# Timber Folded Plate Structures - Folded Form Analysis

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## Summary

This paper analyses the potential of different possible folded form topologies for generating timber folded surface structures. The main advantage of such structures lies primarily in the realm of ecology and sustainability. By offering an integral way of construction, which fulfils both a supporting as well as a covering function, very lightweight structures are achieved. Also, a greater degree of prefabrication is possible which leads to a reduced overall cost. As these structures consist of a large number of discrete, thin plane elements, proper edgewise connection details are needed in order to ensure an efficient load bearing system. For structures made of wood products this presents a great challenge using the state-of-the art joining techniques. For this reason, the use of timber folded plate structures in civil engineering applications has been very limited. However, recently new technical solutions have been proposed for efficient edgewise joining of thin timber panels. In this paper focus is put on integrated mechanical attachment technique which utilises digital prefabrication to integrate connectors through panel geometry. Considering the material and the chosen connection detail fabrication as well as assembly related constraints, feasible folded forms for folded surface structures are explored. Their influence on structural performance is studied and different forms are compared by means of Finite Element Analysis. Furthermore, based on the obtained results, form improvements are proposed and observations made on a case study of a built prototype structure are presented.

**Keywords:** folded plates, folded surface structures, engineered timber panels, integral mechanical attachments, folded form, anti-prismatic, rhombus-based folded plane

## 1. Introduction

Architectural and technical applications of Origami *inspired* structures (the resulting forms need not necessarily be developable) employ the structural potential of the folding principle with regard to material saving and structural efficiency. By placing the material further away from the axis of flexure, i.e. folding, the moment of inertia is increased which inherently leads to higher structural stiffness. The common terminology for describing such structures which utilize the benefits of folding usually employs "folded plates", "folded slabs" or "corrugated structures". However, the structures considered in this paper are composed of discrete inclined plane structural surfaces, where the load bearing behaviour combines both a slab and a plate mechanism. These individual surfaces are then joined together to form a globally folded surface form, which is why the terminology "folded surface structures" is preferred [1].

Several structures made from timber engineered panels which utilize the structural benefits of folding have been realised [2] and [3]. In both examples the width of the structure was spanned with a single horizontal element. Due to manufacturing and transportation constraints, timber elements are only available in limited sizes. Consequently, covering larger spans requires the realisation of efficient connection details between adjacent plane elements. The choice of the folded form, in this case, has a large influence on the connection load-bearing requirements. A folded surface structure made of prefabricated cross-laminated timber panels assembled with screwed miter joints was proposed by [2]. The folded form geometry chosen was based on folded plane rhombic elements. After examining its load-bearing performance it was concluded that the screwed miter joint connections were not sufficiently resistant to withstand the resulting transverse bending moments, and that the joints for such large panel assemblies have to be improved. This issue of designing adequate thin panel edgewise joining details presents a major challenge in timber engineering as it is difficult to address by using standard, state-of-the art timber panel joining techniques. However, the performance of folded surface structures depends on these linear edge-to-edge panel connections and they are considered a key design component. Since on-site gluing is not possible due to a lack of constant conditions for curing of the adhesive, jointing is usually realised by using metal fasteners. Still, according to current regulations, the minimal distance from the screw to the panel edge is set to $4d$ ($d$ = diameter of the screw) which restricts the minimal value of panel thickness depending on the size of the fastener [4]. Consequently, the final panel thickness is usually not dimensioned according to the structural requirements regarding the load-bearing capacity, but more due to the minimal requirements imposed by the connection detail.

Very recently the re-discovery of integral mechanical attachments provided an innovative method for edgewise jointing of timber panels [5] and [6]. In this paper, we focus on integrated mechanical attachment technique developed by [6] which utilises digital prefabrication to integrate connectors through panel geometry. The main advantage of using these form-fitting joints is that, in addition to their load bearing function (connector feature), they also integrate features for the fast and precise positioning of thin elements (locator feature). In addition, these joints do not impose any constraints on the panel thickness. However, a certain number of constraints concerning the folded form geometry need to be taken into consideration. This paper analyses the potential of different possible folded form topologies for generating timber folded surface structures consisting of integrally attached discrete elements.

## 2. Topology of the folded form

The topology of any form can be defined with a set of vertices and their surrounding faces. These faces can further be defined by their bounding edges, which can then be used to represent the connectivity of the folded form [7]. Additionally, the spatial arrangement of vertices as well as their connectivity can be regular or irregular, thus creating a form which is either composed of identical or diverse base element geometry. This geometry is essential in determining the stiffness and structural potential of the folded form.

### 2.1 Joint constraints

The chosen integral attachment technique for connecting adjacent panels imposes a certain number of constraints considering the overall folded surface geometry. One of the main constrains considered is robotic accessibility, where due to the limited tool inclination the dihedral angle between the connecting panels, $\varphi$, is restrained between 50° and 140°. Furthermore, due to the local joint geometry the orthogonal as well as non-orthogonal assembly of individual elements, also presents specific restrictions concerning the number of panel edges which can be simultaneously connected. The simultaneous assembly of multiple edges requires the panel assembly direction to be chosen within the intersection of the so called "individual edge rotation windows". These "windows" represent all possible assembly directions for each edge. In order to ensure that the assembly of multiple edges is possible, i.e. that respective rotation windows intersect, the assembly direction of the panel must not deviate from the edge normal direction for more than 20 degrees [6]. The latter is only possible for panels which have no more than three simultaneously assembled edges.

## 2.2 Folded form analysis

Figure 1 shows the classification of folded surface structures by their folded form topology as well as their possibility to form flat and singly curved surfaces. Due to previously mentioned assembly constraints, only forms based on polygons with either three or four edges are considered. The first category (Fig. 1a) contains simply corrugated surfaces composed of long rectangular plate elements. Such folded forms are most commonly used and are obtained simply by creating a succession of parallel or oblique folds. However, the span of such structures is limited, considering the material size constraints, as they contain only one element along the transverse axis of the structure. For spanning larger widths, which exceed the length of single longest available timber element, it is necessary to mutually connect a number of discrete elements along the span. By doing so, bidirectional folded forms are obtained (Fig. 1b,c,d). Such forms consist of multiple elements in two distinct directions of the structure. First bidirectional folded form category considered (Fig. 1b), includes spot or facet folded forms which consist of vertices where several folds intersect like a bunch in one single spot [8]. Such forms are obtained by taking a basic polygon and vertically rising its centroid point. This vertex is then connected to the polygon edges, consequently forming triangular faces. For such forms there are only three types of polygons which enable regular tessellations of a surface: equilateral triangles, squares and regular hexagons. Other require either the use of semi-regular tessellations or irregular polygon geometry. In the presented classification only regular tessellations are shown, as irregular ones offer a manifold of different geometric configurations. The following category (Fig. 1c) consists of rhombus based folded plane forms composed of quadrilateral elements folded along their adjoining edge. In literature this form is also known as the *Miura Ori* or *Herringbone pattern.* The final bidirectional folded form considered is the anti-prism based one (Fig. 1d), where series of anti-prisms are joined at their face-polygons. Such folded forms, also known as *Yoshimura* or *Diamond pattern,* cannot form a flat surface and are limited to shapes of cylindrical cross-section [9].

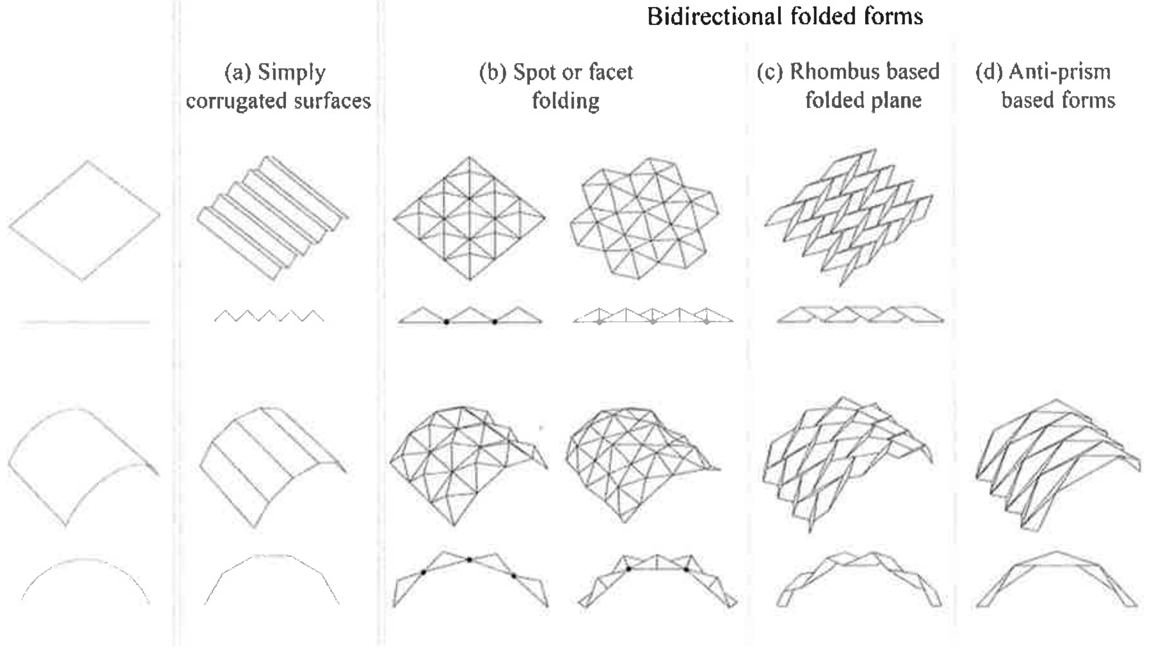

*Fig. 1: Classification of folded surface structures; perspective and transverse cross section view.*

The above classified bidirectional folded forms are further compared according to their utilization of the folding principle, making them more or less feasible for structural application in timber folded surface structures. To that extent all structures are considered to have pinned supports at the sides, while connections between the plates are regarded as line hinges, allowing rotations about the face edge direction. A cutting-plane is positioned perpendicular to the longitudinal axis and the obtained transverse cross section profile is observed (Fig. 1). It can be noted that for quadrilateral based facet folded form, both flat and singly curved, a continuous longitudinal hinge line is formed in every second vertex of the transverse cross-section. Even with a sufficient rise to span ratio of the structure, the number of such hinges mustn't exceed three (including the ones at side supports) [10]. If so, the system transforms into a mechanism and is no longer stable for more than two quadrilateral polygons per span. Also, the static height of the structure at this section point is very low and equal only to the panel thickness. Concerning other polygonal facet folded forms, due to the offset of each transversal line of polygon elements according to the previous line, the system remains stable regardless of the number of hinges. Each individual line is still a mechanism by itself, but kept stable by the neighbouring offset element lines. Furthermore, as the number of polygon sides in facet folded forms increases, the dihedral angle between the faces surrounding the raised centroid point increases as well. The value of this angle is constrained by the previously mentioned tool inclination ability. It is considered that angles close to 90° work particularly well for folded surface systems, whereas the system will lack in stiffness when incrementally increasing the angle. As the value of this dihedral angle gets very obtuse already for five-sided polygons, such forms are regarded as unsuitable for the studied purpose. Respectively, only two of the considered bidirectional folded forms are further examined for use in timber folded surface structures; the rhombus base folded plane form and the anti-prism based one.

## 3. Geometric and structural considerations for comparing anti-prismatic and rhombus based folded plane forms

### 3.1 Folded form geometry

The two chosen bidirectional forms were examined by comparing their ability to build up a folded surface structure with a set of previously defined parameters. For each folded form the base element geometry was kept the same throughout the whole system so that the form remains regular. Apex angles, $\beta$, were kept within the limits imposed by assembly constraints and restrained to a minimum of 140° for anti-prismatic and a minimum of 70° for the rhombus based folded plane form (Fig. 2).

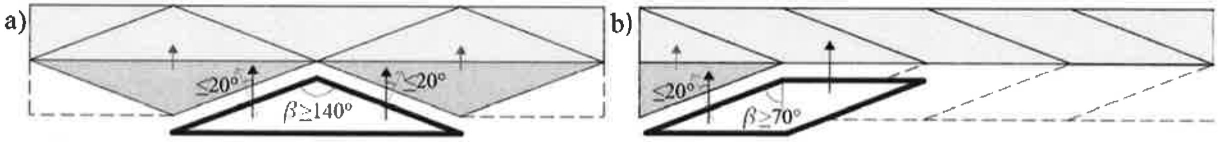

*Fig. 2: Minimal value of the apex angle, β, and the maximal value for the assembly direction deviation from the edge normal for the anti-prismatic (a) and rhombus-based folded plane form (b).*

The panel thickness, $t$, widths of elements, $w$, and the dihedral angle, $\varphi$, were also taken as invariables for both forms. Reverse folding was done by reflecting identical base elements about a plane which is perpendicular to the interior angle bisector of the main crease edges. This resulted in structures that follow a constant transverse cross-sectional curvature. The radius of this curvature was set to $R_{ext}=2{,}5m$ and the span to $S=3m$ in order to obtain an optimal structure height of $h=0{,}5$m, as well as a favourable height to span ratio of 1/6. Additionally, for restricting the maximal static height of the system, $h_s$, radius of the intrados curvature was set to $R_{int}=2{,}3m$. For both forms the number of individual elements along a transverse cross-section, which can be used to approximate a set curvature, with a defined maximal static height and structure span, can be determined according to the following formulae:

$$n=\left\lfloor \frac{\psi_{tot}}{\cos^{-1}\left(\dfrac{R_{ext}-h_s}{R_{ext}}\right)} \right\rfloor \quad (1) \qquad \tilde{n}\geq\left\lceil \frac{\psi_{tot}}{\cos^{-1}\left(\dfrac{R_{ext}-h_s}{R_{ext}-(h_s/2)}\right)} \right\rceil,\quad n\geq\begin{cases}\tilde{n} & \textit{if } \tilde{n} \textit{ is even}\\ \tilde{n}+1 & \textit{if } \tilde{n} \textit{ is odd}\end{cases} \quad (2)$$

Where $\psi_{tot}$ is the central angle of the circular segment observed, $R_{ext}$ the radius of curvature and $h_s$ the static height. For anti-prismatic based forms there is a unique result for a required number of whole, uncut triangular elements (1). On the other hand, for rhombus-based folded plane forms the result is defined as a minimal one necessary. Additionally, only an even number of elements is considered, as any odd number would result in end segments of different orientation, leading to a height difference between side supports. Therefore, the result is to be rounded up to the first greater even number (2). For the set parameters, we obtained three segments for anti-prismatic and a minimum of six segments for the rhombus based folded form. An additional rhombus based form with eight cross-section segments was also considered (Fig. 3). It can be noted that approximating the same curvature with a defined height resulted in smaller spans, as well as reduced maximal static height of the system, for rhombus based folded plane forms.

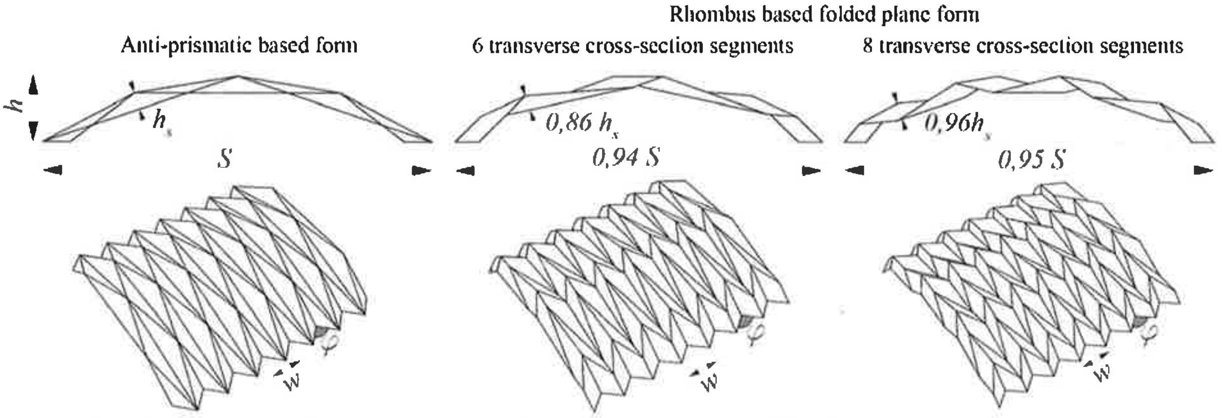

*Fig. 3: Transversal cross section and perspective view of the three examined structures.*

### 3.2 Finite Element Analysis (FEA)

The three obtained structures were modelled in FE analysis software (ABAQUS) and their performance under structural self-weight was observed. They were compared based on vertical displacements of the mid transversal cross-section and bending moments about the longitudinal axis of the structure. The plates were modelled as conventional shell elements with the reference surface which is coincident with the plate's mid surface. This way a three-dimensional continuum was approximated with a two-dimensional theory due to the fact that the plate thickness is small compared to its remaining dimensions. Material properties of simplified orthotropic, 21 mm thick, structural LVL timber panels were implemented into the model with values according to [11]. Boundary conditions were modelled as pinned, allowing rotations but no movement in all three directions. A simplified model of the complex semi-rigid behaviour of the chosen connection detail was chosen. As bending moments are the critical forces transferred between the adjacent plates the structural behaviour of the system and its overall stiffness depends on the connection detail capacity to resist them. For that reason the joints between the plates were considered as completely rigid in order to obtain maximal values of the bending moments for each folded form considered.

Figure 4 shows vertical displacements along a normalised span of the observed structures and an overlay plot of the deformed and undeformed structure shape, uniformly scaled for all three forms. The lowest vertical displacements under dead load were found for the anti-prismatic form. Respectively, the six segment rhombus based folded plane form showed around 3,5 times higher values, while for the eight segment one they were about 4,5 times higher. Similar discrepancies were found for the values of the observed bending moments about the longitudinal axis of the structure. The 6-segment rhombus based form showed more than 5 times higher and the 8-segment one more than 7 times higher values, with respect to the bending moment values obtained form the anti-prismatic folded form.

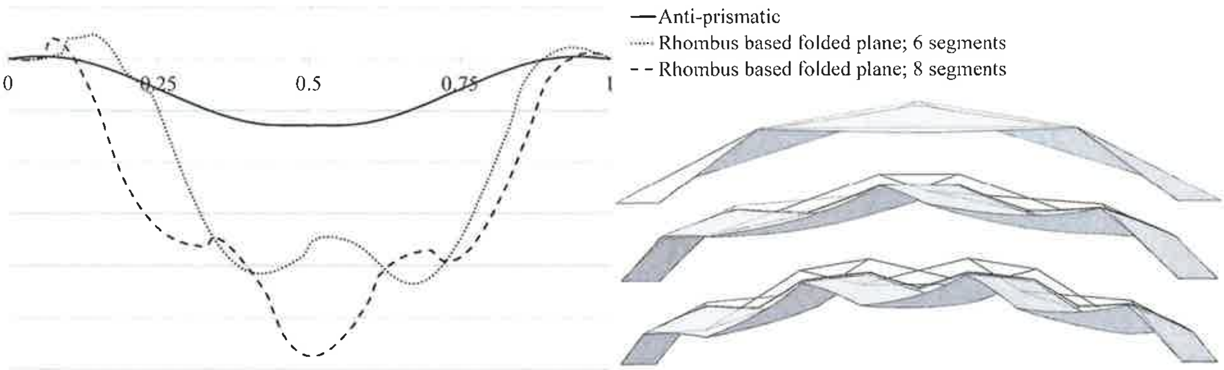

*Fig. 4: Vertical displacements along a normalised span (left) and an overlay plot of the deformed and undeformed structure's shape, uniformly scaled for all three observed forms (right).*

The reason for this can be found if a comparison is done between each folded form transverse cross section central line and a funicular curve of the equivalent smooth shell (Fig.5). The magnitude of moments generated at the connection edges depends on the degree of deviation of the folded form central line from the funicular. Validating the results obtained from FEA, it was observed that the central line of the anti-prismatic form approximates the one of the smooth shell the best. This is due to the fundamental topology difference between the two considered folded forms. The rhombus based form consists of quadrilateral faces with vertices which alternately lie on the intrados and extrados of the circumscribed arc, i.e. the arc thickness is taken as equal to the maximal static height of the system. On the other hand, the anti-prism based form consists of isosceles triangle faces, with all vertices lying on the extrados of the circumscribed arc. This makes the anti-prismatic form faces more uniformly distributed around the cross section central line. Considering the difference in the number of segments between the two rhombus-based folded forms it can be noted that dihedral angles between the plane elements are reduced as the number of segments rises. However, this benefit regarding the folding principle leads to a greater deviation of the forms central line. Inherently this contributes to higher bending stresses along the edges and higher displacements, as well as reduced load bearing capacity.

As a result of the presented analysis it is concluded that, when compared to shells, even though there is no such form which can completely eliminate bending stresses in folded surface structures, they can be significantly reduced by choosing the appropriate folded form geometry.

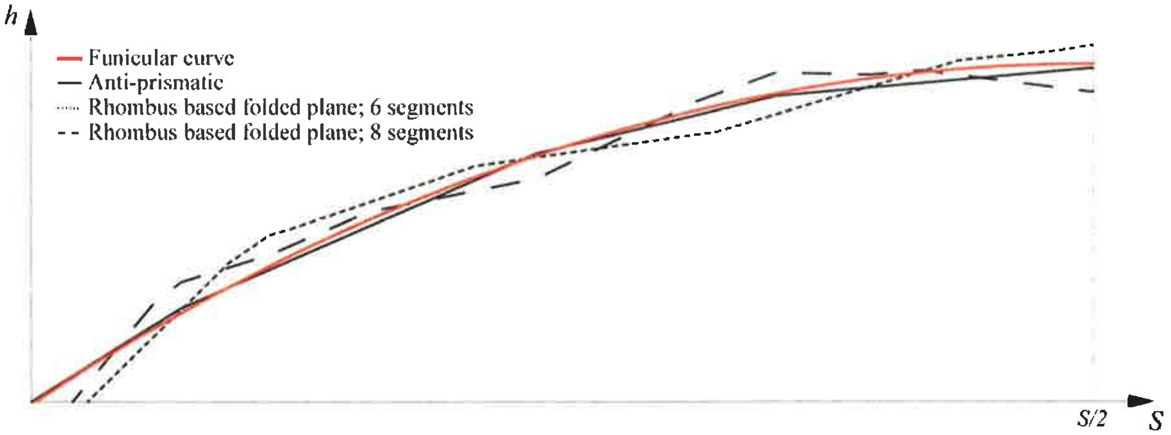

*Fig. 5: Deviation of folded form central line from the equivalent smooth shell funicular curve*

# 4. Conclusion

## 4.1 Observations on a prototype structure

A prototype structure was constructed and tested in order to explore folded surface structures behaviour under load and collect first experiences concerning the fabrication and assembly of its constituting elements. Using the RhinoPython application programming interface, a computational tool which instantly generates both the geometry of the individual components and the machine G Code required for fabrication was developed. Regarding the material, two types of engineered wood products were considered, Kerto-Q structural grade Laminated Veneer Lumber (LVL) and Cross-Laminated Timber (CLT) panels. It was noticed that the more homogenous and mechanically strong peeled-veneer laminates offer particular advantages compared to CLT panels. Firstly, considerably thinner cross-sections are possible enabling for more lightweight structures. Secondly, an important shortcoming of CLT was recognised while milling the panel edges. As CLT panels consist of several layers of longitudinal timber planks which are not mutually glued on the sides, depending on the angle of the joint with respect to the individual layer plank orientation, considerable pieces of the joint can simply chip off after manufacturing.

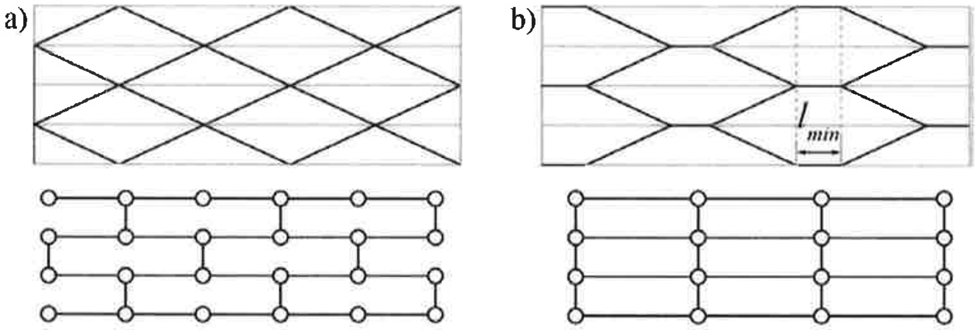

*Fig. 6: Edge-to-edge connectivity between the faces of an Anti-prismatic folded form (a) and its isosceles trapezoid version (b).*

Another issue of using integral mechanical attachments was recognised very early in the prototype design process. Due to specific joint geometry the efficient connecting length of each edge is always shorter than its total length. This can pose a problem when trying to achieve an efficient connection of a number of plane elements which meet at one vertex. Consequently it was decided to reduce the vertex valence number by modifying the anti-prismatic folded form and changing its isosceles triangle elements into isosceles trapezoids. An additional edge line was added at the vertex opposite to the triangle base. Its minimal length was restricted in order to secure the minimum connection between panels and provide at least one pin on each adjoining plate edge. As a result, the edge-to-edge connectivity between every element along the span and the neighbouring line element was realized (Fig. 6b). In the previous, isosceles triangle solution, this connection was possible only for every second element (Fig. 6a). Moreover, by using isosceles trapezoids a wider span was realised with the same number of elements along a transverse cross section.

Figure 7 shows the completed folded surface prototype. The built structure spanned 3 meters and was constructed with 21 mm thick, Kerto-Q structural grade LVL panels (7-layer, I-III-I). The structure's total weight amounted to only 192 kg. Boundary conditions that restrain displacements of the supports in every direction, but allow rotations, were applied on both sides. A longitudinal line load was introduced along the top of the shell and vertical displacement was measured at centre point. The results obtained from the testing showed that the load of 25kN, which corresponds to the proportional limit of the load-displacement curve, causes a vertical displacement of 23mm. Also a high structural efficiency (ratio of the maximal load over the dead weight of the structure) which reaches 23,44 when loaded with 45kN, was obtained.

*Fig. 7: Timber folded surface prototype*

### 4.2 Outlook

The final proposed timber folded surface structure form successfully combines the structural advantages of timber panels with the efficiency of folded plates, while conforming to the constraints of integrated mechanical attachments. The presented prototype demonstrated the realization of a very lightweight structure with a weight to surface area ratio of only 11,5 kg/m$^2$. Furthermore it verified the structural capacity of the chosen folded form along with the fabrication and assembly methods used for its construction. However, it should be noted that further research is required regarding the physical connections used. Their mechanical behaviour and load-bearing capacity still need to be confirmed. Nevertheless a great structural potential is recognized in the proposed timber panel structural systems and the forms discussed within this paper are considered to be of significant influence in establishing these structures on a building scale.

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