Measurements of Cavitation Compliance in the Draft Tube Cone of a Reduced Scale Francis Turbine Operating at Part Load

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Abstract

Francis turbines operating at part load conditions typically experience a cavitation vortex rope immediately at the runner outlet. As the compliance of this cavitation flow is much higher than the one in cavitation-free conditions, a decrease in the value of the eigen frequencies of the hydraulic circuit is observed. One of the eigen frequencies can eventually match the frequency of the vortex precession, which acts as an excitation source for the hydraulic system, inducing strong pressure pulsations and output power swings. To predict such undesirable resonance conditions in hydropower plants, the cavitation compliance in the draft tube cone can be identified beforehand at the model scale. This paper presents the identification of the cavitation compliance on a reduced scale model of a Francis turbine over a wide range of part load operating conditions, for different Thoma and Froude numbers. The first eigen frequency of the test rig is firstly identified by modal analysis. The cavitation compliance is then defined by adjusting a 1-D numerical model of the test rig to match this first eigen frequency. These compliance results could then be transposed to the prototype scale, enabling the prediction of the first eigen frequency of the hydropower plant in any part load condition.

Keywords: Cavitation vortex rope; resonance; Francis turbine; hydro-acoustic model; cavitation compliance; pressure pulsations

Introduction

At off-design conditions, Francis turbines experience the development of a swirling flow in the draft tube, which intensity depends mainly on the value of the flow discharge. At part load conditions, with a discharge ranging usually from about 50 % to 90 % of the rated value, a vortex breakdown arises immediately at the runner outlet and produces a flow recirculation zone in the draft tube, accompanied by a helical cavitation vortex rope.

The precession of this vortex acts as a pressure excitation source for the system, leading to the propagation of synchronous pressure pulsations in all the hydraulic circuit [1]. Further consequence of the cavitation vortex development is the strong impact on the eigen frequencies of the hydraulic circuit, as it increases the flow compliance inside the draft tube [2]. One of these eigen frequency values may match the frequency of the synchronous pressure pulsations, causing resonance phenomena and potentially electrical output power swings of the generating unit [3]. To predict these undesirable resonance conditions at the prototype scale, the hydro-acoustic parameters describing the draft tube cavitation flow in transient 1-D numerical simulations must be known. They can be determined by performing hydro-acoustic modal analysis of a test rig facility containing the reduced scale physical model of the Francis turbine prototype operating in similar conditions. Once the experimental eigen frequencies of the test rig are known, the parameters of the 1-D hydro-acoustic numerical model of the test rig, including the draft tube cavitation flow, can be adjusted to match the measurements [4].

In this paper, the hydro-acoustic parameters of the hydraulic circuit are initially adjusted by the experimental results obtained in cavitation-free conditions. Measurements are then performed at part load conditions, for which the cavitation vortex rope is experienced. The compliance of the draft tube cavitation flow is then modified in the 1-D numerical model to match the experiments, while the hydro-acoustic parameters of the other parts of the test rig remains unchanged. Finally, the obtained values of cavitation compliance are presented and comparisons are made between different IEC speed and discharge factors, n_{ED} and Q_{ED} respectively, and Thoma and Froude numbers, σ and Fr respectively (definition of these factors and numbers can be found in [5]). The possibility of using the swirl number S to reduce the number of measurements at different n_{ED} values is then analyzed and discussed.

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Resonance in part load conditions

For a given longitudinal position x along the draft tube cone center-line, the local plane wave speed a can be calculated as in Equation (1). The variable C_{eq} represents the equivalent compliance of the draft tube, including the cavitation vortex, the water and the pipe, as presented in Equation (2). The cavitation compliance C_c is defined in Equation (3), [6].

$$a(x)=\sqrt{\frac{gA_p(x)dx}{C_{eq}(x)}} \tag{1}$$

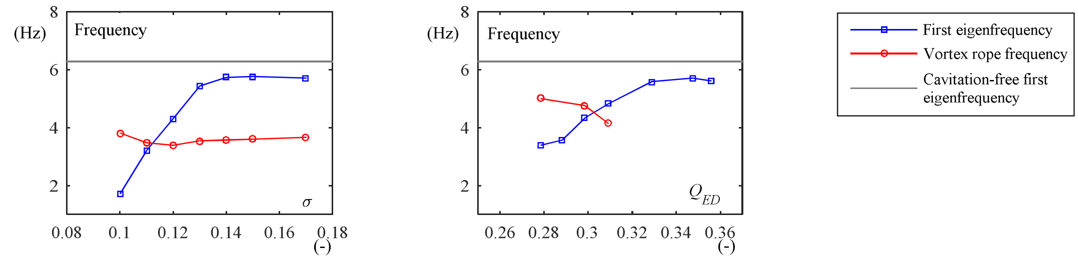
$$C_{eq}=C_c+C_{water}+C_{pipe} \tag{2}$$

$$C_c(x)=-\frac{\partial V_c(x)}{\partial h}=-dx\frac{\partial A_c(x)}{\partial h} \tag{3}$$

where g is the local gravity, A_p is the pipe cross-section, A_c is the cavitation vortex cross section and h is the piezometric head.

Equation (1) implies that higher values of C_{eq} lead to lower wave speeds. As the wave speed in cavitation conditions is much smaller than the one in cavitation-free conditions, the approximation $C_{eq}\approx C_c$ can be made. Consequently, the cavitation compliance becomes the most important factor determining the wave speed that, in its turn, will have a major impact on the value of test rig eigen frequencies.

In Figure 1(a), the influence of the cavitation Thoma number on both the first eigen frequency of the test rig and the vortex rope frequency is presented. The influence of the discharge factor on both frequency values is given in Figure 1(b). For the conditions presented in this figure, the vortex rope frequency remained approximately constant for any σ and presented a small decrease with the increase in Q_{ED}. Furthermore the first eigen frequency of the test rig presents an important decrease for both lower σ and Q_{ED} values. In both cases, resonance occurs when the first eigen frequency of the test rig and the precession frequency of the vortex match, notably for $\sigma=0.112$ and $Q_{ED}=0.303$ in the shown test cases. It is reminded, however, that this kind of resonance at part load is commonly encountered during reduced scale model tests at low σ and does not necessarily occur on the prototype, as the plant and the turbine can be properly designed to avoid this type of risk.



(a) $Q_{ED}=0.282$, $n_{ED}=0.478$, Fr=5.6 (b) $\sigma=0.15$, $n_{ED}=0.532$, Fr = 5.6

Figure 1 – Influence of the Thoma number (a) and discharge factor (b) on both the first eigen frequency of the test rig and the vortex rope frequency.

Experimental setup and cavitation compliance determination

The methodology to determine the first eigen frequency and the cavitation compliance C_c is similar to the one described by Landry et al. in [4]. For the present study, the two 400 kW centrifugal pumps are installed in parallel and the test case is a reduced scale model of a Francis turbine with specific speed N_{QE} = 0.28 and a reference diameter D = 0.35m.

An external excitation system is used to identify the first eigen frequency of the test rig. It consists of a rotating valve that is installed connecting one pipe at the turbine upstream pipe to a high-pressure reservoir, allowing the injection of periodical fluctuations at a given frequency into the test rig. By using pressure sensors installed along the hydraulic circuit, the first eigen frequency is identified as being the frequency leading to the maximum response.

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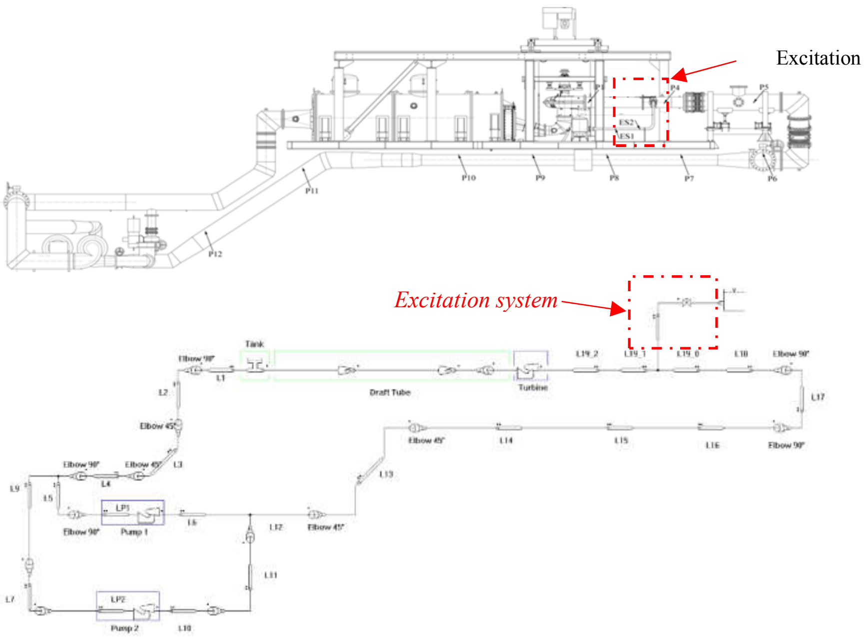

a)

b)

Figure 2 – Complete test rig and excitation system experimental setup (a) and the 1-D numerical model (b).

The compliance and bulk viscosity values of the test rig pipes are first identified by adjusting their values in the 1-D numerical model to match the first eigen frequency and the system response of the test rig obtained experimentally in a cavitation-free condition. The value of first eigen frequency in cavitation-free conditions is 6.28 Hz and is indicated by the grey line in Figure 1.

In cavitation conditions, a lower eigen frequency is made apparent and the cavitation compliance values in the first part of the draft tube of the numerical model is adjusted to obtain the same value of eigen frequency. The cavitation compliance is applied to the first one meter long ($2.86 \times D$) upper part of the draft tube, starting from the runner outlet until the beginning of the draft tube elbow. The remaining parts of the draft tube (roughly half of the elbow and the diffuser), and the rest of the test rig keep the same compliance as in cavitation-free condition. The bulk viscosity remains unchanged and equal to 0.4818 MPa·s in the full circuit. More details on the 1-D model applied in this paper can be found in [6] and [7]. General aspects and discussions on modeling the cavitation compliance can be found in [8].

The investigated operating conditions are summarized in Table 1. Firstly, it presents the cavitation-free operating condition used to calibrate the model. Secondly, it presents the tests where the influence on the discharge factor variation is focused. These tests are made for a combination of two values of Thoma number, $\sigma_{\min}$ and $\sigma_{\max}$, two Froude numbers, and five different speed factors. The two Thoma numbers, $\sigma_{\min}$ and $\sigma_{\max}$, are defined using a similar approach to the one adopted in [9]. The last line of Table 1 presents additional measurements focused on the effects of the Thoma number variation on the cavitation compliance.

Swirl number

The swirl number S is defined as the ratio between the axial flux of angular momentum and the axial flux of axial momentum and was originally proposed by Gupta et al. [10]. As the swirl intensity and the cavitation compliance in a vortex rope are strongly related, Favrel et al. proposed Equation (4) for the calculation of the swirl number at the Francis turbines outlet [9]. As presented in [9], points with similar σ, same Froude number and varying n_{ED} presented resonance conditions when operating at the same swirl number.

$$S = n_{\mathrm{ED}} \frac{\pi^2}{8} \left(\frac{1}{Q_{\mathrm{ED}}} - \frac{1}{Q_{\mathrm{ED}}^{S=0}} \right) \tag{4}$$

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Table 1 – Tested operating conditions

	Fr (-)	n_{ED} (-)	Q_{ED} (-)	σ (-)
Cavitation-free test	5.6	0.478	0.282	0.917
Q_{ED} variation tests	5.6 and 7.10	0.457	0.183 → 0.287	$\sigma_{min} = 0.110$ and $\sigma_{max} = 0.128$
		0.478	0.204 → 0.322	$\sigma_{min} = 0.120$ and $\sigma_{max} = 0.140$
		0.506	0.229 → 0.314	$\sigma_{min} = 0.134$ and $\sigma_{max} = 0.157$
		0.532	0.245 → 0.356	$\sigma_{min} = 0.149$ and $\sigma_{max} = 0.174$
σ variation tests	5.6	0.478	0.220 and 0.282	0.10 → 0.17

Measurements results and discussion

The first eigen frequency measured on the test rig and the average wave speed in the draft tube cone as a function of the cavitation compliance are presented in Figure 3. The relation between C_c and the first eigen frequency is provided by the 1-D numerical model and the local wave speed is calculated as in Equation (1). The graph contains all the operating conditions described in the Table 1. With the exception of the result in cavitation-free condition, all the eigen frequency data collapses into one single curve, regardless of variations in discharge, rotation speed, Froude and Thoma numbers. The presented average wave speed values are below 50 m/s, far lower than the value in cavitation-free conditions, for which a wave speed above 750 m/s is expected.

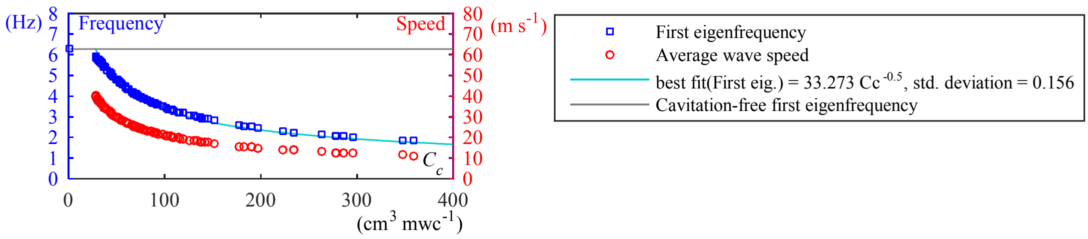

Figure 3 – First eigen frequency of the test rig and mean wave speed as a function of the cavitation compliance.

In Figure 4, the influence of the discharge factor (Figure 4(a)) and the swirl number (Figure 4(b)) on the value of C_c is presented for four different n_{ED} values with $\mathrm{Fr} = 5.6$ and $\sigma = \sigma_{min}$. While using the swirl number instead of Q_{ED}, with the exception for $n_{ED} = 0.457$, the measurements are slightly better collapsed into a single curve. This deviation for $n_{ED} = 0.457$ may be related to the presence of inter-blade vortices, expected to be more intense for this speed factor. Even though this type of cavitation is not developing in the draft tube cone, it affects the value of the first eigen frequency and is then assimilated by the numerical model into the value of cavitation compliance in the draft tube.

The value of C_c tends to grow exponentially with an increase in the value of the swirl number. For each tested n_{ED} value, measurements are performed until a Q_{ED} value for which the cavitation vortex dynamics lose its coherence and the first eigen frequency cannot be determined accurately. This phenomenon is discussed in more details in [9].

Figure 5 presents the cavitation compliance as a function of the swirl number for different values of σ. The speed factor and the Froude number are kept constant ($n_{ED} = 0.478$, $\mathrm{Fr} = 5.6$). As expected, smaller values of σ are characterized by higher values of cavitation compliance. The cavitation compliance increases exponentially with the decrease in σ, explaining the sharp drop in the first eigen frequency shown in Figure 1(a) for low σ values. It can also be noticed that σ has a greater influence on C_c as the swirl increases.

The effect of the Froude number is presented in Figure 6 and Figure 7. The influence of the Froude number on the cavitation compliance for σ_{min} and σ_{max} at $n_{ED} = 0.506$ is given in Figure 6. It can be noticed that for higher swirl numbers, an increase in the value of Froude number tends to decrease the cavitation compliance. The same observation is made for all n_{ED} values in Figure 7 for σ_{min} and σ_{max}.

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The tested σ values were rather low when compared to typical rated values of turbines with a similar specific speed. For this reason, inter-blade cavitation was observed even at swirl-free conditions, keeping all the cavitation compliance curves slightly above zero.

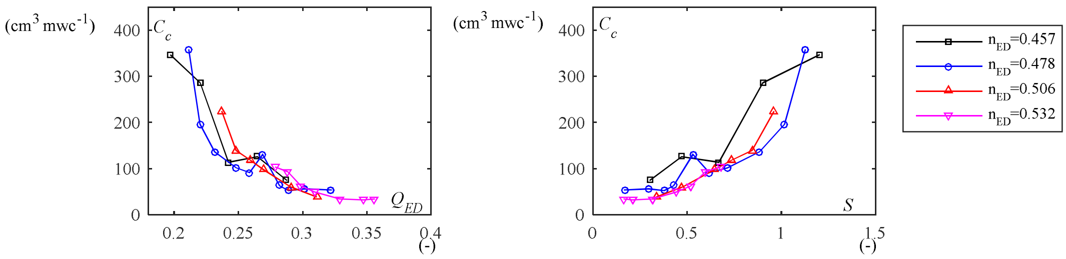

Figure 4 – Cavitation compliance for four different values of n_{ED} as a function of (a) Q_{ED} and (b) swirl number S for $\mathrm{Fr} = 5.6$ and $\sigma = \sigma_{\min}$.

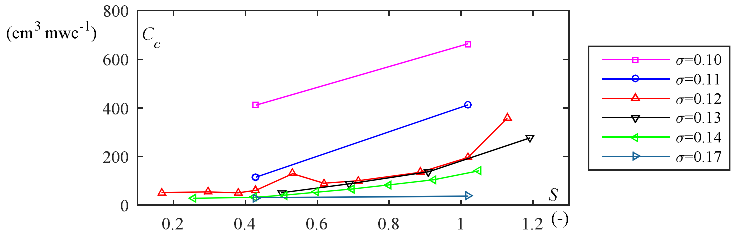

Figure 5 – Influence of σ on the cavitation compliance for a constant Froude number and n_{ED} value.

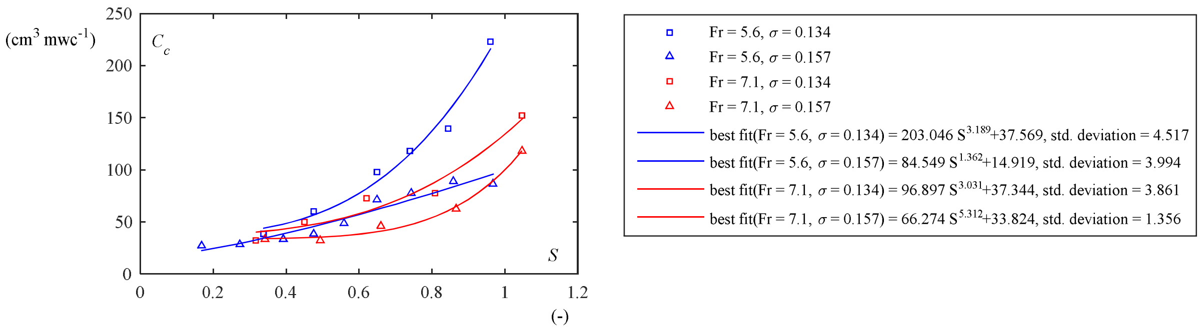

Figure 6 – Influence of Froude number and σ on the cavitation compliance for a constant n_{ED} value ($n_{\mathrm{ED}} = 0.506$).

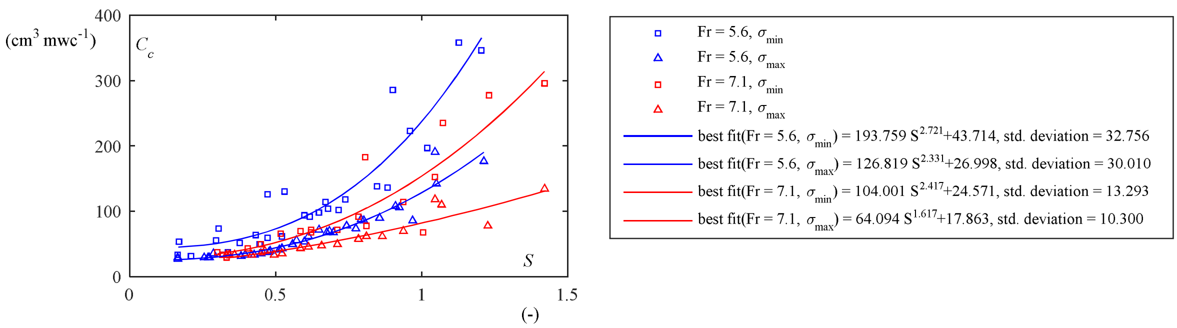

Figure 7 – Influence of the Froude number and σ on the cavitation compliance for all the tested n_{ED} values.

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Conclusion

This paper presents the results of measurements of the cavitation compliance in the draft tube cone of a Francis turbine model operating in part load conditions. The results are intended to improve the understanding of the influence of the operating parameters on the compliance, so that resonance conditions can be predicted in the future.

A simple relation between the eigen frequency and the cavitation compliance is first presented. A similar behavior is expected in a prototype hydraulic circuit. By performing measurements such as the one presented in this paper, combined with a proper transposition from model to prototype of the cavitation compliance, the first eigen frequency of the power plant can then be predicted for any part load operating condition.

Furthermore, the swirl number can be used to obtain a good overall tendency of the cavitation compliance behavior at different n_{ED} values. However, additional phenomena that do not depend directly on the swirl number, such as inter-blade cavitation vortices development, affect the calculated cavitation compliance, making the only use of the swirl number an approximate approach in certain cases.

Assuming a constant cavitation compliance located only in the upper part of the draft tube does not reflect the real size and evolution of the cavitation volume along the draft tube. In addition, other types of cavitation can develop in the runner. However, the proposed approach largely simplifies the comparison of the cavitation compliance between different operating points and, eventually, different turbine designs. The results presented in this paper can be quickly compared with measurements performed with other test cases. This methodology can in the future become a standard procedure during model testing for the assessment of risk of resonance in any hydropower plant.

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