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AN EQUIVALENT ELECTRIC CIRCUIT OF A PIEZOELECTRIC BAR RESONATOR WITH A LARGE PIEZOELECTRIC PHASE ANGLE

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The effect of the piezoelectric phase angle on an equivalent electric circuit of a piezoelectric resonator is discussed. A new equivalent circuit is derived which contains components associated with the piezoelectrically coupled energy losses. The impedance of the derived equivalent circuit is in excellent agreement with the experimentally determined impedance of a piezoelectric ceramic resonator having a significant piezoelectric phase angle.

Keywords: equivalent electric circuit, piezoelectric relaxation

1. INTRODUCTION

In the past several years there has been increased interest in relaxation of the piezoelectric properties of some single crystals,[1] ceramics,[2] and composite materials.[3] Although the piezoelectric relaxation in these materials is not yet fully understood, it is clear that it does not originate from an independent loss mechanism in the material, but is a result of an electromechanical coupling between the dielectric and mechanical losses operating in the material.[4] A particularly interesting consequence of this coupling is that the piezoelectric contributions to the total energy dissipation in a material may be either positive or negative, depending on the sign of the piezoelectric phase angle.[5]

In the vicinity of the resonant frequency, the relevant properties of a piezoelectric resonator may be described by an equivalent electric circuit. In practice, the usage of equivalent circuits greatly simplifies the analysis and design of piezoelectric transducers. The equivalent circuit, in standard form, for an unloaded resonator is shown in Figure 1a, and contains components which are associated with the mechanical losses (R_1) and dielectric losses (R_2) in the resonator.[6,7] Land *et al.*,[7] have noted that a resonator with a nonzero piezoelectric phase angle can no longer be represented by the standard equivalent circuit or its simple generalization in which the resistors R_1 and/or R_2 are replaced by a series of two resistors, one of which would then account for the piezoelectrically coupled energy losses. Martin[8] also discussed the effect of the piezoelectric losses on an equivalent circuit of a resonator, however, a new equivalent circuit which would contain elements associated with the electromechanically coupled energy dissipation processes was not derived.

In a recent study of ceramics based on lead titanate, which exhibit highly anisotropic piezoelectric properties,[9] a significant piezoelectric phase angle associated

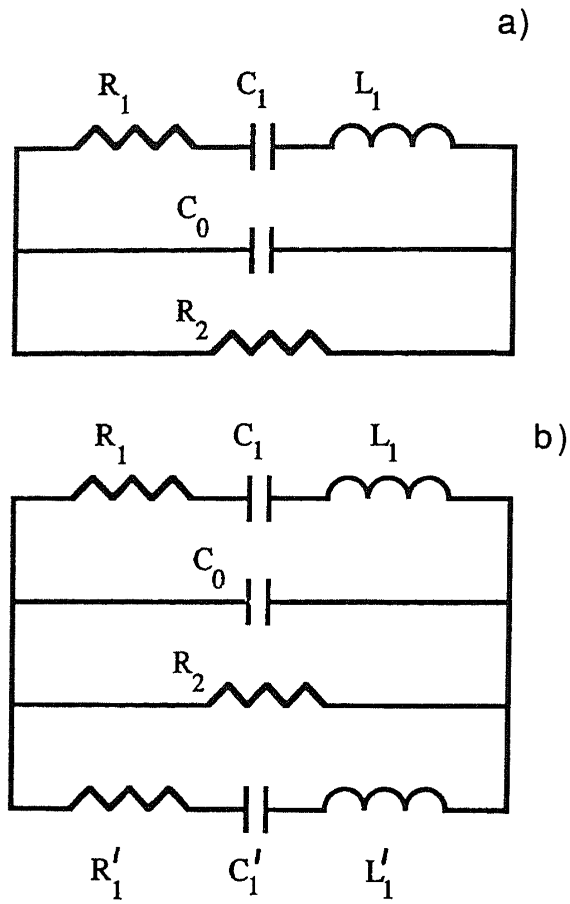

FIGURE 1 a) Standard equivalent circuit of an unloaded piezoelectric resonator. b) Equivalent circuit of a piezoelectric bar resonator with significant piezoelectric phase angle. All symbols are identified in the text. Components of the standard circuit are identical to the corresponding components of the new circuit in the limit $d''_{31} \rightarrow 0$.

with the transverse (d_{31}) piezoelectric coefficient was observed. Some relevant material properties of the lead titanate ceramics modified with samarium used for this study are given in Table I. In the course of this work an attempt was made to derive an equivalent circuit which included the elements that represented the piezoelectrically coupled energy losses. With the relatively large piezoelectric phase angle of our lead titanate based ceramics, we were able to compare the impedance of the derived equivalent circuit with the measured impedance of the resonator. In addition, these modified lead titanate ceramics exhibit a change of sign of the real component of d_{31} with temperature, while the imaginary component of d_{31} remains the same in sign.[10] Thus, the piezoelectric phase angle changes its sign with temperature, permitting us to observe whether the derived circuit is sensitive to the sign of the piezoelectric phase angle.

2. DERIVATION OF THE CIRCUIT AND DISCUSSION

For simplicity, the bar-shaped resonator (Figure 2), with an electric field perpendicular to the length of the resonator is considered. For a ceramic resonator, the

TABLE I

Selected properties of modified lead titanate ceramics used for this study. The data are for ceramic samples in a shape of a thin disk and a bar, at 25°C and poled at 60 kV/cm.

Property	Symbol (units)	
Composition	$(Pb_{0.85}Sm_{0.10})(Ti_{0.98}Mn_{0.02})O_3$	
Lattice constants[1]	a (Å)	3.902
	c (Å)	4.072
Density[2]	ρ (kg/m^3)	7420
Transition temperature[3]	T_C (°C)	310
Elastic properties		
Poisson ratio	σ (−)	0.177
Elastic compliance	s^E_{11} (m^2/N)	7.65×10^{-12}
Mechanical quality factor[4]	Q_m (−)	900
Dielectric properties		
Relative dielectric permittivity[5]	$\varepsilon^T_{33}/\varepsilon_0$ (−)	175
Dielectric loss factor[6]	$tan\delta_\varepsilon$ (−)	0.01
Piezoelectric properties		
Transverse coefficient	d_{31} (C/N)	0.27×10^{-12}
Longitudinal coefficient	d_{33} (C/N)	60×10^{-12}
Piezoelectric loss factor[7]	$tan\delta_p$ (−)	0.15
Planar coupling coefficient	k_p (−)	0.0038
Transverse coupling coefficient	k_{31} (−)	0.0025
Thickness coupling coefficient	k_t (−)	0.45

[1] Point group *4mm*
[2] 97% of the theoretical
[3] Cubic (*m3m*) to tetragonal (*4mm*)
[4] $Q_m = s'_{11}/s''_{11}$
[5] at 1 kHz; $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
[6] at 1 kHz; $tan\delta_\varepsilon = \varepsilon''_{33}/\varepsilon'_{33}$
[7] $tan\delta_p = d''_{31}/d'_{31}$

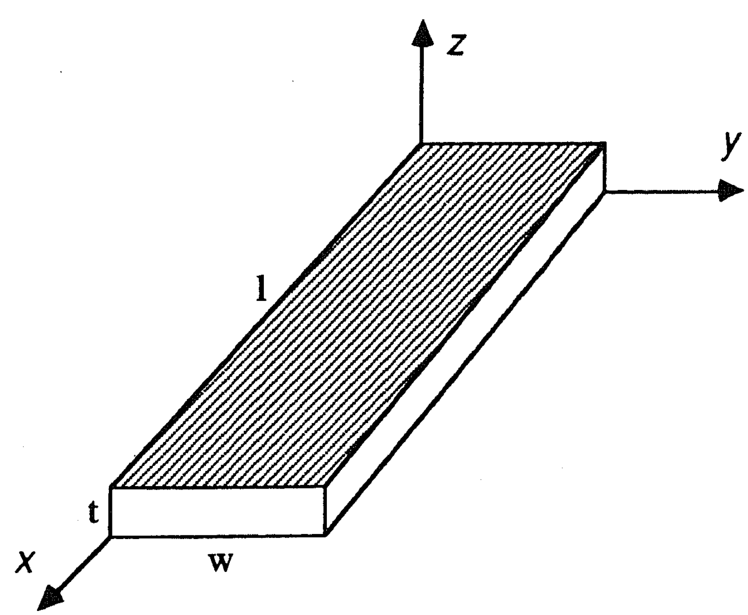

FIGURE 2 Piezoelectric bar resonator, The top and bottom surfaces are electroded.

direction of polarization is parallel to the driving electric field which is applied parallel to z-axis. The admittance Y of such a resonator which oscillates in the transverse mode is given by:[11]

$$Y = i\,\frac{\omega wl}{t}\left(\varepsilon_{33}^{T} - \frac{d_{31}^{2}}{s_{11}^{E}}\right) + i\,\frac{2wd_{31}^{2}}{\sqrt{\rho s_{11}^{E}}s_{11}^{E}t}\tan\frac{\omega l\sqrt{\rho s_{11}^{E}}}{2} \qquad (1)$$

where l, t and w are the length, thickness and width of the bar, respectively, and ρ its density. ε_{33}^{T} is the dielectric permittivity at constant stress T and s_{11}^{E} is the elastic compliance at constant electric field E. The superscripts E and T are omitted hereafter for simplicity. Near the resonant frequencies, Equation (1) can be approximated by:[12]

$$Y = i\,\frac{\omega wl}{t}\left(\varepsilon_{33}^{T} - \frac{d_{31}^{2}}{s_{11}^{E}}\right) + i\,\frac{8wd_{31}^{2}}{\rho l s_{11}^{2}t}\sum_{n}\frac{1}{\dfrac{n^{2}\pi^{2}}{\rho s_{11}l^{2}} - \omega^{2}} \qquad (2)$$

where summation goes over $n = 1, 3, 5, \ldots$ and n is the number of the overtone. In order to describe the effects of energy loss on the resonator's impedance, it is necessary to introduce the complex elastic and dielectric coefficients: $s_{11} = s_{11}' - is_{11}''$ and $\varepsilon_{33} = \varepsilon_{33}' - i\varepsilon_{33}''$. Similarly, in order to account for the effects of piezoelectric coupling on total energy losses, the piezoelectric coefficient is also assumed to be complex: $d_{31} = d_{31}' - id_{31}''$. The ratio d_{31}''/d_{31}' then defines the tangent of the piezoelectric phase angle. Equation (2) can be simplified by defining the fundamental resonant frequency as $\omega_0^2 = \pi^2/l^2\rho s_{11}'$ and writing $d_{31}^2/s_{11} = Reds + iImds$. Thus, one obtains, near the fundamental resonance ($n = 1$):

$$Y = i\omega(\varepsilon' - Reds)\frac{lw}{t} + \omega(\varepsilon'' + Imds)\frac{lw}{t} + i\omega\frac{8wl}{t\pi^{2}}(Reds + iImds)\left(\frac{\Omega - iM}{\Omega^{2} - iM^{2}}\right) \qquad (3)$$

where $\Omega = (\omega_0^2 - \omega^2)/\omega_0^2$, $M = s_{11}''/s_{11}'$ and assuming $\omega^2/\omega_0^2 \approx 1$. A further manipulation of Equation (3) leads to:

$$\mathrm{Y} = i\omega(\varepsilon' - Reds)\frac{lw}{t} + \omega(\varepsilon'' + Imds)\frac{lw}{t} + i\omega\frac{8wl}{t\pi^{2}}Reds\left(\frac{\Omega - iM}{\Omega^{2} + iM^{2}}\right) + i\omega\frac{8wl}{t\pi^{2}}(iImds)\left(\frac{\Omega - iM}{\Omega^{2} + iM^{2}}\right) \qquad (4)$$

The third term in Equation (4) can be written in terms of the components of an electric circuit in the following way:

$$\mathrm{Y}_3 = \frac{1}{R_1 + i\left(\omega L_1 - \dfrac{1}{\omega C_1}\right)} \qquad (5)$$

where $R_1 = M/\omega C_1$, $C_1 = 8wlReds/t\pi^2$ and $L_1C_1 = 1/\omega_0^2$. As expected, the resistance R_1 is associated with the mechanical losses,[11] and the product of the inductance L_1 and the motional capacitance C_1 defines the resonant frequency ω_0. Similarly, the first term defines the clamped capacitance $C_0 = (\varepsilon'_{33} - Reds)lw/t$ and the second term, related to the dielectric losses, is represented by $1/R_2 = \omega(\varepsilon''_{33} + Imds)lw/t$.

The last term in Equation (4) is proportional to the third term with the proportionality constant being $i(Imds/Reds)$. Therefore, one can use the following abbreviations: $R'_1 = R_1/i(Imds/Reds)$, $L'_1 = L_1/i(Imds/Reds)$ and $C'_1 = i(Imds/Reds)C_1$ and the last term in Equation (4) may be, then, rewritten as:

$$Y_4 = \frac{1}{R'_1 + i\left(\omega L'_1 - \dfrac{1}{\omega C'_1}\right)} \tag{6}$$

A simple calculation shows that the admittance in Equation (4) is identical to the admittance of the electric circuit shown in Figure 1b, with its components defined as in the text above. In the limit $d''_{31} \rightarrow 0$, the impedances of the standard equivalent circuit, Figure 1a, and the new one, Figure 1b, coincide, assuming $\omega^2/\omega_0^2 \approx 1$. It is interesting to note that, although the values of the "resistance" R'_1, "capacitance" C'_1 and "inductance" L'_1 are purely imaginary numbers, the total impedance $Z_4 = 1/Y_4$ of the corresponding branch of the new equivalent circuit may still be expressed in the same form as the impedance $Z_3 = 1/Y_3$ of the R_1-C_1-L_1 branch (compare Equations 5 and 6). In fact, the imaginary part of the Z_4, $R'_1 = i(Imds/Reds)R_1$, is now simply a reactive component added in parallel to the

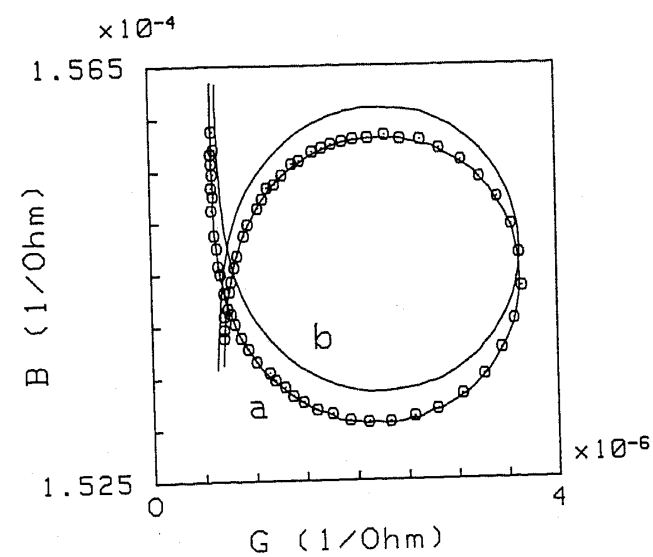

FIGURE 3 Admittance loop for a (Pb, Sm)TiO_3 resonator at 60°C. The circles represent experimental data, and the curves a) and b) correspond to admittance of the new and standard equivalent circuit, respectively. Dimensions of the resonator are: $l = 1.435 \times 10^{-2}$ m, $w = 1.454 \times 10^{-3}$ m, $t = 1.99 \times 10^{-4}$ m, $\rho = 7420$ kgm^{-3}. The values of the relevant coefficients are: $s'_{11} - is''_{11} = 7.694 \times 10^{-12} - i1.171 \times 10^{-14}$ m²/N, $d'_{31} - id''_{31} = 6.703 \times 10^{-13} - i6.318 \times 10^{-14}$ C/N, $\varepsilon'_{33} - i\varepsilon''_{33} = 1.612 \times 10^{-9} - i6.764 \times 10^{-12}$ F/m.

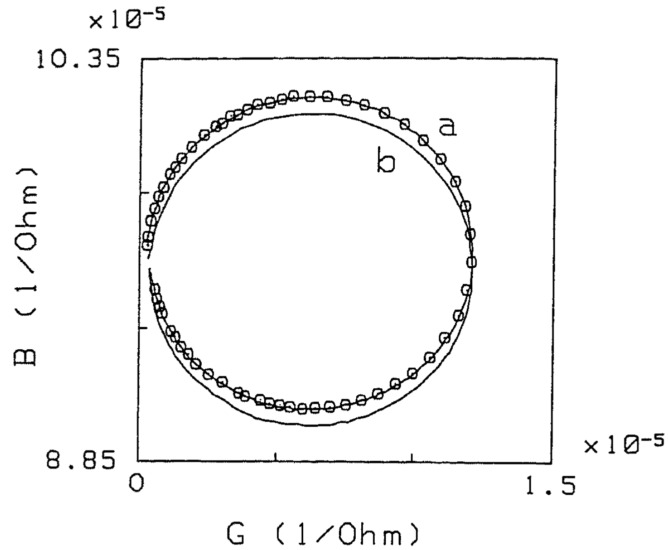

FIGURE 4 Same as for Figure 3, at −180°C. The values of the coefficients are: $s'_{11} - is''_{11} = 7.544 \times 10^{-12} - i8.569 \times 10^{-15}$ m²/N, $d'_{31} - id''_{31} = -1.133 \times 10^{-12} - i5.712 \times 10^{-14}$ C/N, $\varepsilon'_{33} - i\varepsilon''_{33} = 9.869 \times 10^{-10} - i3.747 \times 10^{-12}$ F/m.

impedance of the standard circuit. The real component of Z_4, which now contains C'_1 and L'_1, is a frequency dependent resistive term, added in parallel to the impedance of the standard circuit, in which the resistance R_1 also depends on frequency ($R_1 = M/\omega C_1$).[11]

Two examples were chosen to illustrate the validity of the new equivalent circuit. The admittance of a piezoelectric resonator made of lead titanate ceramics modified with samarium is measured at two different temperatures.[13] The sample was poled at 80 kV/cm. At these temperatures, +60°C and −180°C, the piezoelectric phase angle of the examined resonator was positive and negative, respectively. Figures 3 and 4 show the admittance ($Y = G + iB, G$ − conductance, B − susceptance) loops of this resonator. The circles represent experimental points. Curve (a) is calculated using the new equivalent circuit in Figure 1b while curve (b) is obtained using the standard equivalent circuit shown in Figure 1a. The values of the complex piezoelectric d_{31}, elastic s_{11} and dielectric ε_{33} coefficients were calculated using an iterative method[14] from the experimental data[13] and these coefficients were used to calculate the components of the equivalent circuits in Figures 1a ($d''_{31} = 0$) and 1b. It is clear that the derived circuit, as opposed to the standard, accurately describes the admittance of the investigated resonator, by taking into consideration the piezoelectric phase angle. The new equivalent circuit is also sensitive to the sign of the piezoelectric phase angle.

Finally, it is interesting to see the effect of the piezoelectric phase angle on the power dissipation of the derived circuit. The power dissipation P over one cycle is given by $P = (U_0 I_0 \cos \phi)/2$, where $U = U_0 \exp(i\omega t)$ and $I = I_0 \exp(i\omega t + \phi)$ are the AC driving voltage and current across the resonator, respectively. It follows that $P = \frac{1}{2} U_0^2$ Real $(1/Z_{tot})$, where the Z_{tot} is the total impedance of the circuit. Then:

$$P \propto \mathrm{Real}(1/Z_{tot}) = \frac{R_1}{R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1}\right)^2} + \frac{1}{R_2} - \frac{1}{\frac{Imds}{Reds}}\left(\frac{\omega L_1 - \frac{1}{\omega C_1}}{R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1}\right)^2}\right) \tag{7}$$

The origin of the last term in Equation (7) is in the piezoelectric coupling between the dielectric and mechanical losses in the resonator. Depending on the sign of the ratio *Imds*/*Reds* (which in turn depends on the sign of the piezoelectric phase angle, d''_{31}/d'_{31}) and frequency, this piezoelectric contribution to the power dissipation may be either positive or negative. The above result agrees with the predictions made by Holland[5] and Mezheretskii[15] that the electromechanically coupled losses in a piezoelectric material may increase or *decrease* the total energy dissipation.

3. SUMMARY

In summary, we have derived an equivalent circuit of a piezoelectric bar resonator which exhibits a significant piezoelectric phase angle. It is experimentally verified that the new circuit is necessary for an accurate description of the impedance of such a resonator near its resonance. The predictions that the piezoelectric contributions to the total energy losses in a piezoelectric resonator may be either positive or negative are confirmed by calculating the power dissipation in the derived circuit. The new equivalent circuit is sensitive to the sign of the piezoelectric phase angle.

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