTPR values.

It can be concluded that the Wiener model and the presented algorithm using harmonic analysis is an expedient method to map the non-linear dynamics of large scale analog circuits with high accuracy to a fast and easy to handle discrete-time model.

6. REFERENCES

[1] S. Boyd and L. O. Chua, "Fading memory and the problem of approximation nonlinear operators with Volterra series," *IEEE Trans. on Circuits and Systems*, vol. 32, no. 11, pp. 1150–1161, November 1985.

[2] R. D. Novak and B. Van Veen, "Random and pseudorandom inputs for Volterra filter identification," *IEEE Transactions on Signal Processing*, vol. 42, no. 8, pp. 213–313, August 1994.

[3] S. Boyd and L. O. Chua, "Measuring Volterra kernels," *IEEE Trans. Circuits and Systems*, vol. 30, no. 8, pp. 571–577, August 1983.

[4] W. H. Press and S. A. Teukolsky, *Numerical Recipes in C : The Art of Scientific Computing*, Cambridge University Press, London, 1993.

[5] B. Zojer, R. Koban, J. Pichler, and G. Paoli, "A broadband high-voltage SLIC for a splitter- and transformerless combined ADSL-Lite/POTS linecard," *IEEE Journal of Solid State Circuits*, vol. 35, no. 12, pp. 1976–1988, December 2000.