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Comminution limit (CL) of particles and possible implications for pumped storage reservoirs

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Abstract Comminution (fragmentation) of solid particles is important in a range of technologies. An interesting effect is the so-called comminution limit (CL), which is effectively a brittle/ductile transition. Above the CL particles fail by fracture. However, as particle size decreases the amount of stored energy in the particle also decreases and eventually there is no longer sufficient stored energy in the particle to propagate a crack and the particle flows plastically. The CL depends on the hardness, H, and the toughness, K_{Ic}. In mountainous countries, two-reservoir systems are used to generate and store power. When power is needed, water runs through the turbines to the lower reservoir. If there is excess power, water is pumped to the upper reservoir. This recycling of liquid through the turbines can break up entrained particles. Previous work in this area has been primarily concerned with sedimentation of the particles. The research reported in this paper uses the CL to calculate the particle sizes produced for different materials including different rock types. Interestingly, the particle sizes predicted mainly fall in the range where they sediment near the upper water surface. In such cases, the surface layers become opaque to sunlight and plant and animal life will be affected. It is suggested that the CL provides additional information which would assist research in this area. Where H and K_{Ic} are not known for a particular rock type they should be measured.

Introduction

Pumped storage power plants (PSPPs) allow storing and generating electricity by moving water back and forth between upper and lower reservoirs through a single reversible turbine. Such highly flexible hydraulic schemes fit well in today's liberalized electricity market. Reversible turbines may be operated in generating mode when electricity demand is high and may be switched to pumping mode within a few minutes when the electricity price is low. PSPPs also offer storage capacity for new renewable energy production such as solar and wind power. For these reasons, PSPPs are expected to play a major role in the future energy mix with a growing number of planned projects worldwide. Nevertheless, PSPP development is raising concerns related to its impact on the environment due to the fate of solid particles exchanged between the reservoirs and possible chemical and physical changes produced as they undergo multiple passages through the turbines. Solid particles may impact the turbine blades or other surfaces at high speed. They may also experience large stresses due to cavitation. As a result, solid particles can fracture and their size distribution may move to lower values. The particles will then be acted upon by gravity and will sediment. Large particles will sink quickly to the bottom of the reservoir. But, other particles will sediment in layers or be so small that they remain in the surface layers; see, for example [1]. All of this may lead to a deep change of the reservoir's geomorphology and water turbidity with dramatic consequences on the fauna and flora.

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Not surprisingly, sedimentation processes have been studied in some detail including the effect of the lakes being frozen in winter, the temperature of the layers changing by ~20 °C over the year and particularly large amounts of sediment being added when streams and rivers flood.

A recent paper by Bonalumni et al. [2] gives reliable data and is an excellent summary of earlier research on sedimentation effects reported in [3–14].

However, there is little discussion in the literature published on this topic about particle fragmentation (comminution) and, in particular, about the so-called comminution limit (CL). The present paper discusses some of these issues, including the benefits of being able to predict better the response of the particles and their sizes at the CL.

It should be noted that although the loading of a particle during impact is predominantly compressive, there are various mechanisms that can give rise to tensile failure, for example, Hertzian fracture around the contact area [15, 16], indirect tensile stresses produced by axial compression as in the Brazilian test [17–19] and spall caused by compressive stress waves reflecting at boundaries [20–22].

Comminution

Comminution of solids is important in a range of technologies, for example, in rock blasting and mining which produce particles of centimetre size to be used for the extraction of metals, coal, cement manufacture, etc. At the other extreme, the pharmaceutical industry requires powders of chosen size, micron to millimetre dimensions, for the manufacture of tablets or additives to liquids to control the properties as in suntan lotions. A supermarket has packages containing a wide variety of powders, sugars, flour, coffee granules, etc., where the particle size of the powder is all important.

The disciplines needed for understanding particles and their production involve those of fracture and strength including techniques for measurement of the effects of strain rate on strength properties. Finally, the acquisition of data is required for modelling comminution.

Comminution limit (CL)

One of the more interesting properties of comminution is the so-called 'comminution limit' which has been researched by many groups both theoretically and practically [18, 19, 23–32]. The basic physics is that as particles of brittle materials are stressed to failure they eventually become impossible to break and instead fail plastically. This brittle-ductile transition is the CL. One way of considering the process is that there is no longer sufficient stored energy in the particle to propagate a crack across the particle and so the particle flows plastically. Steier and Schönert [25] used a micro-compression test apparatus and scanning electron microscopy to study particles of quartz, limestone and polystyrene of micron dimensions. As their micrographs show, there is increasing evidence of plastic flow as the particles become smaller.

A particle which has deformed plastically will have changed shape but not size (material has not been lost). The CL for a particular material depends on the hardness, H, or yield strength, Y, as $H \sim 3Y$ (see [33]) and toughness, K_{Ic}. The magnitude of the limit can vary from a few tens of microns down to ~30–50 nm for diamond. It should be emphasized that diamond only deforms plastically under very particular loading conditions. However, in a study of diamond polishing, Hird and Field [31] obtained micrographs showing spherical particles on the polished surface of ~30 nm size. They also showed that there are plastic grooves of similar width on polished diamond surfaces. Diamond has the lowest CL for any material due to its very high hardness.

Models for the CL

Various models have been proposed for the size of the limit based on somewhat different assumptions. As mentioned above, H can be replaced by $3Y$, where Y is the yield stress.

(a) Kendall's [19, 27] calculates the critical diameter of the crack produced, d_c.

$$d_c \approx 48(K_{\mathrm{Ic}}/H)^2 = 32ER/3Y^2 \tag{1}$$

where E is Young's modulus and is the fracture energy.

The analysis depends on solving a quadratic and choosing between large and small values of d_c.

(b) Hagan's [29, 30] is based on nucleation, where d is the size of crack produced.

$$d \approx 30(K_{\mathrm{Ic}}/H)^2. \tag{2}$$

(c) Ours involves an energy argument. Consider a cubical particle of side length d, which is loaded in compression.

$$\text{The stored energy, } \xi = 0.5(\sigma\varepsilon d^3). \tag{3}$$

But, $E = \sigma/\varepsilon$, so $\xi = 0.5(\sigma^2 d^3/E)$ where σ is the stress, ε is the strain and E is the modulus. The energy required to fracture the particle and produce two fracture surfaces is

$$\xi_F = 2\gamma d^2, \tag{4}$$

where γ is the fracture surface energy.

For $\xi < \xi_F$, there is not enough energy to fracture the particle and so plastic flow results. Using Eqs. (3) and (4), this criterion may be written as

Table 1 Comparison of the CL for different materials

Materials	Hagan's model for d (μm)
Polystyrene	2800
KCl	295
NaCl	32
Sapphire/Al_2O_3	3.3
Quartz/SiO_2	1.1
Soda-lime glass	0.5
MgO	0.6

Table 2 CL, d, for different rock types (minerals) using Hagan's model, and the K_{Ic} and H figures from Whitney et al. [34]

Name	Formula	Micro-hardness H (GPa)	Toughness K_{Ic} (MPa $m^{1/2}$)	d (μm)
Orthoclase	$KAlSi_3O_8$	6.9	1.1	0.76
Sillimanite	Al_2SiO_5	11.0	1.6	0.63
Quartz	SiO_2	12.1	1.5	0.46
Garnet	$Ca_3Al_2SiO_3O_{12}$	13.2	1.2	0.24
Cubic zirconia	ZrO_2	16.7	1.5	0.24

$$\frac{\sigma^2 d^3}{2E} > 2\gamma d^2. \tag{5}$$

Thus,

$$d > \frac{4\gamma E}{\sigma^2}. \tag{6}$$

But, $K_{Ic}^2 = 2E\gamma$.

Therefore, $d = 2K_{Ic}^2/\sigma^2$.

At yield, $\sigma = Y$ with $H \approx 3Y$.

Therefore,

$$d = (K_{Ic}/Y)^2 = 18(K_{Ic}/H)^2. \tag{7}$$

One of the referees has pointed out that the above analysis has the benefit of being a fairly straightforward dimensional argument, but is not formally correct as fracture does not only depend on energy.

Typical values for CL d/μm

The Tables below give values of the CL, d, for a range of materials. Table 1 contains data quoted by Hagan [29, 30].

The data in the Tables are interesting since they show a wide range of values for d. Not surprisingly, the largest value of d is for a polymer (polystyrene) and the smallest diamond of 30–50 nm. The underlined numbers are mean values.

Table 3 Other materials (Hagan's model used)

	H (GPa)	K_{IC}	d (μm)
Diamond	90	3.4–5 ($\underline{4.2}$)	0.04
Ge	8.5	0.6	0.115
Si	2.2	0.94	5.5
ZnS	1–2.5 ($\underline{1.8}$)	0.75–1 ($\underline{0.9}$)	7.5
Al_2O_3	12	2.5–4 ($\underline{3.3}$)	2.2
Calcite	Kendall	Calcite	0.8–1.0

Solids such as ionic or covalent crystals have reasonably precise values since their hardness and toughness values are well documented. However, values for minerals and other geological materials are not always easily obtained. There are a great many papers which contain data on hardness or toughness, but those that give both are relatively small in number. The paper by Whitney et al. [34] is an exception, and their results are analysed in Table 2.

Sedimentation velocities in water

Sedimentation is a key concept which has been extensively used for studying lakes and reservoirs. If we know the particle size, sedimentation equations can be used to calculate sedimentation velocities. Large particles will progress to the bottom of lakes, while smaller particles will remain in the upper levels [1, 2] and potentially affect the turbidity and ecology of the surface layers. The relevant equations can be found in many books. The one used here is due to Zapryanov and Tabakova [35] (Table 3).

Stokes' drag equation gives,

$$\text{Force} = 6\pi\mu a u, \tag{8}$$

where μ is the kinematic viscosity, a is the particle radius and u is the sedimentation velocity.

The sedimentation velocity is given by,

$$\frac{\Delta u}{u_0} = \frac{3\sqrt{2}}{2}\Phi^{0.5}, \tag{9}$$

where $\Delta u = u_0 - u$, u is the mean sedimentation velocity of the particles, u_0 is the velocity for an isolated particle and Φ is the volume occupied by the particles. Solving for u gives:

$$u = u_0\left(1 - \frac{3\sqrt{2}}{2}\Phi^{0.5}\right). \tag{10}$$

However, the second term can usually be ignored so that $u \approx u_0$.

Now, a rigid spherical particle of radius a and density ρ_p falls under gravity in an unbounded fluid with velocity

Table 4 Effect of particle size on sedimentation velocity and time for particle fall

Particle size ($a = d/2$)	Sedimentation velocity, u (m s^{-1})	Time (s) for particle to fall 10 m
25 nm	2.1×10^{-9}	5×10^{9}
0.5 μm	8.3×10^{-7}	1.2×10^{7}
1.0 μm	3.3×10^{-6}	3×10^{6}
5.0 μm	8.3×10^{-5}	1.2×10^{5}
10.0 μm	3.3×10^{-4}	3×10^{4}
100 μm	3.3×10^{-2}	3×10^{2}
300 μm	0.3	3

Note that 1 day = 8.64×10^{4} s

$$u_0 = \frac{2a^2(\rho_\mathrm{p} - \rho)g}{9\mu}, \tag{11}$$

where ρ_p and ρ are the particle and fluid densities assumed to be $\rho_\mathrm{p} = 2.5 \times 10^3$ kg m^{-3} and $\rho = 1.0 \times 10^3$ kg m^{-3} throughout, g is the acceleration due to gravity and μ is the dynamic viscosity = 1.002×10^{-3} Pa s.

The values in Table 4 were obtained using Eq. (11). They give sedimentation velocities ranging from diamond with critical radius $a = 25$ nm up to $a = 300$ μm. The third column in Table 4 gives the times for particles to sink 10 m. 5 μm particles take about 1 day, 1 μm particles about 1 month and smaller particles very long times.

Conclusions

In cases where liquid is pumped up and down between reservoirs, multiple impacts of entrained particles with the turbine surfaces will take place. Earlier research in this area has been mainly focussed on sedimentation effects. However, the so-called comminution limit has not been applied to this particular problem, although it is used in many other technologies.

This paper notes that attempts have been made to analyse the CL effect which is essentially a brittle/ductile transition. When expressed in terms of H and K_Ic, the different models are reasonably close. Calculations for the size of the particles at the CL range from ≈50 nm for diamond to 2800 μm for polystyrene. For various other brittle materials and rock types, the range is from ≈0.24 to 16 μm.

Using sedimentation equations, it is shown that in many cases the particles at the CL will remain in the surface layers causing opaque and turbid layers which will adsorb sunlight and affect the ecology of plant and animal life.

There are several cases where the values of H and K_Ic are either not known or only one is for a particular material. In such cases, measurement is needed.

If the present ideas are correct, particles will reduce in size down to the CL limit. This suggests that efforts to reduce the impact stresses between particles and the turbine surfaces would be worth making since the larger particles sediment at lower levels.

In conclusion, it is suggested that research in this area could usefully consider the CL approach which gives added physical insight. Overall, the effect of opaque layers in lakes and reservoirs needs further study.

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