ON THE USE OF INPUT-OUTPUT FEEDBACK LINEARIZATION TECHNIQUES FOR THE CONTROL OF NONMINIMUM-PHASE SYSTEMS

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Avant-propos

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Abstract

The objective of the thesis is to study the possibility of using input-output feedback linearization techniques for controlling nonlinear nonminimum-phase systems. Two methods are developed. The first one is based on an approximate input-output feedback linearization, where a part of the internal dynamics is neglected, while the second method focuses on the stabilization of the internal dynamics.

The inverse of a nonminimum-phase system being unstable, standard input-output feedback linearization is not effective to control such systems. In this work, a control scheme is developed, based on an approximate input-output feedback linearization method, where the observability normal form is used in conjunction with input-output feedback linearization. The system is feedback linearized upon neglecting a part of the system dynamics, with the neglected part being considered as a perturbation. Stability analysis is provided based on the vanishing perturbation theory. However, this approximate input-output feedback linearization is only effective for very small values of the perturbation. In the general case, the internal dynamics cannot be crushed and need to be stabilized.

On the other hand, predictive control is an effective approach for tackling problems with nonlinear dynamics, especially when analytical computation of the control law is difficult. Therefore, a cascade-control scheme that combines input-output feedback linearization and predictive control is proposed. Therein, input-output feedback linearization forms the inner loop that compensates the nonlinearities in the input-output behavior, and predictive control forms the outer loop that is used to stabilize the internal dynamics. With this scheme, predictive control is implemented at a re-optimization rate determined by the internal dynamics rather than the system dynamics, which is particularly advantageous when internal dynamics are slower than the input-output behavior of the controlled system. Exponential stability of the cascade-control scheme is provided using singular perturbation theory.

Finally, both the approximate input-output feedback linearization and the cascade-control scheme are implemented successfully, on a polar pendulum 'pendubot' that is available at the Laboratoire d'Automatique of EPFL. The pendubot exhibits all the properties that suit the control methodologies mentioned above. From the approximate input-output feedback linearization point of view, the pendubot is a nonlinear system, not input-state feedback linearizable. Also, the pendubot is nonminimum phase, which prevents the use of standard input-output feedback linearization. From the cascade control point of view, although the pendubot has fast dynamics, the input-output feedback linearization separates the input-output system behavior from the internal dynamics, thus leading to a two-time-scale systems: fast input-output behavior, which is controlled using a linear controller, and slow reduced internal dynamics, which are stabilized using model predictive control. Therefore, the cascade-

control scheme is effective, and model predictive control can be implemented at a low frequency compared to the input-output behavior.

Résumé

L'objectif de la thèse est d'étudier la possibilité d'utiliser les techniques de linéarisation en entrée-sortie par retour d'état afin de commander les systèmes non linéaires à non minimum de phase. Dans cette optique, deux méthodes de commandes ont été développées. La première est basée sur une approximation de la linéarisation en entrée-sortie du système, où une partie de la dynamique interne du système est compensée, La deuxième méthode se concentre sur la stabilisation de la dynamique interne.

Vu que l'inverse d'un système non linéaire à non minimum de phase est instable, la technique standard de linéarisation par retour d'état n'est pas efficace pour commande ce type de systèmes. Dans ce travail de thèse, une technique de commande a été développée, basée sur une méthode d'approximation de la linéarisation par retour d'état. Le système est linéarise via un retour d'état, en négligeant une partie de la dynamique du système. La partie négligée est alors considérée comme perturbation. La preuve de stabilité d'un tel schéma de commande est établie sur la base de la théorie des perturbations qui s'annulent (vanishing perturbations). Cependant, cette technique de linearisation approximative n'est efficace que pour des valeurs considérablement faibles des perturbations. Ainsi, en règle générale il est préférable si possible de stabiliser tout le système, en incluant la dynamique interne.

D'un autre cote, la commande prédictive est une approche efficace pour traiter de la stabilisation de systèmes non linéaires, particulièrement lorsque le calcul d'une loi analytique de commande est difficile. Ainsi, un schéma de commande en cascade a été développé, combinant les techniques de linéarisation en entrée-sortie par retour d'état et la commande prédictive. Dans la boucle interne de la commande en cascade, la linéarisation du système en entrée-sortie est appliquée au système afin de compenser les non linéarités existantes dans le comportement en entrée-sortie du système. La boucle externe de la commande en cascade consiste en une commande prédictive, qui a pour objectif de stabiliser la dynamique interne du système.

Avec ce schéma en cascade, la fréquence de ré-optimisation de la commande prédictive est fixée par la vitesse de la dynamique interne, et non par celle de la dynamique entière du système. Aussi, le schéma de cascade peut être très avantageux dans le cas de dynamique interne lente par rapport à la dynamique en entrée-sortie du système. L'étude de stabilité de la commande de cascade a été menée, sur la base de la théorie des perturbations singulières. Une extension du schéma de cascade a été établie, en utilisant des concepts de théorie des extremums au voisinage (neighboring extremals), afin de robustifier la commande, mais aussi de permettre l'utilisation de la commande prédictive à une fréquence de ré-optimisation lente.

Les deux méthodologies de commande développées dans ce travail ont été appliquées sur un double pendule polaire inversé 'pendubot', disponible au laboratoire d'automatique. Les

résultats d'implémentation ont été très concluants. Le pendubot est un exemple approprié pour les deux méthodologies de commande développées dans ce travail. Du point de vue de la commande par approximation du linéarisé par retour d'état, le pendubot est un système qui n'est pas entièrement linéarisable par retour d'état, et la dynamique interne résultant de la linéarisation en entrée-sortie standard est instable. Du point de vue de la commande en cascade, bien que le pendubot a une dynamique rapide, la linéarisation en entrée-sortie par retour d'état sépare le comportement en entrée-sortie du système de sa dynamique interne. Ceci donne lieu à un système a deux échelles de temps: un sous système rapide, qui consiste en le comportement en entrée-sortie, et qui est commandé via une commande linéaire; et un sous système lent, qui consiste en la dynamique interne, et qui est stabilisée en utilisant une commande predictive. Ainsi, le schéma de commande en cascade est efficace et la commande prédictive implantantée à une faible fréquence de ré-optimisation.

Contents

General Notations

Here are the main acronyms used in this document. The meaning of an acronym is usually indicated the first time it occurs in the text. English acronyms are also used for the French summary.

SISO	Single-input single-output
MIMO	Multiple-inputs multiple-outputs
FL	Feedback linearization
IOFL	Input-output feedback linearization
ISFL	Input-state feedback linearization
MPC	Model predictive control
NMPC	Nonlinear model predictive control
NMP	Nonminimum phase
QSS	Quai-steady state
NE	Neighboring extremals
LQR	Linear quadratic regulator
$(\cdot)^T$	Transpose operator
s.t.	Such that
R	The set of real numbers
R^n	The set of real vectors of length n
t	Time variable
h	Sampling time

u	System input
x	Vector containing the state variables
y	System output
$\dot{x}$	Time derivative of the vector x
$\|\cdot\|$	Norm 2 operator
$\frac{\partial}{\partial x}$	Partial derivative with respect to x

Chapter 1

Introduction

1.1 Motivation and Problem Statement

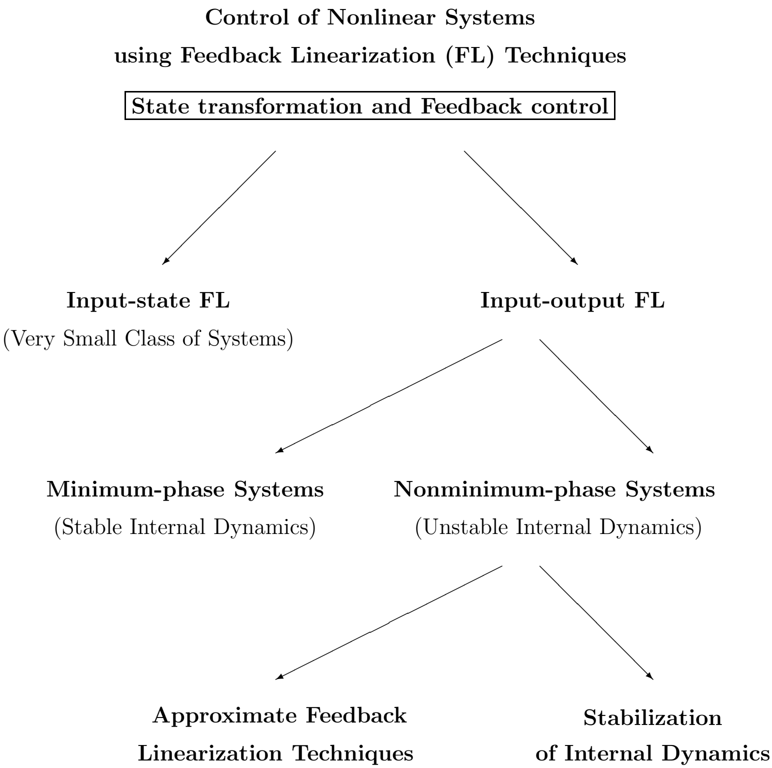
Control is a very wide and common concept. The term 'control' is used to refer to: (i) purely human activity (self control...), (ii) a specific human-machine interaction (driving a car...), (iii) an activity without human presence (automatic piloting of a plane...). The automatic control field focuses on the third category, dealing with the analysis and control of dynamical systems.

In industry, most machines or processes are controlled using PID-type controllers. The control parameters are tuned on-line on the machines in order to reach a prescribed range of performance. However, in some domain applications, such as aeronautics, robotics or machine too industry, high-precision control is needed. In this case, more sophisticated control strategies are required. For this task, the system needs first to be modeled, in order to describe mathematically the system behavior. Depending on the system nature, the mathematical model can be continuous-time, discrete-time, hybrid (contains both continuous-time and discrete-time elements), stochastic, algebraic, static, dynamic, etc ...

For a continuous-time dynamical system, a mathematical model of the plant consists of a set of differential and algebraic equations linking the system inputs to the system outputs. Two major classes of mathematical models for continuous-time systems are commonly distinguished in the literature: linear models and nonlinear models. Analysis and control of linear models have been widely addressed in the literature, and strong theoretical tools have been developed to cover this class of models. However, this is not the case for nonlinear models, as the theoretical tools for their analysis and control are very limited. Thus, the use of linear models is usually preferred to describe nonlinear systems, in order to make the analysis easier.

An effort has been made to transform a nonlinear system into a linear one using feedback control. The idea of using feedback to achieve (or enhance) system linearity is all but a new one. It can be traced back to the origin of feedback system theory in 1934: *"... feeding the output back to the input in such a way as to throw away the excess gain, it has been found possible to effect extraordinary improvements in constancy of amplification and freedom from nonlinearity"* [1, 2].

The control of nonlinear systems using feedback linearization techniques is summarized in the following diagram:


Control of Nonlinear Systems
using Feedback Linearization (FL) Techniques
State transformation and Feedback control
Input-state FL
(Very Small Class of Systems)
Input-output FL
Minimum-phase Systems
(Stable Internal Dynamics)
Nonminimum-phase Systems
(Unstable Internal Dynamics)
Approximate Feedback
Linearization Techniques
Stabilization
of Internal Dynamics

In the early 1980s, the exact conditions under which a nonlinear plant can be input-state linearized by static state feedback and coordinate transformation were stated [3, 4, 5, 6]. Such a scheme is referred to *input-state feedback linearization*, where after the coordinate transformation and static state feedback, the state equations are completely linearized. However, the conditions for input-state linearization of a nonlinear system are only satisfied for a very small class of systems.

This has given rise to the idea of linearizing only part of the dynamics, i.e. the dynamics between the inputs and the outputs [4]. In such a scheme, referred to as *input-output feedback linearization*, the input-output map is linearized, while the state equation may be only partially linearized. Residual nonlinear dynamics, called internal dynamics, occur. These dynamics do not depend explicitly on the system input, and thus are not controlled. The main limitation of the input-output linearization is that the internal dynamics, which can be unstable at the equilibrium point, are not controlled. Such systems are called *nonminimum-phase* systems.

Two different ideas for the control of nonlinear nonminimum-phase systems will be exploited in this work:

(i) The internal dynamics are neglected such that the resulting approximate system is input-state feedback linearizable.

(ii) The internal dynamics are taken into account and are stabilized simultaneously with the control of the input-output behavior of the system. Nonlinear model predictive control will be used as a systematic way to control the internal dynamics.

1.2 State of the Art

In this section, the control of nonlinear systems, first using approximate input-state feedback linearization, then using stabilization of internal dynamics, are detailed. Although this work considers the case of affine-in input nonlinear systems, this section treats nonlinear systems in a general case, in order to provide a complete overview of the existing techniques available in the literature.

1.2.1 Approximate Feedback Linearization Techniques

Given a nonlinear system, the objective of input-state feedback linearization is to find a state feedback control and a change of variables that transform the nonlinear system into an equivalent linear one [4, 7, 8]. The main difficulties with input-state feedback linearization of nonlinear systems are its limited applicability, as well as the complexity, sensitivity and design difficulties of the exact linearizing compensators [9]. When the system is not input-state linearizable, an alternative is the input-output linearization. For nonminimum-phase systems such an alternative is not effective.

As an alternative to the input-state feedback linearization, there has been an increasing amount of work searching for approximate solutions to the problem of linearizing nonlinear systems by state or output feedback. The main approximate linearization methods available in the literature can be classified according to the following scheme (Figure 1.1):

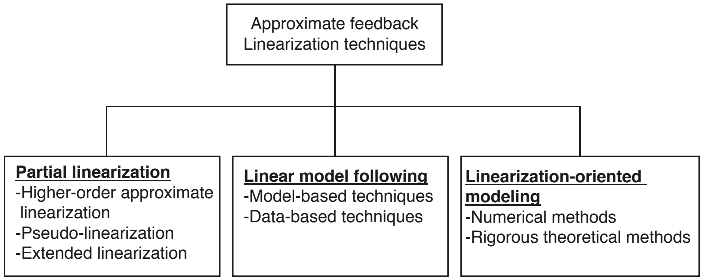


Figure 1.1: Approximate feedback linearization methods

(a) Partial linearization

These methods consist of local approximations of the system. Therein, a Taylor series expansion of the system dynamics is first computed. Then, state feedback and local coordinate transformation are computed to transform the system locally around the origin, or within a set of operating points, to an approximately linear one. The main disadvantage of partial linearization techniques is that the stability results are only local.

Partial linearization techniques presented in the literature are known as *higher-order approximate linearization*, *pseudo linearization* and *extended linearization*:

Higher-order approximate linearization local state feedback and local coordinate transformation are computed such that a number of higher terms of the Taylor series expansion around the operating point become zero.

Some important results of higher-order approximate linearization reported in the literature can be summarized in Table 1.1.

Necessary and sufficient conditions for the existence of static state feedback and coordinate transformation	[10, 11, 12]
Extension to dynamic feedback	[13]
Extension to discrete systems	[14, 15, 16]
Numerical algorithms to compute state feedback and coordinate transformation	[17, 18, 19]

Table 1.1: Higer-order approximate linearization

The disadvantage of nonlinear control design by linearization about a constant operating point is that the characteristics of the linearized closed-loop system will change with changes in the operating point.

Pseudo linearization: In opposite to higher-order approximate linearization, in pseudo linearization, state feedback and coordinate change are computed such that the Taylor series linearization of the transformed system at a constant operating point has input-output behavior that is independent of the considered operating point, within a local set that contains it.

Some important results of pseudo linearization reported in the literature can be summarized in Table 1.2.

Necessary and sufficient conditions for the existence of static state feedback and coordinate transformation	[20, 21, 22]
Numerical methods for designing the linearizing feedback	[23, 24, 25]
Generalization, extensions and specific methods to construct state feedback laws	[26, 27, 28]

Table 1.2: Pseudo linearization

Extended linearization The objective therein is similar to that of pseudo linearization. However, extended linearization deals with the input-output feedback linearization problem while pseudo linearization does not require the definition of an output since it focuses on the input-to-state behavior of the control system.

Some important results of extended linearization reported in the literature can be summarized in Table 1.3.

First results for the design of an output-feedback linearizing controller	[29, 30]
Extension to single-input multiple-outputs nonlinear systems	[31]
Extended linearization using parameterized linear systems	[32, 33]

Table 1.3: Extended linearization

(b) Linear model following

Partial linearization techniques lead to local results, since the system is linearized at an operating point or a local set of operating points. In linear model following techniques, the idea is to use a feedback to match a prescribed linear behavior. In other words, the problem is to find a feedback compensator such that the behavior of the closed-loop system gets as close as possible, in a sense to be specified, to a linear model referred to as *reference model*. The design of such a feedback compensator is based on optimization methods.

Linear model following techniques can be tackled by either *model-based techniques*, if the available description of the plant is a mathematical model, or *data-based techniques*, if the description of the plant is only a set of input-output data.

Some important results of linear model following reported in the literature can be summarized in Table 1.4.

Model-based techniques	[34, 35, 36, 37]
Data-based techniques	[38, 39, 40, 41, 42, 43, 44]

Table 1.4: Linear model following

Since the design of the feedback compensator in linear model following techniques is based on optimization methods, the main difficulty which occurs using these techniques is the possibility of nontrivial optimization tasks to be solved and very demanding computations.

(c) Linearization-oriented modeling

In the linearization-oriented modeling, the system is approximated everywhere, in contrast to the partial linearization techniques. The approximation can be achieved using *numerical methods*, or *rigorous theoretical methods*.

Some important results of linearization-oriented modeling reported in the literature can be summarized in Table 1.5.

Numerical methods	[45, 46, 47, 48, 49, 50, 51]
Rigorous theoretical methods	[52, 53, 54, 55]

Table 1.5: Linearization oriented modeling

However, as for linear model following techniques, numerical methods can be very demanding computations. Thus, the linearization-oriented modeling using theoretical methods is more interesting to approximate globally a nonlinear system to a linear one.

Linearization-oriented modeling techniques applied to the particular case of nonminimum-phase systems have been widely addressed in the literature. In [52], a stable but non-causal inverse is obtained off-line, and is incorporated into a stabilizing controller for dead-beat output tracking. In [53], the control design uses synthetic outputs that are statically equivalent to the original process outputs and make the system minimum phase. A systematic procedure is proposed for the construction of statically equivalent outputs with prescribed transmission zeros. These calculated outputs are used to construct a model-state feedback controller.

Kravaris and Soroush have developed several results on the approximate linearization of nonminimum-phase systems [56, 54, 55, 57]. In particular, in [55, 57], the system output is differentiated as many times as the order of the system, instead of stopping at the relative degree as would be done with standard input-output feedback linearization. As an approximation, the input derivatives that appear in the control law are set to zero, in the computation of the state feedback input. [58] extends the above results to the multiple-inputs multiple-outputs case. The particular case of second-order nonminimum-phase systems is considered in [59]. The main disadvantage of Kravaris and Soroush results is that they require the open-loop system to be stable. In [60], an extension of the controller is developed for unstable systems. However, the feedback is dynamic, and only asymptotic stability has been provided therein. In [61], new outputs are introduced, so as to approximately factor the nonlinear system as a minimum-phase factor and an all-pass factor. The system is then linearized using a dynamic state feedback. The main disadvantage of this method is its restrictive applicability domain as the approximate factorization can only be derived for maximum-phase systems, i.e. systems with only anti-stable zero dynamics. In [62], the system is first input-output feedback linearized. Then, the zero dynamics are factorized into stable and anti-stable parts. The anti-stable part is approximately linear and independent of the coordinates of the stable part. The main difficulty with this method is that very conservative conditions have been stated under which the zero dynamics can be separated into stable and anti-stable parts, so that only a few practically-relevant nonminimum-phase systems can be transformed into this form.

1.2.2 Stabilization of Internal Dynamics

A cascade structure involving feedback linearization and stabilization of the internal dynamics has been considered in the literature. The system is first input-output feedback linearized, and then the internal dynamics are stabilized. In [63], the desired output trajectory is redefined so as to maintain the internal dynamics stable. [64] tackles the problem of designing a nonlinear multiple-inputs multiple-outputs voltage and speed regulator for a single machine infinite bus power system. Therein, the internal dynamics are stabilized using a Lyapunov-based scheme. In [65], the internal dynamics are stabilized using output redefinition and repetitive learning control. [66] addresses the problem of swinging up an inverted pendulum and controlling it around the upright position. The internal dynamics are stabilized using elements of energy control and Lyapunov control. Moreover, [67] considers the problem of controlling a planar vertical takeoff and landing (PVTOL) aircraft. The internal dynamics are stabilized using a Lyapunov-based technique and a minimum-norm strategy. However, none of the above results provides a systematic stabilization procedure. For this purpose, the present work makes use of predictive control.

The interplay between input-output feedback linearization and model predictive control has been addressed in the literature. For example, it is shown in [57] that the input-output feedback linearization is in fact a continuous version of an unconstrained model predictive controller with a quadratic performance index and an arbitrarily small prediction horizon. In [58], a control law is derived based on approximate input-output feedback linearization and used for predictive control of stable nonminimum-phase nonlinear systems. In [68], a control law is derived based on extended linearization and predictive control. The main issues therein are robustness and the use of predictive control with convenient calculation. In [69, 70, 71, 72], input-output feedback linearization is chosen as a systematic way of pre-stabilizing the possible unstable system dynamics in order to use predictive control. The emphasis therein is on handling constraints [69, 70, 71] or robustness [72], while the present work emphasizes on the possibly unstable internal dynamics.

Model predictive control (MPC) originated in the late 1970s [73, 74, 75]. It refers to a class of computer control algorithms that use a process model to predict the future response of a plant. At each implementation interval, an MPC algorithm attempts to optimize future plant behavior by computing a sequence of future manipulated variable adjustments. The first input of the optimal sequence is then applied to the plant, and the entire calculation is repeated at subsequent implementation intervals. Originally developed to meet the specialized control needs of power plants and petroleum refineries, MPC technology can now be found in a wide variety of application areas including chemicals, food processing, automotive, and aerospace applications [76, 77].

Several recent publications provide a good introduction to theoretical and practical issues associated with MPC technology. [78] provides a good introductory tutorial aimed at control practitioners. [79] provides a comprehensive review of theoretical results on the closed-loop behavior of MPC algorithms. Notable past reviews of MPC theory include those of [80, 81, 82, 83]. Also, several books on MPC have been published [84, 85, 86].

Predictive control presents a series of advantages over other methods [85, 78, 87], namely:

- It can deal with a very large variety of processes, such as nonlinear systems, nonmininmum-phase systems, and multiple-inputs multiple-outputs systems.
- It can easily deal with constraints.
- The resulting control law is easy to implement.
- It introduces a natural feedforward control action to compensate measurable disturbances.
- Since it is a numerical method of control, it can be more effective when the analytical computation of a control law is difficult.

1.3 Objectives of the Thesis

This work aims to address the problem of synthesizing a global control for nonlinear nonminimum-phase systems. Two directions are explored:

- Neglecting the internal dynamics, thus approximating the system by an input-state feedback linearizable one; and

- Stabilizing the internal dynamics, while controlling the input-output behavior of the system.

1.3.1 Neglecting the internal dynamics

This work seeks global stability results. This precludes the use of partial linearization techniques, as these would only provide local stability results. Moreover, linear model following methods are not exploited because they lead to non trivial optimizations and are computationally very demanding.

Linearization-oriented modeling techniques are preferred for the control of nonlinear nonminimum-phase systems. These techniques provide an approximate input-state feedback linearization using static state feedback and coordinate transformation. The objective is to develop an approximate feedback linearization of the system which (i) guarantees global stability of the closed-loop form, and (ii) does not require any precondition on the stability or instability of the open-loop system or on the structure of the zero dynamics.

1.3.2 Stabilizing the internal dynamics

The objective is to control the input-output behavior, as well as to stabilize the internal dynamics using a systematic method. This is done by designing a cascade-control scheme, where input-output feedback linearization forms the inner loop and model predictive control forms the outer loop. The use of model predictive control provides a systematic way of stabilizing the internal dynamics in the outer loop.

Predictive control is chosen as a systematic way of stabilizing the internal dynamics because it is a numerical control method, i.e. it is more effective when the analytical computation of a control law is difficult.

1.4 Organization of the Thesis

This thesis is organized as follows. Chapter 2 introduces the main definitions and concepts used throughout the document. It also reviews the tools and material that are necessary to understand the following chapters. Chapter 3 presents the approximate input-output feedback linearization of nonlinear systems using the observability normal form. It also provides the details of the control structure and a stability analysis. Chapter 4 addresses the theoretical results of the cascade-control scheme. These results are based on input-output feedback linearization, model predictive control and singular-perturbation theory. Stability issues are discussed, and a robust extension of the cascade control based on the neighboring extremal theory is introduced. Chapter 5 addresses the problem of controlling the pendubot, which is a double inverted polar pendulum. The approximate input-output feedback linearization and the cascade-control scheme with its robust extension are applied to the pendubot. The corresponding experimental results are presented and discussed. Finally, Chapter 6 draws concluding remarks and presents future prospects.

Chapter 2

Preliminaries

The aim of this chapter is to give the main definitions and concepts used in the thesis. It sets the notations used throughout the document and gives the necessary tools and material to understand contents of the forthcoming chapters.

This chapter is divided into two parts: Section 2.1 emphasizes nonlinear systems analysis, while Section 2.2 is dedicated to the problem of nonlinear systems control. On the analysis side, different classes of nonlinear systems are described and Lyapunov stability results are presented. On the control side, concepts of nonminimum-phase systems and zero dynamics are first defined. Then, two classical methods to control nonlinear single-input single-output affine-in-input systems: input-output feedback linearization and nonlinear model predictive are reviewed.

2.1 Analysis

2.1.1 Nonlinear Dynamical Systems

This work considers continuous, nonlinear, affine-in-input single-input single-output (SISO) systems. However, different forms of nonlinear systems are discussed in this section. In particular, autonomous systems are defined to address concept of Lyapunov stability, and singularly perturbed systems are defined for their stability properties, which will be used later in this work.

Nonlinear Affine-in-input Systems

A nonlinear, affine-in-input, single-input single-output system of order n consists of n coupled first-order ordinary differential equations, and is represented as follows:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t), \qquad x(0) = x_0 \\ y(t) &= h(x(t)) \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is a vector of states, $u \in R$ is the input, $y \in R$ is the system output, x_0 is a vector containing the value of vector $x(t)$ at time $t = 0$ (initial conditions of vector x) and $f : R^n \to R^n$ and $g : R^n \to R^n$ are nonlinear functions describing the system dynamics. $\dot{x}(t)$ denotes the derivative of the vector of states $x(t)$ with respect to time t and $h : R^n \to R$ is a nonlinear function giving the output expression y. In what follows, the time index is dropped where there is no confusion. In all this work, functions f, g and h are assumed to be sufficiently smooth in R^n, and therefore the mappings $f : R^n \to R^n$ and $g : R^n \to R^n$ are vector fields on R^n.

Equilibrium Point A point (x_{eq}, u_{eq}), where $x_{eq} \in R^n$ and $u_{eq} \in R$, is said to be an *equilibrium point* of System (2.1) if it satisfies the following equation:

$$f(x_{eq}) + g(x_{eq})u_{eq} = 0 \tag{2.2}$$

Without loss of generality, it will be assumed everywhere in this work that $(x, u) = (0, 0)$ is an equilibrium point of System (2.1), with $f(0) = 0$ and $h(0) = 0$ such that, at the equilibrium, the output is zero.

To verify this assumption, consider the change of states $\bar{x} = x - x_{eq}$ and the change of input $\bar{u} = u - u_{eq}$. Replacing them in (2.1), then using (2.2) yields:

$$\begin{aligned} \dot{\bar{x}} &= \dot{x} - \dot{x}_{eq} = \dot{x} \\ &= \bar{f}(\bar{x}) + \bar{g}(\bar{x})\bar{u} \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} \bar{f}(\bar{x}) &= f(\bar{x} + x_{eq}) + g(\bar{x} + x_{eq})u_{eq} \\ \bar{g}(\bar{x}) &= g(\bar{x} + x_{eq}) \end{aligned} \tag{2.4}$$

At the equilibrium of System (2.3), $(x, u) = (x_{eq}, u_{eq})$. Therefore, $(\bar{x}, \bar{u}) = (0, 0)$. Replacing the point $(\bar{x}, \bar{u})$ in (2.3) with its value $(0, 0)$, the right hand side of (2.3) is found to be zero. Then, the point $(\bar{x}, \bar{u}) = (0, 0)$ is an equilibrium point of (2.3) with $\bar{f}(0) = 0$.

Nonlinear Autonomous Systems

Consider a nonlinear affine-in-input system (2.1) and let the input u be specified as a given function of the states, $u = \gamma(x)$. Substituting $u = \gamma$ in (2.1) eliminates u and yields an unforced state equation:

$$\dot{x} = f(x), \qquad x(0) = x_0 \tag{2.5}$$

where $x_0 \in R^n$ is the initial condition of the vector of states.

If the function f does not depend explicitly on time, System (2.5) is said to be autonomous.

As seen in the above section, without loss of generality, the origin $x = 0$ is considered as an equilibrium point of System (2.5). The autonomous systems are used for stability analysis.

Singularly Perturbed Systems

A singularly perturbed system is one that exhibits a two-time-scale behavior, i.e. it has slow and fast dynamics, and is modeled as follows:

$$\dot{\eta} = F_\eta(\eta, \xi, u, \epsilon) \qquad \eta(0) = \eta_0 \tag{2.6}$$

$$\epsilon\dot{\xi} = F_\xi(\eta, \xi, u, \epsilon) \qquad \xi(0) = \xi_0 \tag{2.7}$$

where $\xi \in R^r$, $\eta \in R^{n-r}$, $u \in R$ and $\epsilon \geq 0$ is a small parameter. Functions F_η and F_ξ are assumed to be continuously differentiable. ξ_0 and η_0 are the initial conditions of vectors ξ and η respectively. The origin is considered as an equilibrium point, such that $F_\eta(0, 0, 0, \epsilon) = 0$ and $F_\xi(0, 0, 0, \epsilon) = 0$.

The term singular comes from the fact that System (2.6)-(2.7) contains a singularity when the perturbation parameter ϵ is equal to zero. In fact, setting $\epsilon = 0$, the dynamic equation (2.7) degenerates into the algebraic equation $0 = F_\xi(\eta, \xi, u, 0)$.

2.1.2 Mathematical Tools

In this section, some mathematical definitions are stated, which are necessary in this work.

a) Lipschitz Functions [7]

In this section, the conditions of existence and uniqueness of solution of ordinary differential equations will be discussed.

Let the following equation:

$$\dot{x} = f(x), \qquad x(0) = x_0 \tag{2.8}$$

be the mathematical model of a physical system, where $x \in R^n$ is a vector containing the states, x_0 the initial conditions on the state variables and f is a nonlinear function describing the system dynamics. For the model (2.8) to predict the future state of the system from its current state at initial time $t = 0$, the equation in (2.8) must have a unique solution.

In the next Theorem, sufficient conditions for the existence and uniqueness of a global solution to the equation (2.8) will be presented. It will be assumed that the function $f(x)$ is continuous in x. Therefore, the solution $x(t)$ if it exists, is continuously differentiable.

Theorem 2.1 Global Existence and Uniqueness *Given System (2.8) with function $f(x) \in R^n$ continuous in x. If f satisfies the Lipschitz condition:*

$$\forall x_1, x_2 \in R^n : \; \exists L > 0 : \; \|f(x_1) - f(x_2)\| \leq L\|x_1 - x_2\| \tag{2.9}$$

then, the equation (2.8) has a unique solution over R^n.

Definition 2.1 *A function $f(x)$ satisfying the Lipschitz condition (2.9) is said to be global Lipschitz in x and the real positive number L is called the* Lipschitz constant.

From the Lipschitz condition in Theorem 2.9, it is easily seen that the property of Lipschitz functions provides a limit on the growth or decay of the states. In fact, the Lipschitz condition (2.9) implies that on a plot of $f(x)$ versus x, a straight line joining any two points of $f(x)$ cannot have a slope whose absolute value is greater than L, and the solution of the differential equation of System (2.8) is bounded. The next theorem gives these bounds.

Theorem 2.2 Lipschitz Property[7] *Consider System (2.8). If $f(x)$ is Lipschitz in x on R^n with the Lipschitz constant L, then*

$$\|x_0\| \; \exp[-L\,t] \leq \|x(t)\| \leq \|x_0\| \; \exp[L\,t] \tag{2.10}$$

In this manuscript, global Lipschitz property is assumed everywhere.

b) Stability of Linear Systems

Consider the following linear system:

$$\dot{x} = Ax \qquad with\; x(0) = x_0 \tag{2.11}$$

with $A \in R^n$ is a matrix of order n, x a vector containing the states and $x(0) = x_0$ the initial conditions of the vector of states x at time $t = 0$. Let define the following matrix:

$$\Phi = \int_0^\infty e^{A^T t} M e^{At} dt \tag{2.12}$$

where $M \subset R^{r\times r}$ is a positive definite matrix.

The following results shows important properties of the matrix Φ:

Theorem 2.3 Positivity Property of Φ [88] *If all eigenvalues of the matrix A have negative real parts, then the matrix Φ defined in (2.12) is positive definite and satisfies the following Lyapunov equation:*

$$A^T\Phi + \Phi A = -M \tag{2.13}$$

2.1.3 Lyapunov Stability Theorems

Stability plays an important role in system theory and engineering. In this section, the focus is on the stability of equilibrium points of nonlinear dynamical systems with emphasis on Lyapunov's method [89].

This work focuses on exponential stability, although it is a strong property. The interest in exponential stability is justified by the fact that if a system is exponentially stable, then there exists a margin which can be used to accommodate eventual disturbances or model errors.

In order to illustrate the stability of the origin $x = 0$ of System (2.5), the following definition and theorems are introduced.

Definition 2.2 Lyapunov Function [7] *Let $x = 0$ be an equilibrium point of System (2.5). A continuously differentiable function $V(x) : R^n \rightarrow R$ is called a Lyapunov function if it satisfies*

- $V(0) = 0$;
- $V(x) > 0$ *for* $x \in R^n - \{0\}$,
- $\dot{V}(x) \leq 0$ *for* $x \in R^n$.

Theorem 2.4 Stability and asymptotic stability of nonlinear systems using Lyapunov functions [7] *Let $x = 0$ be* *an equilibrium point for System (2.5). If there exists a Lyapunov function* $V(x)$*, then the origin* $x = 0$ *is stable. If in addition* $\dot{V}(x) < 0$ *in* $R^n - \{0\}$*, then* $x = 0$ *is asymptotically stable.*

Theorem 2.5 Exponential stability of nonlinear systems using Lyapunov functions [**7**] *Let* $x = 0$ *be an equilibrium point for System (2.5). If there exist a Lyapunov function* $V(x)$ *and positive real constants* c_1*,* c_2 *and* c_3 *such that*

- $c_1\|x\|^2 \leq V(x) \leq c_2\|x\|^2$,
- $\dot{V}(x) \leq -c_3\|x\|^2$,

then the origin $x = 0$ *is exponentially stable.*

Theorem 2.6 Converse theorem on exponential stability of nonlinear systems using Lyapunov functions [**7**]

Consider System (2.5). If the origin $x = 0$ *is exponentially stable, then there exists a Lyapunov function* $W(x)$ *and positive constants* c_1*,* c_2*,* c_3 *and* c_4 *such that:*

- $c_1\|x\|^2 \leq W(x) \leq c_2\|x\|^2$,
- $\dot{W}(x) \leq -c_3\|x\|^2$,
- $\|\frac{\partial W}{\partial x}\| \leq c_4\|x\|$.

From an analysis point of view, the above theorems provide sufficient stability conditions, where a Lyapunov function is chosen to be an energy of the system. The energy is dissipative such that function $V(x)$ decreases to zero along the solution of the equations of System (2.5) subject to an initial condition $x(0) = x_0$.

2.1.4 Vanishing Perturbation Theory

In this section, results on Lyapunov stability will be applied to the stability analysis of perturbed autonomous systems.

Consider the autonomous system (2.5), and assume that it constitutes the nominal model for the following perturbed system:

$$\dot{x} = f(x) + \Delta(x), \qquad x(0) = x_0 \tag{2.14}$$

where $f(x)$ represents the nominal dynamics, with $f(0) = 0$, and $\Delta(x)$ represents the perturbed dynamics. Both f and Δ are Lipschitz in x.

Since $\Delta(x)$ is Lipschitz then

$$\exists \delta > 0 : \quad \|\Delta(x)\| \leq \delta \|x\| \tag{2.15}$$

The vanishing perturbation theory is based on the assumption that $\Delta(0) = 0$, such that the perturbation vanishes to zero at the origin. The main result is that, if the nominal system is exponentially stable and δ is smaller than a predetermined limit, then the perturbed system is also exponentially stable.

Theorem 2.7 *[7] Let $x = 0$ be an exponentially stable equilibrium point of the nominal system (2.5). Let $V(x)$ be a Lyapunov function for the nominal system satisfying*

$$\frac{\partial V}{\partial x} f(x) \leq -c_1 \|x\|^2 \tag{2.16}$$

$$\left\| \frac{\partial V}{\partial x} \right\| \leq c_2 \|x\| \tag{2.17}$$

where c_1 and c_2 are positive real constants. Also, let $\|\Delta(x)\| \leq \delta \|x\|$, where δ is a nonnegative constant. If $\delta < \frac{c_1}{c_2}$, then, the origin is an exponentially stable equilibrium point of the perturbed system (2.14).

Based on Lyapunov analysis, the above theorem provides sufficient exponential stability conditions for the perturbed system (2.14). If an autonomous system (2.5) is exponentially stable, then there exists a margin in the reduction of the Lyapunov function, which, in turn, can be used to accommodate the perturbations $\Delta(x)$.

2.1.5 Singular Perturbation Theory

a) Approximation of a Singularly Perturbed System

Singular perturbation theory provides an asymptotic method to approximate the solution of nonlinear differential equations.

Consider a singularly perturbed system (2.6)-(2.7). The asymptotic solution of its differential equation consists of computing an approximate trajectory, based on the decomposition of the singularly perturbed model, which exhibits a two-time-scale behavior, into a reduced (slow) model and a boundary-layer (fast) model.

In fact, as $\epsilon \to 0$, the dynamics of ξ in (2.7) vary rapidly. This leads to a time-scale separation, where the vector η represents the slow states and the vector ξ represents the fast states. Thus, ξ can be approximated by its quasi-steady state solution:

$$\bar{\xi} = \phi(\eta, u) \tag{2.18}$$

which is obtained solving the following algebraic equation:

$$F_\xi(\eta, \bar{\xi}, u, 0) = 0 \tag{2.19}$$

Replacing the state vector ξ by its quasi-steady state solution $\bar{\xi}$, the reduced (slow) dynamics arise:

$$\begin{aligned} \dot{\eta} &= F_\eta(\eta, \phi(\eta, u), u, 0) \\ &= \bar{F}_\eta(\eta, u), \qquad \eta(0) = \eta_0 \end{aligned} \tag{2.20}$$

System (2.20) is called the *reduced subsystem* and is an approximation of Subsystem (2.6) under the quasi-steady state assumption of the state vector ξ.

Using the changes of variables $\tilde{\xi} = \xi - \bar{\xi}$ and $\tau = \frac{t}{\epsilon}$, Subsystem (2.7) can be rewritten as:

$$\frac{d\tilde{\xi}}{d\tau} = F_{\xi}(\eta, \tilde{\xi} + \bar{\xi}, u, \epsilon) - \epsilon\, \dot{\bar{\phi}}(\eta, u) \qquad (2.21)$$

Setting $\epsilon = 0$ yields

$$\begin{aligned} \frac{d\tilde{\xi}}{d\tau} &= F_{\xi}(\eta, \tilde{\xi} + \bar{\xi}, u, 0) \\ &= \bar{F}_{\xi}(\eta, \tilde{\xi} + \bar{\xi}, u), \qquad \tilde{\xi}(0) = \tilde{\xi}_0 \end{aligned} \qquad (2.22)$$

System (2.22) is called the *boundary-layer subsystem*, and consists of the quasi-steady state model of System (2.6), considered at a different (fast) time scale than the one (slow) of the reduced subsystem (2.20).

Singular perturbations cause a multi-time scale behavior of System (2.6)-(2.7): the slow response (2.6) is approximated by the reduced model (2.20), while the discrepancy between the response of the reduced model and the response of the overall system (2.6)-(2.7) is the fast transient.

Since ϵ in (2.6)-(2.7) is a small parameter, the quasi-steady state solution $\bar{\xi}$ of the fast transient (2.7) is considered instead of the real state vector ξ. Therefore, System (2.6)-(2.7) is approximated by System (2.20)-(2.22). The solution of the above system subject to the initial conditions $\tilde{\xi}(0) = \tilde{\xi}_0$, $\eta(0) = \eta_0$ is then an approximation of the solution of System (2.6)-(2.7) subject to the initial conditions $\xi(0) = \xi_0$, $\eta(0) = \eta_0$.

b) Exponential Stability of a Singularly Perturbed System

Stability is an important property of the approximated form (2.20)-(2.22) of a singularly perturbed system (2.6)-(2.7). Once the input u is fixed $u = k(\eta, \bar{\xi})$, stability analysis of the singularly perturbed system in its closed-loop form can be obtained from the analysis of the stability of its approximated form. The following theorem gives the dependency between the stability of the singularly perturbed system in its approximate form (2.20)-(2.22) and the stability of the singularly perturbed system in its original form (2.6)-(2.7).

Theorem 2.8 *[7] Consider System (2.6)-(2.7) and assume that the following conditions are satisfied:*

- *The origin $(\eta, \xi) = (0, 0)$ is an equilibrium point,*
- *The solution $\phi(\eta, k(\eta, \bar{\xi}))$ is unique, and $\phi(0, 0) = 0$,*
- *The functions F_η, F_ξ and their partial derivatives up to order 2 are bounded for ξ in the neighborhood of $\bar{\xi}$,*
- *The origin of the boundary-layer subsystem (2.22) is exponentially stable for all η,*
- *The origin of the reduced subsystem (2.20) is exponentially stable.*

Then, there exists $\epsilon^* > 0$ such that, for all $\epsilon < \epsilon^*$, the origin of 2.6)-(2.7) is exponentially stable.

The above theorem indicates that the exponential stability analysis of System (2.6)-(2.7) directly follows from the stability results of the reduced subsystem (2.20) and the boundary-layer subsystem (2.22). Indeed, if (2.20)-(2.22) is exponentially stable, then the singularly perturbed system in its original form (2.6)-(2.7) is exponentially stable.

2.2 Control of Nonlinear Systems

The above section tackled the analysis of nonlinear systems, while the present section describes the control of nonlinear systems.

2.2.1 Input-Output Feedback Linearization

A classical way to control System (2.1) is to compute a linear controller using the first-order approximation of the system dynamics around the origin $x = 0$, which gives a local linear approximation of the system. A non-approximating linearization called *input-output feedback linearization* (IOFL) is discussed here. It consists of inverting the system dynamics given a nonlinear change of coordinates and a feedback law. IOFL is a systematic way to linearize globally part of, or all, the dynamics of System (2.1).

In order to compute the input-output feedback linearization of System (2.1), some definitions and geometric tools are recalled.

Lie Derivatives

Consider System (2.1). Differentiating the output y with respect to time yields:

$$\begin{aligned} \dot{y} &= \tfrac{\partial h}{\partial x}\dot{x} \\ &= \tfrac{\partial h}{\partial x}[f(x) + g(x)u] \\ &= L_f h(x) + L_g h(x)u \end{aligned} \tag{2.23}$$

where $L_f h(x) = \frac{\partial h}{\partial x} f(x), \qquad L_g h(x) = \frac{\partial h}{\partial x} g(x)$

The function $L_f h(x)$ is called the *Lie Derivative* of $h(x)$ with respect to $f(x)$ or along $f(x)$, and corresponds to the derivative of h along the trajectories of the system $\dot{x} = f(x)$. Similarly, $L_g h(x)$ is called the *Lie Derivative* of h with respect to g or along g, and corresponds to the derivative of function $h(x)$ along the trajectories of the system $\dot{x} = g(x)$.

Continuing the differentiation of the output $y = h(x)$ of System (2.1) with respect to time, the derivatives of the output y along the different possible directions of the trajectories of System (2.1) can be computed.

Relative Degree of a Nonlinear System

For linear systems, the input-output property of *relative degree* corresponds to the difference between the number of poles and the number of zeros of the system. For nonlinear systems, the relative degree r of System (2.1) corresponds to the number of times the output $y = h(x)$ has to be differentiated with respect to time before the input u appears explicitly in the resulting equations.

Definition 2.3 Relative Degree of a Nonlinear System *System (2.1) is said to have a relative degree* r*,* $1 \leq r \leq n$ *in* R^n *if* $\forall x \in R^n$*:*

$$L_g L_f^{i-1} h(x) = 0 \qquad i = 1, 2, \cdots, r-1 \tag{2.24}$$

$$L_g L_f^{r-1} h(x) \neq 0 \tag{2.25}$$

where $L_g L_f^i h(x) = L_g[L_f^i h(x)]$*,* $L_f^i h(x) = L_f[L_f^{i-1} h(x)]$*,* $i = 1, 2, \cdots, r-1$ *and* $L_f^0 h(x) \triangleq h(x)$*.*

Input-Output Feedback Linearization

System (2.1), with relative degree $r < n$ in R^n, where n is the order of the system, is input-output feedback linearized into *Byrnes-Isidori normal form* (Figure 2.1), according to the following steps [4]:

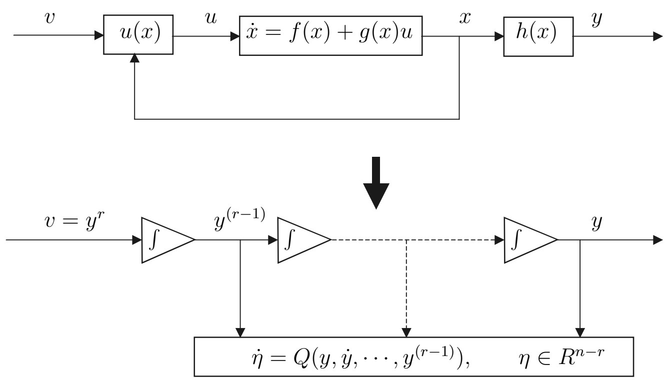


Figure 2.1: Input-output feedback linearization

1. Apply a state feedback law that compensates the nonlinearities in the input-output behavior:

$$u = \frac{v - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \tag{2.26}$$

   Since r is the relative degree, then

$$\begin{aligned} \forall\, x \in R^n : &\quad L_f^{r-1} L_g h(x) \neq 0 \\ \forall\, i < r-1 : &\quad L_f^i L_g h(x) = 0 \end{aligned}$$

2. Use the nonlinear transformation $z = T(x)$, $z = [\xi^T \ \eta^T]^T$, where $(\cdot)^T$ is the transpose operator, $\xi = [y \ \dot{y} \cdots y^{(r-1)}]^T$ is of length r and η is of length $n - r$. System (2.1) is then rewritten as:

$$y^{(r)} = v, \qquad y(0) = y_0 \tag{2.27}$$

$$\dot{\eta} = Q(y, \dot{y}, \cdots, y^{(r-1)}, \eta), \qquad \eta(0) = \eta_0 \tag{2.28}$$

   where η is chosen so that $T(x)$ is a *diffeomorphism* [1]. Q is a nonlinear function defining the time derivative of η, and $y^{(i)}$ is the i^{th} time derivative of y. Note that the dynamics (2.28) do not depend on the input u. In fact, it is always possible to choose a vector η such that its time derivative is independent of the input u [4].

The new form (2.27)-(2.28) of System (2.1) is called the Byrnes-Isidori normal form, and the dynamics (2.28) are called the internal dynamics of System (2.1).

Input-output feedback linearization decouples the input-output behavior of a nonlinear system (2.1) from the internal dynamics,

[1] Function $T : R^n \rightarrow R^n, x \longmapsto z = T(x)$ is a diffeomorphism if $\forall x \in R^n,\ \exists T^{-1} : R^n \rightarrow R^n,\ z \longmapsto x = T^{-1}(z)$

i.e. η has no effect on y. System (2.27)-(2.28) can be rewritten as a function of the new coordinates as follows:

$$\dot{\xi}_i = \xi_{i+1}, \quad i = 1, \dots, r-1 \tag{2.29}$$

$$\dot{\xi}_r = v \tag{2.30}$$

$$\dot{\eta} = Q(\eta, \xi) \tag{2.31}$$

$$\text{with } \xi_1(0) = y_0 \text{ and } \eta(0) = \eta_0$$

System (2.1) is then globally linearized in its input-output behavior, and a simple linear controller v can be used to control the system output. However, the internal dynamics are not controlled, which can be problematic if they are unstable.

Zero Dynamics

Consider System(2.1), with relative degree $r < n$. Since the equilibrium point of System (2.1) is the origin $x = 0$, then the first r coordinates ξ_i $(i = 1 \cdots r)$ of the transformed System (2.29)-(2.30) are equal to 0. At $x = 0$, $\eta = 0$ as its value can be fixed arbitrarily [4]. Therefore, $(\xi, \eta) = (0, 0)$ is an equilibrium point of System (2.1) written in its transformed form (2.27)-(2.28) with $L_f^r h(0) = 0$ and $Q(0, 0) = 0$.

Consider now the problem of zeroing the output. This is equivalent to finding all the pairs (x, u) of initial states x and input u, defined for all t in the neighborhood of $t = 0$, such that the corresponding output $y(t)$ of System (2.1), which is zero at time $t = 0$, stays at zero in the neighborhood of $t = 0$. The pair $(x, u) = (0, 0)$ is a trivial solution of the problem. However, all the pairs satisfying this property have to be found.

Let $b(\xi, \eta) = L_f^r h(x)$, $a(\xi, \eta) = L_f^{r-1} L_g h(x)$ denote the Lie derivatives of the transformed System (2.27)-(2.28), written with the new coordinates (ξ, η). Since $y(t) = 0$ in the neighborhood of $t = 0$, then $\dot{y}(t) = \ddot{y}(t) = \cdots = y^{(r-1)}(t) = 0$, yielding $\xi(t) = 0$ and $\dot{\xi}(t) = 0$. Therefore, the new input $v(t)$ in (2.27)-(2.28) is zero and the input vector of the original system u must satisfy the following equation:

$$u = -\frac{b(0, \eta^*)}{a(0, \eta^*)} \tag{2.32}$$

where $\eta^*(t)$ is the solution of the following equation:

$$\dot{\eta}^* = Q(0, \eta^*) \tag{2.33}$$

The dynamics defined in (2.33) are called the *zero dynamics* of the nonlinear system (2.1). It correspond to the internal dynamics of System (2.1) when the value of u constrains the output $y = h(x)$ to remain 0.

The concept of zero dynamics of a nonlinear system is related to that of zeros of a linear system. Indeed, the poles obtained when linearizing (2.33) at the origin $\eta = 0$ are equal to the zeros obtained when linearizing System (2.1) around the equilibrium point $(x, u) = (0, 0)$.

Nonminimum-phase Systems

Nonminimum-phase property (NMP) is an input-output property. The output $y = h(x)$ of System (2.1) is said to be nonminimum phase if System (2.27)-(2.28) contains unstable zero dynamics (2.33). In the case of linear systems, a system is nonminimum phase if it contains unstable zeros. When input-output feedback linearization is applied to a nonminimum-phase system, the unstable zero dynamics prevent the use of the standard IOFL technique for the state control.

2.2.2 Nonlinear Model Predictive Control (NMPC)

Model predictive control (MPC) is an effective feedback control approach for tackling problems with constraints and nonlinear dynamics [75, 87]. MPC makes use of a model in order to predict the future system output, based on past and current values and on the proposed optimal future actions [87, 85, 78]. These future actions are obtained by minimizing an objective function, taking future tracking errors as well as constraints into consideration.

Formulation of NMPC

The current control $u(t)$ at time t is obtained by computing online a finite-horizon open-loop optimal control $u(\tau)$ over the interval $\tau \in [t, t+T]$, where T is called the *prediction horizon*. A part of the obtained signal $u([t, t+\delta])$ is then applied to System (2.1) during the interval $[t, t+\delta]$, where $\delta < T$ is the *implementation interval*. This calculation is repeated continuously at the frequency rate $\frac{1}{\delta}$, called *re-optimization frequency*, less than or equal to the sampling frequency f_e. Since $u(t)$ depends on the states $x(t)$, this procedure leads to feedback control. The design of a model predictive control is based on the following elements:

Prediction Model A model (2.1) of the real system is used to calculate the predicted output. Therefore, it has to be as close as possible to the real system. Different strategies are used in the literature to define the prediction model [85]. For example, a disturbance model can be taken into account in order to describe the behavior that is not reflected by the basic model, such as the effect of model errors or noise.

Formulation of the optimization problem At time t, the optimization problem consists of the minimization of a cost function, which depends on the states $x(\tau)$ and input $u(\tau)$, over the interval $\tau \in [t, t+T]$. The optimization is performed under equality or inequality constraints (Figure 2.2).

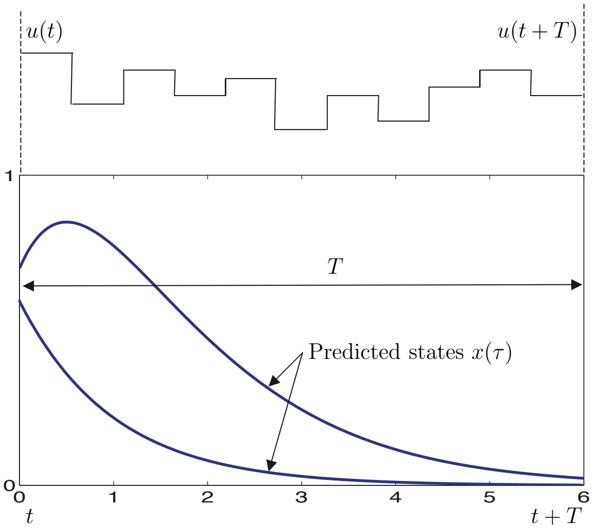


Figure 2.2: Model predictive control strategy

In this work, the selected cost function consists of two terms:

- An integral term, which consists of the time integral over the interval $[t, t+T]$ of the norms of the state vector $x(\tau)$ and the input $u(\tau)$. The integrands are respectively weighted by positive definite matrices S, R, to enhance the importance of certain elements in the computation of the optimal input. The integral term is expressed as:

$$J_1 = \frac{1}{2}\int_t^{t+T} \left(x(\tau)^T S\, x(\tau) + R\, u^2(\tau)\right) d\tau \tag{2.34}$$

- A final term, which consists of the value of the states at the end of prediction $x(t+T)$, weighted by a positive definite matrix, to enhance the importance of final states. The expression of the final term is:

$$J_2 = \frac{1}{2} x(t+T)^T P\, x(t+T) \tag{2.35}$$

Then, the overall cost function used in this work for the model predictive control of System (2.1) is given as:

$$\begin{aligned} J &= J_1 + J_2 \\ &= \frac{1}{2} x(t+T)^T P\, x(t+T) \\ &\quad + \frac{1}{2} \int_t^{t+T} \left(x(\tau)^T S\, x(\tau) + R\, u^2(\tau) \right) d\tau \end{aligned} \tag{2.36}$$

$$\begin{aligned} \text{such that} \qquad & \dot{x} = f(x) + g(x)u \\ & x(t) = x_t \\ & \text{and } u(\cdot) \in \mathcal{U},\ x(\cdot) \in \mathcal{N},\ x(t+T) \in \mathcal{N}_f \end{aligned}$$

where x_t is the vector of measured or estimated states at time t; P, S, and R are positive-definite weighting matrices of appropriate dimensions; T is the prediction horizon; $\mathcal{N}$ is the set of eligible states; $\mathcal{U}$ is the set of eligible inputs; and $\mathcal{N}_f \subset \mathcal{N}$ is a closed set containing the origin ($(x = 0) \in \mathcal{N}_f$).

The cost function can also contain equality or inequality constraints on the state variables $x(\tau)$ or the input $u(\tau)$. However, in this work, the effect of additional constraints to the optimization problem is not studied. Therefore, *unconstrained* model predictive control is used everywhere.

Computing the Control Law: The control law is obtained by solving the following minimization problem :

$$\min_{u([t,t+T])} \{J = \frac{1}{2} x(t+T)^T P\, x(t+T) + \frac{1}{2}\int_t^{t+T} \left(x(\tau)^T S\, x(\tau) + R\, u^2(\tau)\right) d\tau\} \quad (2.37)$$

$$\text{such that} \quad \dot{x} = f(x) + g(x)u$$
$$x(t) = x_t$$
$$\text{and } u(\cdot) \in \mathcal{U},\ x(\cdot) \in \mathcal{N},\ x(t+T) \in \mathcal{N}_f$$

To solve the optimization problem (2.37), input u needs to be parameterized. This is usually performed using piecewise-constant functions, which is the case in this work. However, alternative functions such as exponential functions or state feedback can be used.

Solving the minimization problem (2.37) yields the control sequence $u^*([t, t+T])$, of which only the part $u^*([t, t+\delta])$ is applied in an open-loop fashion to the plant on the interval $[t, t+\delta]$. Then, numerical optimization is repeated every δ seconds. Therefore, although System (2.1) is continuous, model predictive control is applied in a discrete time.

Asymptotic Stability of Nonlinear Model Predictive Control

In general, nonlinear predictive control design does not guarantee stability. However, an appropriate choice of parameter settings in (2.36) can guarantee stability. The most frequently used methods to guarantee the stability of model predictive control are listed in the following.

(i) End-point constraint: The states are constrained to be equal to the desired ones at the end of the prediction horizon [90, 91]. This method can make the problem unfeasible, especially if the input parameterization is not adequate.

(ii) Infinite horizon: The prediction horizon is infinite [92, 93]. However, the integration for prediction of the system dynamics over an infinite horizon can be computationally difficult, especially if the dynamics are unstable.

(iii) Contraction constraint: A contraction constraint on the states is introduced [94, 95]. Note that the choice of the contraction domain is of crucial importance for this method.

(iv) Final set constraint: Using an appropriate final cost, the output at the end of prediction belongs to an invariant set where a local controller of the system exists [96, 97]. This method is less restrictive than the end-point constraint method

In this work, stability analysis is conducted using the final set constraint method.

The issue of stability becomes more complicated in the case of constrained model predictive control. The main problem therein is ensuring feasibility, and the existence of a stabilizing control law is not trivial. However, in this work, only unconstrained nonlinear model predictive control is studied.

The results of Jadbabaie [96, 97] on asymptotic stability of nonlinear model predictive control using final set constraint are presented here. In [96, 97], asymptotic closed-loop stability of nonlinear predictive control requires the following three elements:

- An invariant set $\mathcal{N}_f$ and a control law, where the state evolves within the set,

- A function $F(x) = \frac{1}{2}x^T P x$ defined in $\mathcal{N}_f$ and a positive function $q(x)$, with $q(0) = 0$, satisfying $\dot{F} + q \leq 0$,
- The existence of a prediction horizon T such that, at the end of the prediction horizon, the system is inside $\mathcal{N}_f$.

The first two elements presented above state that, within the invariant set $\mathcal{N}_f$, there exists a control law (not necessarily the one computed from predictive control) such that the system is exponentially stable. The last element states that, even if $\mathcal{N}_f$ is arbitrarily small, the prediction horizon is chosen such that the system can reach $\mathcal{N}_f$ at the end of the prediction horizon. Using the above elements, the following theorem ascertains the asymptotic stability of the predictive control part.

Theorem 2.9 *[96] Let $(x, u) = (0, 0)$ be an equilibrium point for the nonlinear system (2.1). If the following statements are satisfied:*

- *the function $f(x)$ is Lipschitz in x,*
- *P, S and R used in (2.36) are positive definite,*
- *there exists a local controller $k(x)$ of (2.1), defined in $\mathcal{N}_f$, for which $q(x, k(x))$ satisfies the inequality $\dot{F} + q \leq 0$, with $q(0, k(0)) = 0$,*
- *the set $\mathcal{N}_f$ is positively invariant [2] with respect to u,*
- *the prediction horizon T is sufficiently large to ensure that $x^*(t + T) \in \mathcal{N}_f$, where $x^*(.)$ represents the states obtained under predictive control,*

[2] See [7] for calculation details

then, Controller (2.36) stabilizes System (2.1) asymptotically.

In this work, these results are extended to the case of exponential stability, and the pertaining results are presented in the next chapter.

Chapter 3

Approximate Input-output Feedback Linearization of Nonlinear Systems Using the Observability Normal Form

3.1 Introduction

The previous chapter has shown that input-output feedback linearization allows compensating for the nonlinearities of the system using nonlinear state feedback and nonlinear state transformation. A linear controller can then be designed to control the linearized system [4]. However, there exist many systems for which the entire system nonlinearity cannot be compensated for. Hence, various ideas related to linearization have been explored in the literature, for systems that are not input-state feedback linearizable [9].

In this work, two methods will be discussed to solve this problem. The first method is presented in this chapter. The idea is to neglect a part of the system dynamics so as to make the approximate system input-state feedback linearizable. The neglected part is then considered as a perturbation. A linear controller is designed so as to account for the nonlinearities that have not been compensated for but simply ignored. Stability is analyzed using the vanishing perturbation theory.

Section 3.2 develops the proposed approximate linearization approach, while the stability analysis of the control scheme is provided in Section 3.3. The obtained results are discussed in Section 3.4. Section 3.5 concludes the chapter.

3.2 Approximate IOFL Control Scheme

The proposed controller is computed in three steps:

1. System (2.1) is transformed into its observability normal form;
2. The transformed system is approximated as a chain of integrators, with the neglected part considered as a perturbation;
3. A linearizing feedback controller is applied to the approximate system, leading to a linear system that can be controlled using linear feedback.

3.2.1 System Transformation

Consider the nonlinear SISO affine-in-input system (2.1), and assume that:

$$\forall x \in R^n : \quad \text{span}\left\{ \left[\begin{array}{ccccc} \frac{\partial h}{\partial x} & \frac{\partial L_f h}{\partial x} & \frac{\partial L_f^2 h}{\partial x} & \dots & \frac{\partial L_f^{n-1} h}{\partial x} \end{array} \right] \right\} = n \quad (3.1)$$

The above condition states that $\frac{\partial h}{\partial x}, \frac{\partial L_f h}{\partial x}, \frac{\partial L_f^2 h}{\partial x}, \cdots, \frac{\partial L_f^{n-1} h}{\partial x}$ are linearly independent[1].

Condition 3.1 will be referred to as "strong observability". It is a more restrictive condition than the one required for the observability of System (2.1), where there is no restriction on the number of Lie derivatives necessary to span the n-dimensional space. Strong observability implies that the time-varying linearized system is observable around all operating points [98]. Consider (2.1) which satisfies the strong observability condition (3.1), with relative degree r strictly less than the system order n. Consider also the state transformation

$$z = T(x) = \begin{bmatrix} h \\ L_f h \\ L_f^2 h \\ \vdots \\ L_f^{n-1} h \end{bmatrix} \tag{3.2}$$

Note that the strong observability condition is essential here as it ensures that the transformation T is a diffeomorphism, implying that the inverse transformation $x = T^{-1}(z)$ exists and is unique for all $x \in R^n$.

Using the state transformation (3.2), System (2.1) takes on

[1] See [98] for calculation details

the following form:

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
&\vdots \\
\dot{z}_r &= z_{r+1} \quad + L_g L_f^{r-1} h \quad u \\
\dot{z}_{r+1} &= z_{r+2} \quad + L_g L_f^{r} h \quad u \\
&\vdots \\
\dot{z}_n &= L_f^n h \quad + L_g L_f^{n-1} h \quad u, \qquad z(0) = z_0
\end{aligned}
\tag{3.3}
$$

where $L_g L_f^i h(x) = 0, \forall i < r-1$ and $L_g L_f^i h(x) \neq 0$, for $i = r-1$, since System (2.1) is of relative degree r. System (3.3) corresponds to the observability normal form of System (2.1).

3.2.2 Approximation of the Transformed System

Note that, even if $L_g L_f^i h(x) \neq 0$, for $r-1 \leq i < n$, it can still be close to zero and therefore neglected. The approximation $L_g L_f^i h(x) = 0$ for $i = r-1, \cdots, n-2$ is then introduced in (3.3), yielding the following approximate system:

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
&\vdots \\
\dot{z}_r &= z_{r+1} \\
\dot{z}_{r+1} &= z_{r+2} \\
&\vdots \\
\dot{z}_n &= L_f^n h \quad + L_g L_f^{n-1} h \quad u, \qquad z(0) = z_0
\end{aligned}
\tag{3.4}
$$

The resulting system (3.4) consists of a chain of n integrators. However, the following question arises: “How close to zero does $L_g L_f^i h(x)$, $r-1 \leq i < n$ need to be in order for the approximation to be effective?”. The next section shows how to design a controller to accommodate the perturbation resulting from the approximation. Then, the subsequent section provides a sufficient condition on the perturbation size that ensures the stability of the controlled system.

3.2.3 Linearizing Feedback Control

Consider System (2.1), after transforming it into its observability normal form and approximating it with a chain of integrators. If $L_g L_f^{n-1} h \neq 0$ for all x, then the following "linearizing" feedback can be imposed:

$$u = \frac{v - L_f^n h}{L_g L_f^{n-1} h}, \tag{3.5}$$

Then, System (3.4), with the feedback law (3.5), can be rewritten in the following form:

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
&\vdots \\
\dot{z}_r &= z_{r+1} \\
\dot{z}_{r+1} &= z_{r+2} \\
&\vdots \\
\dot{z}_n &= v, \qquad z(0) = z_0
\end{aligned} \tag{3.6}$$

which can be rewritten as:

$$\dot{z} = Az + Bv, \qquad z(0) = z_0 \tag{3.7}$$

with $A \in R^n \times R^n$ and $B \in R^n$, and are given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdot & 1 \\ 0 & 0 & \cdot & \cdot & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{3.8}$$

The resulting system (3.7)-(3.8) consists of a linear system, and corresponds to an approximate input-state feedback linearization of System (2.1). Therefore, the following linear controller

$$v = -Kz, \quad K = \left[\begin{array}{ccc} k_1 & \cdots & k_n \end{array}\right] \tag{3.9}$$

can be applied to the approximate linearization (3.7)-(3.8), yielding the following linear closed-loop system:

$$\dot{z} = A_{cl}\, z, \qquad z(0) = z_0 \tag{3.10}$$

with

$$A_{cl} = \begin{bmatrix} 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdot & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_n \end{bmatrix} \tag{3.11}$$

The parameters of the vector of gains K are chosen such that the real parts of the eigenvalues of the matrix A_{cl} in (3.11) are all negative.

3.3 Stability Analysis

Applying the linear controller (3.9) on the transformed system (3.3) leads to:

$$\dot{z} = A_{cl}\, z + \Delta(z), \qquad z(0) = z_0 \tag{3.12}$$

with

$$\Delta(z) = -\begin{bmatrix} 0 \\ . \\ . \\ L_g L_f^{r-1} h \\ . \\ L_g L_f^{n-2} h \\ 0 \end{bmatrix} \frac{(Kz + L_f^n h)}{L_g L_f^{n-1} h} \tag{3.13}$$

and A_{cl} is given in (3.11).

The main advantage of the developed control method is that, using the observability normal form, the resulting perturbation (3.13) is vanishing, i.e. going to zero at the equilibrium. In fact, replacing (z, u) by their values at the equilibrium $(z, u) = (0, 0)$ in (3.3) gives $L_f^n h(z = 0) = 0$. Replacing it in (3.13) yields $\Delta(0) = 0$. Therefore, the perturbation is indeed vanishing, and the theory of vanishing perturbations presented in Chapter 2 can be used.

The stability analysis of the approximate input-output feedback linearization addresses the existence of a non-zero range of $\Delta(z)$ for which System (3.12) can be stabilized. The following theorem gives sufficient conditions for the exponential stability of System (3.12).

Theorem 3.1 *Consider System (2.1), and let the following assumptions be verified:*

- $\forall x \in R^n : \quad span\left\{\left[\begin{array}{ccccc} \frac{\partial h}{\partial x} & \frac{\partial L_f h}{\partial x} & \frac{\partial L_f^2 h}{\partial x} & \ldots & \frac{\partial L_f^{n-1} h}{\partial x} \end{array}\right]\right\} = n,$
- $\exists \delta_1 > 0 : \ |L_f^n h| \leq \delta_1 \|z\|,$
- $\exists \delta_2 > 0 : \ |L_g L_f^{n-1} h| \geq \delta_2,$

- *The gains $k_i, i = 1, \cdots n$ are chosen such that all the eigenvalues of the matrix A_{cl} in (3.11) have negative real parts,*

- *Consider the function $\Delta_1(z)$ defined as follows:*

$$\Delta_1(z) = \begin{bmatrix} 0 \\ . \\ . \\ L_g L_f^{r-1} h \\ . \\ L_g L_f^{n-2} h \\ 0 \end{bmatrix} \tag{3.14}$$

and a matrix P, the solution of the Lyapunov equation $PA_{cl} + A_{cl}^T P = -I$, where I is the identity matrix and A_{cl} is given by (3.11), with the following inequality being satisfied:

$$\|\Delta_1(z)\| < \frac{\delta_2}{2\lambda_{max}(P)(\delta_1 + \|K\|)} \tag{3.15}$$

Then, the feedback law (3.5)-(3.9) stabilizes (3.3) exponentially.

The above theorem indicates that, if the norm of the perturbation vector $\Delta_1(z)$ defined in (3.14) is smaller than a certain value, then the overall system (3.12) is exponentially stable. In particular, when $\Delta_1(z) = 0$, the perturbed system (3.12) coincides with the nominal system (3.10).

The assumption $|L_f^n h| \leq \delta_1 \|z\|$ used in Theorem 3.1 is a global Lipschitz condition, which is a strong restriction. However, this restriction is necessary to ensure the global stability of the system. If local Lipschitz conditions are imposed instead, local stability results could be obtained.

Proof: Consider the Lyapunov function $V = z^T P z$ for the nominal system (3.10), where P is a positive symmetric matrix which satisfies the Lyapunov equation $PA + A^T P = -I$. Then,

$$\begin{aligned} \tfrac{\partial V}{\partial z} A_{cl} z &= z^T P A z + z^T A^T P z \\ &= z^T (PA + A^T P) z \\ &= z^T(-I)z \\ &= -\|z\|^2 \end{aligned} \tag{3.16}$$

$$\begin{aligned} \tfrac{\partial V}{\partial z} &= z^T P + P z \\ \Rightarrow \left\|\tfrac{\partial V}{\partial z}\right\| &= \left\|z^T P + P z\right\| \\ &\leq \left\|z^T P\right\| + \|Pz\| \\ &\leq 2\,\|z\|\,\|P\| \\ &\leq 2\lambda_{max}\,\|z\| \end{aligned} \tag{3.17}$$

Consider now the same Lyapunov function applied to the perturbed system (3.12). Then

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial z} A_{cl} z + \frac{\partial V}{\partial z} \Delta(z) \\ &\leq -\|z\|^2 + 2\lambda_{max}(P)\|z\|\;\|\Delta(z)\| \end{aligned} \tag{3.18}$$

Noting that $|L_f^n h| \leq \delta_1 \|z\|$ and $|L_g L_f^{n-1} h| \geq \delta_2 > 0$, then using the expression of $\Delta(z)$ (3.13) yields:

$$\|\Delta(z)\| \leq \frac{\delta_1 + \|K\|}{\delta_2} \|\Delta_1(z)\|\;\|z\| \tag{3.19}$$

Replacing (3.19) in the expression (3.18) gives:

$$\dot{V} \leq -\|z\|^2 \left(1 - 2\lambda_{max}(P)\frac{\delta_1 + \|K\|}{\delta_2}\|\Delta_1(z)\|\right) \tag{3.20}$$

Therefore, if (3.15) is satisfied, the term between the parenthesis in (3.20) is positive. Hence, from Theorem 2.5, the feedback law (3.5)-(3.9) stabilizes (3.3) exponentially. ■

The following corollaries follow from Theorem 3.1.

Corollary 3.1 *For a given gain vector K, the closed-loop system (3.12)is exponentially stable for all* $\|\Delta_1(z)\| < \frac{\delta_2}{2\lambda_{max}(P)(\delta_1+\|K\|)}$.

The above corollary states that, for any given gain vector K, there exists a non-zero range of $\Delta(z)$ for which System (3.3) can be stabilized. However, the converse is not true, i.e., given a perturbation $\Delta(z)$, it is not always possible to find a gain vector K that stabilizes the system.

Corollary 3.2 *Let δ_1 and δ_2 be as defined in Theorem 3.1, and*

$$\begin{aligned} \delta^* &= \max_K \tfrac{\delta_2}{2\lambda_{max}(P)(\delta_1+\|K\|)} \\ &\text{s.t. } \operatorname{eig}(A_{cl}) < 0 \end{aligned} \tag{3.21}$$

For $\|\Delta_1(z)\| < \delta^*$, there exists a gain vector K such that the feedback law (3.5)-(3.9) stabilizes System (3.3) exponentially.

The maximization in Corollary (3.2) can be done numerically for given values of δ_1 and δ_2.

3.4 Discussion

There exists in the literature several methods based on an approximation of the input-output feedback linearization of a nonlinear system. In particular, in [55, 57, 58], the outputs are differentiated as many times as the order of the system instead of stopping at the relative degree as would be done with standard input-output feedback linearization. As an approximation, the input derivatives that appear in the control law are set to zero in the computation of the state feedback input. Though global stability can be established, the main disadvantage of this method is that it requires the open-loop system to be stable. In [60], an extension of the controller has been developed for unstable systems. However, the feedback is dynamic, and only asymptotic stability has been provided therein.

The idea of the methodology proposed in this work is similar to [55, 57], though a more standard representation, i.e. the observability normal form will be exploited. The system is transformed into its observability normal form, neglecting part of the dynamics so as to make the approximate system feedback linearizable, which will be considered as perturbation. The main advantages of the approximate input-output feedback linearization technique developed in this work is that:

- Since the idea is to design the linear controller so as to account for the nonlinearities that have not been compensated but simply neglected, there is no necessity that the open-loop system be stable for the stability of the closed-loop system;

- The perturbation is vanishing at the origin, thus allowing the use of the vanishing perturbation theory to prove the global exponential stability of the synthesized state feedback.

In Soroush work [55, 57], the stability analysis is based on the *small gain theorem* [2], which requires the open-loop system behavior to be stable. In this work, the objective of the approximate input-output feedback linearization is that no precondition on the stability of the open-loop system behavior is needed. Thus, vanishing perturbation theory for the stability analysis is used instead of the small gain theorem, in order to deal with unstable systems.

However, the main limitation of the developed approximate input-output feedback linearization is essentially due to the use of vanishing perturbation theory for the stability analysis, such that the stability results presented here can be very conservative. In reality, as it will be shown with experimental results in Chapter 5, a much larger perturbation could have been accommodated than that predicted by the theoretical results. On the other side, It may be possible to enlarge the domain of stability by including the perturbation into the control law instead of just neglecting it. This will be investigated in future work.

3.5 Conclusion

This chapter has presented a control scheme based on an approximate input-output feedback linearization technique, where System (2.1) is first transformed into its observability normal form. The latter is in turn approximated as a chain of integrators, neglecting part of the dynamics, and is finally controlled via a linearizing feedback.

[2] See [7, 99]

The neglected part is considered as a perturbation, which is vanishing at the origin. Thus, stability analysis is provided based on vanishing perturbation theory. Unfortunately, the stability results presented here can be very conservative. Indeed, the perturbation predicted by the theoretical results can be much smaller than the perturbation that could otherwise be accommodated.

Chapter 4

Cascade Structure with Input-output Feedback Linearization and Predictive Control

4.1 Introduction

In this thesis, solutions are developed for the control of nonlinear nonminimum-phase systems with relative degree strictly less than the system order. The first one was presented in the previous chapter. It consists of finding a stabilizing controller, based on an approximation of the system, by neglecting the internal dynamics.

The second method to control the considered class of systems is addressed in this chapter. The idea is to stabilize the internal dynamics instead of neglecting them as was done in the case of the approximate input-output feedback linearization in

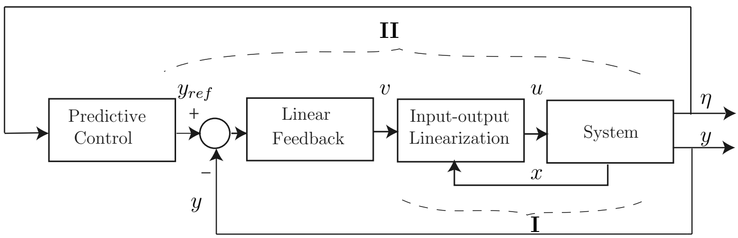
Chapter 3.

The proposed methodology is based on input-output feedback linearization, model predictive control and singular-perturbation theory. The system is first input-output feedback linearized, by separating the input-output system behavior from the internal dynamics. Predictive control is then used to stabilize the internal dynamics, using a reference trajectory of the system output as the manipulated variable. This results in a cascade-control scheme, where the outer loop consists of a model predictive control of the internal dynamics, and the inner loop is the input-output feedback linearization. Model predictive control is then performed on this scheme, at a re-optimization rate determined by the internal dynamics rather than the system dynamics. This can be advantageous if the internal dynamics are slower than the system dynamics. Stability analysis of the cascade-control scheme is provided using results of singular-perturbation theory [7].

This chapter is organized as follows: Section 4.2 develops the cascade-control scheme, while the stability analysis is provided in Section 4.3. In Section 4.4, an extension of the cascade-control scheme is proposed, using neighboring extremal theory, in order to make the cascade scheme more robust. Section 4.5 concludes the chapter.

4.2 Cascade-control Scheme

The idea of the cascade-control scheme is to use the input-output feedback linearization in the inner loop and predictive control in the outer loop. The input-output feedback linearization can be seen as a pre-compensator prior to applying predictive control, whereas predictive control can be viewed as a systematic way of controlling the internal dynamics. The following control structure is proposed (Figure 5.7):



I : $y^{(r)} = v$
II: $\dot{\eta} \quad = Q(\eta, y, \dot{y}, \cdots, y^{(r-1)}, \epsilon) \approx \bar{Q}(\eta, y_{ref}, \dot{y}_{ref}, \cdots, y_{ref}^{(r-1)}, 0)$

Figure 4.1: Cascade-control scheme using IOFL and NMPC

4.2.1 Inner loop: Linear Feedback Based on Input-output Feedback Linearization

Using the control law (2.26), System (2.1) is first input-output feedback linearized as in (2.27)-(2.28). Then, a linear output feedback is applied to the linearized input-output dynamics:

$$y^{(r)} = v, \qquad y(0) = y_0 \tag{4.1}$$

$$v = \frac{k_1}{\epsilon^r}(y_{ref} - y) - \sum_{i=1}^{r-1} \frac{k_{i+1}}{\epsilon^{r-i}} y^{(i)} \tag{4.2}$$

where $y_{ref}(t)$ is the reference trajectory for the output and its successive derivatives, $\epsilon \to 0$ a small parameter, and $k_1, \cdots, k_r$ are coefficients of a Hurwitz polynomial [7].

The gains are chosen as shown since, for any choice of $\epsilon > 0$, the closed loop is stable and ϵ can be used as a single tuning parameter.

System (2.28), (4.2) can be written in the form of a singularly perturbed system (2.6)-(2.7) by defining the fast variables

$$\xi_i = \epsilon^{i-1} y^{(i-1)}, \qquad i = 1, \ldots, r \tag{4.3}$$

Replacing ξ given in (4.3) in (2.28) yields:

$$\dot{\eta} = Q(\eta, \xi, \epsilon), \quad \eta(0) = \eta_0 \tag{4.4}$$

and replacing it in (4.2) yields:

$$\begin{aligned}
\epsilon\dot{\xi}_r &= \epsilon\frac{d}{dt}\left(\epsilon^{r-1} y^{(r-1)}\right) \\
&= \epsilon^r y^{(r)} \\
&= \epsilon^r v \\
&= \epsilon^r \left(\frac{k_1}{\epsilon^r}\left(y_{ref} - y\right) - \sum_{i=1}^{r-1} \frac{k_{i+1}}{\epsilon^{r-i}} y^{(i)}\right) \\
&= k_1\left(y_{ref} - y\right) - \sum_{i=1}^{r-1} k_{i+1}\epsilon^i y^{(i)} \\
&= \sum_{i=0}^{r-1} k_{i+1}\left(\bar{\xi}_{i+1} - \xi_{i+1}\right)
\end{aligned} \tag{4.5}$$

where

$$\bar{\xi} = \begin{bmatrix} y_{ref} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4.6}$$

Therefore, (4.3) can be rewritten as follows:

$$\epsilon\dot{\xi}_i = \xi_{i+1}, \quad i = 1, \ldots, r-1 \tag{4.7}$$

$$\epsilon\dot{\xi}_r = \sum_{i=0}^{r-1} k_{i+1}\left(\bar{\xi}_{i+1} - \xi_{i+1}\right), \qquad \xi(0) = \xi_0 \tag{4.8}$$

4.2.2 Outer loop: Model Predictive Control

The internal dynamics (4.4) depend on the output y and its derivatives $\dot{y}, \ddot{y}, \cdots y^{(r-1)}$, as well as on the small parameter ϵ. Since ϵ is small, quasi-steady-state assumption can be made. The quasi-steady-state (QSS) subsystem is computed when setting ϵ to zero in (4.4). Letting $\epsilon \to 0$ in (4.3) yields:

$$\xi_1 = y, \quad \xi_2 = \cdots = \xi_r = 0 \tag{4.9}$$

Using the above result and letting $\epsilon \to 0$ in (4.8), then:

$$\begin{aligned}\sum_{i=0}^{r-1} \quad & k_{i+1}\left(\bar{\xi}_{i+1} - \xi_{i+1}\right) = 0 \\ & \Rightarrow k_1(\bar{\xi}_1 - \xi_1) = 0 \\ & \Rightarrow \bar{\xi}_1 = \xi_1 \end{aligned} \tag{4.10}$$

Therefore:

$$\epsilon \to 0: \ \xi = \bar{\xi} = \begin{bmatrix} y_{ref} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4.11}$$

The vector $\bar{\xi}$ is the quasi-steady-state value of ξ.

It is important to note that setting ϵ to 0 in the internal dynamics (4.4) leads to undetermined fractions $y^{(i)} = \frac{\xi_{i+1}}{\epsilon^i} = \frac{0}{0}, i = 1, \cdots r-1$, because $y^{(i)} = \frac{\xi_{i+1}}{\epsilon^i}$, $i = 0, \cdots r-1$. However, since y tends to y_{ref} under the quasi-steady-state assumption, then its derivatives $\dot{y}, \cdots, y^{(r-1)}$ tend to their references $\dot{y}_{ref}, \cdots, y_{ref}^{(r-1)}$.

Since the internal dynamics can be rewritten in the following form:

$$\begin{aligned}\dot{\eta} &= Q(\eta, \xi, \epsilon) \\ &= \mathcal{Q}(\eta, y, \dot{y}, \cdots, y^{(r-1)}, \epsilon)\end{aligned} \tag{4.12}$$

then, the quasi-steady-state (QSS) subsystem is computed using the form (4.12) for the internal dynamics. Letting $\epsilon \to 0$ in (4.12), the QSS subsystem yields:

$$\dot{\eta} = \mathcal{Q}(\eta, y_{ref}, \dot{y}_{ref}, \cdots y_{ref}^{(r-1)}, 0), \qquad \eta(0) = \eta_0 \tag{4.13}$$

Then, trajectories $(y_{ref}, \dot{y}_{ref}, \cdots y_{ref}^{(r-1)})$ will be used to stabilize the internal dynamics.

In general, input-output feedback linearization decouples the input-output behavior from the internal dynamics, i.e. η has no effect on the output y. On the other hand, the quasi-steady-state assumption decouples the internal dynamics from the input-output behavior, i.e. y has no effect on the output η, although the profile of y_{ref} (an independent variable) is used to control η. Thus, the two subsystems can be handled separately.

Defining $\bar{\eta} = \begin{bmatrix} \eta, & y_{ref}, & \cdots, & y_{ref}^{(r-2)} \end{bmatrix}^T$ and $w = y_{ref}^{(r-1)} \in R^{n-1}$, the QSS dynamics (4.13) can be rewritten as:

$$\dot{\bar{\eta}} = \bar{Q}(\bar{\eta}, w), \qquad \bar{\eta}(0) = \bar{\eta}_0 \tag{4.14}$$

with

$$\bar{Q}(\bar{\eta}, w) = \begin{bmatrix} Q(\eta, y_{ref}, \cdots, y_{ref}^{(r-2)}) \\ y_{ref} \\ \vdots \\ y_{ref}^{(r-2)} \\ w \end{bmatrix} \tag{4.15}$$

Note that it is important to add additional states $y_{ref}, \cdots, y_{ref}^{(r-2)}$ since they are considered as independent variables, and the last derivative $y_{ref}^{(r-1)} = w$ is considered as the manipulated variable for stabilization.

The value of a stabilizing w^* is computed numerically using predictive control:

$$
\begin{aligned}
w^* = \arg \min_{w([t,t+T])} & \left\{ \frac{1}{2} \bar{\eta}(t+T)^T P\, \bar{\eta}(t+T) \right. \\
& \left. + \frac{1}{2} \int_t^{t+T} \left(\bar{\eta}(\tau)^T S\, \bar{\eta}(\tau) + R\, w^2(\tau) \right) d\tau \right\} \\
\text{s.}t. \quad & \dot{\bar{\eta}} = \bar{Q}(\bar{\eta}, w) \qquad \bar{\eta}(t) = \bar{\eta}_t \\
& w(\cdot) \in \mathcal{Y},\ \bar{\eta}(\cdot) \in \mathcal{N},\ \bar{\eta}(t+T) \in \mathcal{N}_f
\end{aligned}
\tag{4.16}
$$

where $\bar{\eta}_t$ is the vector of measured or estimated states at time t; P, S, and R are positive-definite weighting matrices of appropriate dimensions; T is the prediction horizon; $\mathcal{N}$ is the set of admissible states; $\mathcal{Y}$ is the set of admissible inputs; $\mathcal{N}_f \subset \mathcal{N}$ is a closed set such that it contains the origin $(\bar{\eta} = 0) \in \mathcal{N}_f$.

Under the cascade-control scheme, once System (2.1) is input-output feedback linearized and the input-output behavior is controlled with a linear feedback, then the resulting system is composed of two subsystems:

- a fast subsystem, which is the boundary-layer subsystem (4.19), with input v and output y.
- a slow subsystem, which is the QSS subsystem (4.14), with input w and output η.

The key idea of the cascade control is to stabilize the internal dynamics using y, the output of the initial system (2.1), via a predictive control. This yields a reference on the output trajectory $y_{ref}(t)$ at each optimization. The reference is reached via the linear control of the input-output behavior (4.19) of System (2.1).

Since the internal dynamics are given, thus, the cascade control is effective using large gains in the inner loop to control subsystem (4.19), such that a cascade structure is created artificially.

4.2.3 Advantages of the Cascade-control Scheme

From a feedback linearization point of view, the cascade-control scheme provides an elegant way of handling unstable internal dynamics.

From a predictive control point of view, in the cascade-control scheme, predictive control is applied to the internal dynamics instead of the overall system dynamics. Three different cases have to be distinguished:

- **Stable systems.** There is no advantage in removing the input-output behavior from the prediction problem instead of including it in the prediction, since it is stable. For stable minimum-phase systems the advantage is minor. For stable nonminimum-phase systems, it is counter productive to use the cascade-control scheme, because the input-output feedback linearization of the system can lead to more complicated internal dynamics, which makes the predictive control a difficult task compared to when it is directly applied to the overall system.

- **Unstable minimum-phase systems** In this case, input-output feedback linearization results in stable internal dynamics, and predictive control is used to guarantee performance. The advantage of the cascade scheme for this class of systems is that predictive control is applied to the stable internal dynamics instead of the unstable system dynamics, thus avoiding limitations on the time required for optimization. In other terms, the time required for optimization here can be larger than the sampling period. Examples of this class of systems include a tracking radar system where rotatable antennas are used for precise positioning of large loads [100] and a hydrofoil boat where the angular flaps position disturbed by sea waves needs to be controlled [100].

- **Unstable nonminimum-phase systems** In this case, input-output feedback linearization does not pre-stabilize but only pre-compensates the system dynamics, and predictive control is used to stabilize them. Here, the cascade-control scheme is advantageous when the unstable internal dynamics are slower compared to the unstable system dynamics. Predictive control can then be applied at a lower rate since it is applied at the rate of that of the unstable internal dynamics instead of the unstable system dynamics. For this class of systems, a natural two-time-scale separation should exist between poles and zeros of the open-loop linearized version of System (2.1). The zeros have to be slower than the poles for all possible operating points of System (2.1). Examples of this class of systems include the pendubot [101], which is used as illustrative example in this work, a X29 aircraft which flies at high angles of attack [102] and a flexible space launch vehicle which travels from liftoff at the Earth's surface through the atmosphere to low Earth orbit [103, 104].

The aforementioned time-scale separation between open-loop zeros and open-loop poles should not be confused with the other time-scale separation created using the linear controller. The latter artificial time-scale separation is between the open-loop zeros of the linearized plant (which correspond to the poles of the outer loop) and the closed-loop poles of the inner loop. This condition is necessary for the stability of the cascade-control scheme as discussed in the next section. Since the closed-loop poles of the inner loop are functions of the controller used, they can be made fast by using large gains for the inner-loop.

4.3 Stability Analysis

The stability of the cascade-control scheme is discussed in this section. The key idea is to use the time-scale separation introduced to make the cascade control effective, and which results from the use of sufficiently large gains in the inner loop. Therefore, results from singular perturbation theory can be applied.

As seen in the previous chapter, if a nonlinear system is the combination of a fast and a slow subsystems, exponential stability of both subsystems leads to exponential stability of the overall system. Exponential stability is considered because of the following: if a system is exponentially stable, then there exists a margin in the reduction of the Lyapunov function, $\dot{V}(x) \leq -c_3\|x\|^2 \leq 0$ [7]; this margin, in turn, can be used to accommodate the neglected dynamics. The following procedure will be used in this section to prove the stability of the cascade-control scheme:

- First, exponential stability of the boundary-layer subsystem with the linear controller (4.19)- (4.20) is established;
- Next, the exponential stability of the QSS subsystem (4.14) with the predictive controller (4.16) is addressed;
- Then, using singular perturbation theory, a stability proof for the cascade-control scheme, i.e. System (2.1) with Controller (4.2)-(4.16), is provided.

4.3.1 Stability of the Boundary-layer Subsystem

Using the new variable $\tilde{\xi} = \xi - \bar{\xi}$, subsystem (4.7)-(4.8) becomes:

$$\begin{aligned}
\epsilon\dot{\tilde{\xi}} &= \epsilon\dot{\xi} - \epsilon\dot{\bar{\xi}} \\
&= \begin{bmatrix} \xi_2 \\ \xi_3 \\ \vdots \\ \sum_{i=0}^{r-1} k_{i+1}\left(\bar{\xi}_{i+1} - \xi_{i+1}\right) \end{bmatrix} - \epsilon \begin{bmatrix} \dot{y}_{ref} \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\end{aligned} \tag{4.17}$$

Letting $\tau = \frac{t}{\epsilon}$ yields:

$$\begin{aligned}
\frac{d\tilde{\xi}}{d\tau} &= \begin{bmatrix} \xi_2 \\ \xi_3 \\ \vdots \\ \sum_{i=0}^{r-1} k_{i+1}\left(\bar{\xi}_{i+1} - \xi_{i+1}\right) \end{bmatrix} - \epsilon \begin{bmatrix} \dot{y}_{ref} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\xi}_2 \\ \tilde{\xi}_3 \\ \vdots \\ \sum_{i=0}^{r-1} k_{i+1}\tilde{\xi}_{i+1} \end{bmatrix} - \epsilon \begin{bmatrix} \dot{y}_{ref} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \tilde{\xi}(0) = \tilde{\xi}_0
\end{aligned} \tag{4.18}$$

Letting $\epsilon \rightarrow 0$, the boundary-layer subsystem [7] appears:

$$\frac{d\tilde{\xi}}{d\tau} = A\tilde{\xi}, \qquad \tilde{\xi}(0) = \tilde{\xi}_0 \tag{4.19}$$

with matrix A defined by:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdot & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_n \end{bmatrix} \tag{4.20}$$

Theorem 4.1 *Consider the boundary-layer subsystem (4.19) with the system matrix (4.20). If* $k_1, \cdots, k_r$ *are chosen as coefficients of a Hurwitz polynomial [7], then the origin of the boundary-layer subsystem* $\tilde{\xi} = 0$ *is exponentially stable.*

Furthermore, the following Lyapunov function can be proposed for the system:

$$W = \frac{1}{2}\tilde{\xi}^T \Phi \tilde{\xi}, \quad \Phi = \int_0^\infty e^{A^T t} M e^{At} dt \tag{4.21}$$

where $M \subset R^{r \times r}$ *is a positive definite matrix.*

Proof: Since all the eigenvalues of matrix A are negative, then Φ is positive definite. Thus, letting λ_{max} and λ_{min} denote the maximum and minimum eigenvalues of the matrix Φ respectively, the Lyapunov function candidate W given in (4.21) has the following property:

$$\lambda_{min} \|\tilde{\xi}\|^2 \leq W \leq \lambda_{max} \|\tilde{\xi}\|^2 \tag{4.22}$$

The derivative of the function W with respect to τ is given by:

$$\frac{dW}{d\tau} = \frac{1}{2}\tilde{\xi}^T (A^T \Phi + \Phi A) \tilde{\xi} \tag{4.23}$$

Since $A^T \Phi + \Phi A = -M$, then:

$$\frac{dW}{d\tau} = -\frac{1}{2}\tilde{\xi}^T M \tilde{\xi} \tag{4.24}$$

Since M is positive definite, Theorem 2.5 applies, and the origin of the boundary-layer subsystem is exponentially stable. ■

4.3.2 Stability of the Reduced Subsystem

The stability analysis of the predictive control is based on the work of Jadbabaie [96, 105], where global asymptotic stability is provided (Theorem 2.9). However, in this work, exponential stability of the predictive control is required. Therefore, more restrictive assumptions are made in order to provide the exponential stability of the predictive control of the QSS subsystem (4.14), leading to the following theorem.

Theorem 4.2 *Consider (4.14) controlled using the predictive controller (4.16). If the following assumptions are satisfied:*

1. *System (4.14) is stabilizable considering w as input,*
2. *($\bar{\eta} = 0,\ w = 0$) is an equilibrium point,*
3. *the function $\bar{Q}(\bar{\eta}, w)$ is such that $\|\bar{Q}(\bar{\eta}, w)\| \leq L\|\bar{\eta}\|$,*
4. *P, S, and R given in (4.16) are positive definite,*
5. *$\exists\, w^k = k(\bar{\eta})$ defined in $\mathcal{N}_f$, for which $F(\bar{\eta}) = \frac{1}{2}\bar{\eta}^T P\bar{\eta}$ defined in $\mathcal{N}_f$ and $q(\bar{\eta}, w^k) = \frac{1}{2}\{\bar{\eta}^T S\,\bar{\eta} + R\,(w^k)^2\}$ satisfies*
$$\dot{F}(\bar{\eta}) + q(\bar{\eta}, w^k) = \bar{\eta}^T P\,\bar{Q}(\bar{\eta}, w^k) + \bar{\eta}^T S\,\bar{\eta} + R\,(w^k)^2 \leq 0 \tag{4.25}$$
6. *the set $\mathcal{N}_f$ is positively invariant with respect to w,*
7. *the prediction horizon T is chosen sufficiently large to ensure that $\bar{\eta}^*(t+T) \in \mathcal{N}_f$, where $\bar{\eta}^*(.)$ represents the states obtained under predictive control,*

then, controller (4.16) stabilizes System (4.14) exponentially.

The differences between the above theorem (Theorem 4.2) and the results of Jadbabaie's work (Theorem 2.9) are:

- The third assumption, on the Lipschitz nature of $\bar{Q}(\bar{\eta}, w)$.
- The specific structure of the functions F and q in the fifth assumption.

The Lipschitz assumption is required to provide the lower and upper bounds on the Lyapunov function V and the assumption on the structure of F and q is used to give a bound on $\dot{V}$ as a function of the norm of vector of internal variables $\bar{\eta}$.

Proof: Consider the QSS subsystem (4.14), controlled using the predictive controller (4.16), and let the Lyapunov function candidate $V(t, \bar{\eta})$:

$$V(t, \bar{\eta}) = \min_{y_{ref}} J(t, \bar{\eta}, w) \tag{4.26}$$

where

$$\begin{array}{ll} J(t, \bar{\eta}, w) = & \frac{1}{2}\bar{\eta}(t+T)^T P\, \bar{\eta}(t+T) \\ & +\frac{1}{2}\int_t^{t+T} \left(\bar{\eta}(\tau)^T S\, \bar{\eta}(\tau) + R\, w^2(\tau)\right) d\tau \\ with & \bar{\eta}(t) = \bar{\eta}_t,\ w(\cdot) \in \mathcal{Y},\ \bar{\eta}(\cdot) \in \mathcal{N},\ \bar{\eta}(t+T) \in \mathcal{N}_f \end{array} \tag{4.27}$$

where $\bar{\eta}_t$ is the vector of measured or estimated states at time t; P, S, and R are positive-definite weighting matrices of appropriate dimensions; T is the prediction horizon; $\mathcal{N}$ is the set of admissible states; $\mathcal{Y}$ is the set of admissible inputs; $\mathcal{N}_f \subset \mathcal{N}$ is a closed set that contains the origin $((\bar{\eta} = 0) \in \mathcal{N}_f)$.

The QSS subsystem (4.14) is exponentially stable if the Lyapunov function $V(t, \bar{\eta})$ given in Equation (4.26) verifies the conditions of Theorem 2.5. Therefore, the proof proceeds into two steps:

- First, it is verified that $V(t, \bar{\eta})$ is bounded from above and below by quadratic functions of $\bar{\eta}$. An upper bound and a lower bound are provided.
- Then, the time derivative of the function $V(t, \bar{\eta})$ is shown to be bounded from above by a negative function that is quadratic in the variable $\bar{\eta}$.

Lower bound for V: Since P is a positive definite matrix, then $\bar{\eta}(t+T)^T P \bar{\eta}(t+T) > 0$. Also, $w^2 > 0$. Thus, the following inequality is satisfied:

$$V(t, \bar{\eta}) \geq \frac{1}{2} \int_t^{t+T} \bar{\eta}(\tau)^T S \bar{\eta}(\tau) d\tau \tag{4.28}$$

Also, by assumption in Theorem 4.2, the function $\bar{Q}(\bar{\eta}, w)$ satisfies the inequality:

$$\|\bar{Q}(\bar{\eta}, w)\| \leq L \|\bar{\eta}\| \tag{4.29}$$

where L a positive scalar. Then, it follows from Theorem 2.2 that:

$$\|\bar{\eta}(\tau)\| \geq \|\bar{\eta}(t)\| \exp\left[-L(\tau - t)\right], \qquad \tau \in [t, \infty) \tag{4.30}$$

Therefore, the following inequalities result:

$$\begin{aligned} V(t, \bar{\eta}) &\geq \frac{1}{2} \int_t^{t+T} \bar{\eta}(\tau)^T S \bar{\eta}(\tau) d\tau \\ &\geq \frac{1}{2} \|\bar{\eta}(t)\|^2 \|S\| \int_t^{t+T} \exp\left[-2L(\tau - t)\right] d\tau \\ &\geq \frac{1 - e^{-2LT}}{4L} \|S\| \, \|\bar{\eta}(t)\|^2 \end{aligned} \tag{4.31}$$

thus giving a lower bound on the function $V(t, \bar{\eta})$.

Upper bound for V: Consider the differential equation of System (4.14). Considering $\bar{\eta}(t) = \bar{\eta}_t$ to be the value of the initial state vector and the input $w = 0$ during the interval $[t, t+\tau]$, let $\tilde{\eta}(\tau)$ be the value of the state vector $\bar{\eta}$ at time $\tau > t$ under these conditions.

Since $V(t, \bar{\eta})$ is the minimum of the cost function J with respect to w (4.26), then:

$$V(t, \bar{\eta}) \leq J(t, \tilde{\eta}, 0) \tag{4.32}$$

By assumption in Theorem 4.2, the function $\bar{Q}(\bar{\eta}, w)$ satisfies the inequality:

$$\|\bar{Q}(\bar{\eta}, w)\| \leq L\|\bar{\eta}\| \tag{4.33}$$

with L a positive scalar. Then from Theorem 2.2:

$$\|\tilde{\eta}(\tau)\| \leq \|\bar{\eta}(t)\| \exp\left[L(\tau - t)\right], \qquad \tau \in [t, \infty) \tag{4.34}$$

Therefore, the following inequalities result:

$$\begin{aligned} V(t, \bar{\eta}) &\leq \frac{1}{2}\tilde{\eta}(t+T)^T P \tilde{\eta}(t+T) + \frac{1}{2}\int_t^{t+T} \tilde{\eta}(\tau)^T S \tilde{\eta}(\tau) d\tau \\ &\leq \frac{1}{2}\|P\|\, e^{2LT}\, \|\bar{\eta}(t)\|^2 \\ &\quad + \frac{1}{2}\|\bar{\eta}(t)\|^2 \|S\| \int_t^{t+T} \exp\left[2L(\tau - t)\right] d\tau \\ &\leq \left[\frac{1}{2}\|P\|\, e^{2LT} + \frac{e^{2LT} - 1}{4L}\|S\|\right] \|\bar{\eta}(t)\|^2 \end{aligned} \tag{4.35}$$

thus giving an upper bound on the function $V(t, \bar{\eta})$.

Condition on $\dot{V}$: Let $\tilde{w}(\tau)$, $\tau \in [t, \infty)$, be the input resulting from the concatenation of two signals:

- the input signal $w^*(\tau)$, $\tau \in [t, t+T)$, resulting from the optimization (4.26), with the initial state vector value $\bar{\eta}(t) = \bar{\eta}_t$
- the input $w^k(\tau)$, $\tau \in [t+T, \infty)$, obtained by applying the local controller $w^k = k(\bar{\eta})$ (Theorem 4.2), with the initial state vector value $\bar{\eta}^*(t+T)$ obtained from the optimization (4.26).

Consider a small variation in time t, and the notations $F(\bar{\eta})$ and $q(\bar{\eta}, w^k)$ defined in Theorem 4.2. Therefore, using the concatenate input signal $\tilde{w}$, the cost function J given in (4.27) computed at time $t + \Delta t$ is given by:

$$\begin{aligned} J(t+\Delta t, \bar{\eta}(t+\Delta t), \tilde{w}) = \; & F(\bar{\eta}(t+\Delta t+T)) \\ & + \textstyle\int_{t+\Delta t}^{t+\Delta t+T} q(\bar{\eta}(\tau), \tilde{w}(\tau))d\tau \end{aligned} \tag{4.36}$$

Since $V(t+\Delta t, \bar{\eta}) \leq J(t+\Delta t, \bar{\eta}, \tilde{w})$ (the function V being the minimum of the cost function J), then:

$$\begin{aligned} \dot{V} &= \lim_{\Delta t \to 0} \frac{V(t+\Delta t, \bar{\eta}(t+\Delta t)) - V(t, \bar{\eta}(t))}{\Delta t} \\ &\leq \lim_{\Delta t \to 0} \frac{J(t+\Delta t, \bar{\eta}(t+\Delta t), \tilde{w}) - V(t, \bar{\eta}(t))}{\Delta t} \end{aligned} \tag{4.37}$$

On the other hand:

$$\begin{aligned} J(t+\Delta t, \bar{\eta}, \tilde{w}) - V(t, \bar{\eta}) \;=\; & -\int_{t}^{t+\Delta t} q(\bar{\eta}, w^*)d\tau - F(\bar{\eta}(t+T)) \\ & + \int_{t+T}^{t+\Delta t+T} q(\bar{\eta}, w^k)d\tau \\ & + F(\bar{\eta}(t+\Delta t+T)) \end{aligned} \tag{4.38}$$

By assumption in Theorem 4.2, $\dot{F}(\bar{\eta}) + q(\bar{\eta}, w^k) \leq 0$ for all $\tau \in [T, \infty)$. Then integrating this inequality between times $(t+T)$

and $(t + \Delta t + T)$ yields:

$$F(\bar{\eta}(t+\Delta t+T)) - F(\bar{\eta}(t+T)) + \int_{t+T}^{t+\Delta t+T} q(\bar{\eta}, w^k) d\tau \leq 0 \quad (4.39)$$

Using (4.38) and (4.39), inequality (4.37) becomes:

$$\dot{V} \leq \lim_{\Delta t \to 0} \left\{ -\frac{1}{\Delta t} \int_{t}^{t+\Delta t} q(\bar{\eta}(\tau)), w^*(\tau)) d\tau \right\} \quad (4.40)$$

Since

$$\lim_{\Delta t \to 0} \left\{ -\frac{1}{\Delta t} \int_{t}^{t+\Delta t} q(\bar{\eta}(\tau)), w^*(\tau)) d\tau \right\} = q(\bar{\eta}(t)), w^*(t)) \quad (4.41)$$

then, inequality (4.40) becomes:

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2}\bar{\eta}(t)^T S \bar{\eta}(t) - \frac{1}{2} R w^*(t)^2 \\ &\leq -\frac{1}{2}\bar{\eta}(t)^T S \bar{\eta}(t) \end{aligned} \quad (4.42)$$

Since the three assumptions of Theorem 2.5 are satisfied, then Controller (4.16) stabilizes exponentially System (4.14). ■

4.3.3 Stability of the Cascade-control Scheme

Theorem 4.3 *Consider System (4.4)-(4.8) with the trajectory* $y_{ref}(t)$ *obtained by solving Optimization problem (4.16), and the input* u *computed using (2.26) and (4.2). Suppose that the following assumptions are satisfied:*

- *Function* $Q(\eta, \xi, \epsilon)$ *is such that*

  $$\|Q(\eta, \xi, \epsilon) - Q(\eta, \bar{\xi}, 0)\| \leq a\|\xi - \bar{\xi}\|$$

  with a *being a non-negative constant.*

- *The conditions of Theorem 4.2 are satisfied.*

Then, there exists $\epsilon^* > 0$ such that, for all $\epsilon < \epsilon^*$, the controller (4.2)-(4.16) stabilizes exponentially the origin $x = 0$ of System (2.1).

Proof: Letting $\bar{\eta} = \left[\begin{array}{cccc} \eta & y_{ref} & \cdots & y_{ref}^{(r-2)} \end{array}\right]^T$ and using (4.4), then:

$$\dot{\bar{\eta}} = \hat{Q}(\bar{\eta}, \xi, \epsilon) \tag{4.43}$$

Under the quasi-steady-state assumption, when $\epsilon \longrightarrow 0$, the above equation becomes:

$$\begin{aligned} \dot{\bar{\eta}} &= \hat{Q}(\bar{\eta}, \bar{\xi}, 0) \\ &= \bar{Q}(\bar{\eta}, w) \end{aligned} \tag{4.44}$$

The stability proof of the cascade scheme proceeds by finding a Lyapunov function for the closed-loop system:

$$\begin{cases} \dot{\bar{\eta}} &= \hat{Q}(\bar{\eta}, \tilde{\xi} + \bar{\xi}, \epsilon), \qquad \bar{\eta}(0) = \bar{\eta}_0 \\ \epsilon\dot{\tilde{\xi}} &= A\tilde{\xi}, \qquad \tilde{\xi}(0) = \tilde{\xi}_0 \end{cases} \tag{4.45}$$

where ξ in (4.43) has been replaced by $\tilde{\xi} + \bar{\xi}$, with $\bar{\xi}$ defined by $\bar{\xi} = [y_{ref}\, 0 \, \cdots \, 0]^T$. The following Lyapunov function candidate is proposed:

$$\nu(\bar{\eta}, \tilde{\xi}) = V(\bar{\eta}) + W(\tilde{\xi}) \tag{4.46}$$

The Lyapunov function W is as in Theorem 4.1. Since the QSS subsystem (4.14) is exponentially stable, then there exists V for the reduced internal dynamics (not necessarily the same as the one used in Theorem 4.2) that satisfies the following properties:

$$\frac{\partial V}{\partial \bar{\eta}} \bar{Q}(\bar{\eta}, w) \leq -b_1 \|\bar{\eta}\|^2 \tag{4.47}$$

$$\left\|\frac{\partial V}{\partial \bar{\eta}}\right\| \leq b_2 \|\bar{\eta}\| \tag{4.48}$$

where w is computed using (4.16), and b_1 and b_2 are positive scalars. The existence of one such function V is guaranteed by Theorem 2.6.

Moreover, functions $V(\bar{\eta})$ and $W(\tilde{\xi})$ satisfy the following inequalities:

$$\alpha_1\|\tilde{\xi}\|^2 \ \leq W(\tilde{\xi}) \leq \ \alpha_2\|\tilde{\xi}\|^2 \tag{4.49}$$

$$\beta_1\|\bar{\eta}\|^2 \ \leq V(\bar{\eta}) \leq \ \beta_2\|\bar{\eta}\|^2 \tag{4.50}$$

where α_1, α_2, β_1 and β_2 are positive scalars. Therefore:

$$\gamma_1 \left\| \begin{bmatrix} \bar{\eta} \\ \tilde{\xi} \end{bmatrix} \right\|^2 \leq \nu(\bar{\eta}, \tilde{\xi}) \leq \gamma_2 \left\| \begin{bmatrix} \bar{\eta} \\ \tilde{\xi} \end{bmatrix} \right\|^2 \tag{4.51}$$

where $\gamma_1 = \max(\alpha_1, \beta_1)$ and $\gamma_2 = \min(\alpha_2, \beta_2)$.

The derivative of $\nu(\bar{\eta}, \tilde{\xi})$ with respect to time along the trajectories of (4.45) can be calculated as:

$$\dot{\nu} = \dot{V}(\bar{\eta}) + \dot{W}(\tilde{\xi}) \tag{4.52}$$

The first term of equation (4.52) is as follows.

$$\dot{W} = \frac{1}{\epsilon}\frac{\partial W}{\partial \tilde{\xi}} A\tilde{\xi} \tag{4.53}$$

and represents the derivative of $W(\tilde{\xi})$ along the trajectories of the boundary-layer subsystem (4.19). From Theorem 4.1, this term is negative.

The second term of equation (4.52) is given as:

$$\dot{V}(\bar{\eta}) \ = \ \frac{\partial V}{\partial \bar{\eta}} \left\{ \begin{bmatrix} Q(\eta, \bar{\xi}, 0) \\ y_{ref} \\ \vdots \\ w \end{bmatrix} + \left(\begin{bmatrix} Q(\eta, \xi, \epsilon) \\ y_{ref} \\ \vdots \\ w \end{bmatrix} - \begin{bmatrix} Q(\eta, \bar{\xi}, 0) \\ y_{ref} \\ \vdots \\ w \end{bmatrix} \right) \right\}$$

$$= \frac{\partial V}{\partial \bar{\eta}} \bar{Q}(\bar{\eta}, w) + \frac{\partial V}{\partial \bar{\eta}} \begin{bmatrix} Q(\eta, \xi, \epsilon) - Q(\eta, \bar{\xi}, 0) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4.54}$$

The term $\frac{\partial V}{\partial \bar{\eta}} \bar{Q}(\bar{\eta}, w)$ in (4.54) is the derivative of $V(\bar{\eta})$ along the trajectories of the QSS subsystem (4.14) controlled by the predictive controller (4.16). From Theorem 4.2, this term is negative. The last term in (4.54) $\frac{\partial V}{\partial \bar{\eta}} \left[\begin{array}{cccc} Q(\eta, \xi, \epsilon) - Q(\eta, \bar{\xi}, 0) & 0 & \cdots & 0 \end{array} \right]^T$ expresses the effect of the quasi-steady-state assumption i.e. approximating ξ by $\bar{\xi}$ and ϵ by zero.

From the assumption of Theorem 4.3, the second term of (4.54) satisfies:

$$\|Q(\eta, \xi, \epsilon) - Q(\eta, \bar{\xi}, 0)\| \leq a \|\tilde{\xi}\|, \qquad a \geq 0 \tag{4.55}$$

From the inequalities in (4.48) and (4.55), the time derivative of $V(\bar{\eta})$ (4.54) can be upper bounded as:

$$\dot{V}(\bar{\eta}) \leq -b_1 \|\bar{\eta}\|^2 + a b_2 \|\bar{\eta}\| \|\tilde{\xi}\| \tag{4.56}$$

From Theorem 2.6, there exists some positive constant c such that:

$$\| \frac{\partial W}{\partial \tilde{\xi}} A \tilde{\xi} \| \leq -c \|\tilde{\xi}\|^2 \tag{4.57}$$

Using (4.56) and (4.57), the time derivative of the global Lyapunov function ν in (4.46) satisfies the following inequalities:

$$\begin{aligned} \dot{\nu}(\bar{\eta}, \tilde{\xi}) &\leq -b_1 \|\bar{\eta}\|^2 + a b_2 \|\bar{\eta}\| \|\tilde{\xi}\| - \frac{c}{\epsilon} \|\tilde{\xi}\|^2 \\ &\leq -\left[\begin{array}{cc} \bar{\eta} & \tilde{\xi} \end{array} \right] \begin{bmatrix} -\frac{1}{2} a b_2 & b_1 \\ \frac{c}{\epsilon} & -\frac{1}{2} a b_2 \end{bmatrix} \begin{bmatrix} \bar{\eta} \\ \tilde{\xi} \end{bmatrix} \end{aligned} \tag{4.58}$$

The right hand side of the last inequality is a quadratic form in $\begin{bmatrix} \bar{\eta} \\ \tilde{\xi} \end{bmatrix}$ and is negative for all $\epsilon < \epsilon^*$ such that

$$\epsilon^* = \frac{4b_1c}{a^2b_2^2}$$

Then, from Theorem 2.5, the origin $(\bar{\eta}, \tilde{\xi}) = (0, 0)$ is exponentially stable. Since $\bar{\eta} = \begin{bmatrix} \eta & y_{ref} & \cdots & y_{ref}^{(r-2)} \end{bmatrix}^T$, then the origin $\eta = 0$ is exponentially stable. Therefore, the origin of System (2.1) is exponentially stable. ■

The above theorem indicates that if ϵ is chosen smaller than a certain value, i.e. if the feedback gains of the inner loop are chosen sufficiently large, then the overall system is stable. This means that an effective time-scale separation needs to be created for the stability of the proposed cascade scheme to be guaranteed.

4.4 Cascade-control Scheme using Neighboring Extremal Theory

4.4.1 Robustness Issues

Problems of robustness of the proposed scheme are caused by:

Large gains of the inner loop From an implementation point of view, the use of large gains for the linear controller in the inner loop leads to measurement noise amplification and input saturation. However, it is not always necessary to use very large gains.

Modeling errors in predictive control Due to the low re-optimization frequency used for the predictive control in the cascade scheme, and the open-loop behavior of the predictive control between two optimizations, predictive control can be very sensitive to possible modeling errors.

4.4.2 Extension of the Cascade-control Scheme

In order to reduce the effect of modeling errors or disturbances, a robust extension of the developed methodology is proposed, where additional linear feedback is used whenever the numerical optimizer (nonlinear model predictive control) is unable to compute the optimal input [106].

For small deviations away from the optimal solution, a linear approximation of the QSS subsystem (4.14) and a quadratic approximation of the optimization cost (4.16) are computed. The theory of neighboring extremals (NE) provides a closed-form solution to the optimization problem [107]. Thus, the optimal input can be obtained using state feedback, which approximates the feedback provided by explicit numerical re-optimization.

Including the dynamic constraints of the optimization problem (4.16) in the cost function, the augmented cost function $\bar{J}$ can be written as:

$$\bar{J} = \Phi(\bar{\eta}(t+T)) + \int_t^{t+T} \left(H - \lambda^T \dot{\bar{\eta}}\right) d\tau \qquad (4.59)$$

where $\bar{\eta} \in R^{n-1}$, $\Phi(\bar{\eta}) = \frac{1}{2}\bar{\eta}^T P \bar{\eta}$, $H = \frac{1}{2}(\bar{\eta}^T S \bar{\eta} + Rw^2) + \lambda^T \bar{Q}(\bar{\eta}, w)$, and $\lambda(t) \neq 0$ is the $(n-1)$-dimensional vector of adjoint states or Lagrange multipliers for the system equations.

At the optimal solution, the first variation of $\bar{J}$ is given by [107]:

$$\Delta \bar{J} = \left(\Phi_{\bar{\eta}} - \lambda^T\right) \Delta\bar{\eta}\Big|_{t+T}$$

$$+ \int_t^{t+T} \left[\left(H_{\bar{\eta}} + \dot{\lambda}^T \right) \Delta\bar{\eta} + H_w \Delta w \right] d\tau \qquad (4.60)$$

where $\Delta\bar{\eta}(t) = x(t) - \bar{\eta}^\star(t)$ and $\Delta w(t) = w(t) - w^\star(t)$, with $\bar{\eta}^*$ and $\bar{\eta}^*$ being the optimal state and input, respectively. The notation $a_b = \frac{\partial a}{\partial b}$ is used.

With an appropriate choice of the adjoint states, $\dot{\lambda}^T = -H_{\bar{\eta}}$ with $\lambda^T(t+T) = \Phi_{\bar{\eta}}(t+T)$, the necessary conditions of optimality that are derived from $\Delta\bar{J} = 0$ become:

$$H_w \;=\; R\,w + \lambda^T \bar{Q}_w = 0 \qquad (4.61)$$

The second-order variation of $\bar{J}$ is given by [107]:

$$\begin{aligned} \Delta^2\bar{J} \;&= \tfrac{1}{2}\Delta\bar{\eta}(t+T)^T P\, \Delta\bar{\eta}(t+T) + \\ &\tfrac{1}{2}\textstyle\int_t^{t+T} \left[\Delta\bar{\eta}^T \; \Delta w\right] \begin{bmatrix} H_{\bar{\eta}\bar{\eta}} & H_{\bar{\eta}w} \\ H_{w\bar{\eta}} & H_{ww} \end{bmatrix} \begin{bmatrix} \Delta\bar{\eta} \\ \Delta w \end{bmatrix} d\tau \end{aligned} \qquad (4.62)$$

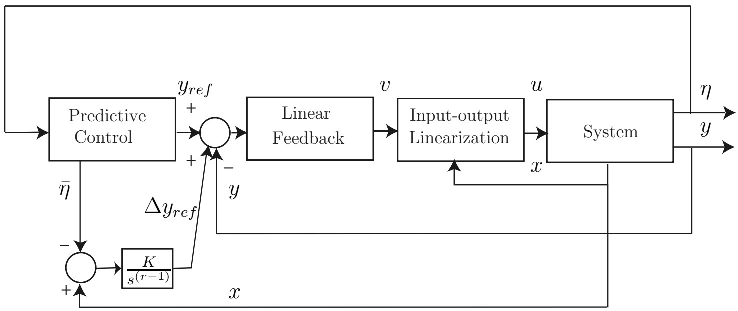
Choosing Δw to minimize $\Delta^2\bar{J}$ under the linear dynamic constraint:

$$\Delta\dot{\bar{\eta}} = \bar{Q}_{\bar{\eta}}\Delta\bar{\eta} + \bar{Q}_w \Delta w \qquad (4.63)$$

represents a time-varying Linear Quadratic Regulator (LQR) problem, for which a closed-form solution is available:

$$\begin{aligned} \Delta w(t) \;&=\; -K(t)\Delta\bar{\eta}(t) && (4.64) \\ K \;&=\; H_{ww}^{-1}\left(H_{w\bar{\eta}} + \bar{Q}_w^T \Omega\right) && (4.65) \\ \dot{\Omega} \;&=\; -H_{\bar{\eta}\bar{\eta}} - \Omega\bar{Q}_{\bar{\eta}} - \bar{Q}_{\bar{\eta}}^T\Omega + H_{\bar{\eta}w}K + \Omega\bar{Q}_w K && \\ \Omega(t+T) \;&=\; P && (4.66) \end{aligned}$$

The above controller, called the neighboring-extremal controller, is used in combination with the nonlinear model predictive control (4.16). The signal which results from their sum $w = \Delta w + w^*$ is considered as a new reference for the inner loop of the cascade-control scheme (Figure 4.2). Although w^* computed from the nonlinear predictive control is updated every δ seconds, the reference resulting from the linearized problem Δw is updated at each sampling time h.



$$y^{(r)} = v$$

$$\dot{\eta} \approx \bar{Q}(\eta, y_{ref}, \dot{y}_{ref}, \cdots, y_{ref}^{(r-1)}, 0)$$

$$\bar{\eta} = [\eta,\ y_{ref},\ \cdots,\ y_{ref}^{(r-2)}]^T$$

Figure 4.2: Cascade-control scheme using IOFL, NMPC and NE

The new control scheme also allows reducing the reoptimization frequency of the MPC problem. In fact, the additional feedback performs optimization implicitly, thereby acting as a standby whenever the numerical optimizer (nonlinear model predictive control) is unable to compute the optimal input. Therefore, model predictive control can be allowed to be re-optimized at a lower frequency.

4.5 Conclusion

This chapter presented a cascade-control scheme that combines input-output feedback linearization and predictive control to control nonlinear systems. From a feedback linearization point of view, this scheme proposes an elegant way of handling unstable internal dynamics. From a predictive control point of view, the proposed scheme has computational advantages for unstable systems. Since the predictive control is applied to the internal dynamics, re-optimization frequency can be reduced if the internal dynamics are slower than the input-output system dynamics. This makes the implementation of the predictive control easier.

Secondly, an additional feedback is combined with the predictive control problem using the neighboring extremal theory. This helps avoid problems of robustness, and also allows predictive control to run at a lower re-optimization frequency.

Clearly, the issue of constraints, which has been one of the main advantages of predictive control techniques, has not been addressed in this work. The presence of constraints would prevent the separation between the input-output dynamics and the internal dynamics, which is crucial to this work. Therefore, the proposed method cannot be directly extended to the constrained case. On the other hand, the method could easily be extended to the multi-input multi-output (MIMO) case.

Chapter 5

Application: Pendubot

5.1 Introduction

This work deals with the control of SISO affine-in-input non input-state feedback linearizable nonlinear systems. So far, two methodologies to control such systems using input-output feedback linearization techniques have been proposed. The first one is based on an approximation of the system, neglecting the internal dynamics, and finding a stabilizing controller; the second one takes the internal dynamics into account and stabilizes them simultaneously with the control of the input-output behavior of the system.

In this chapter, both approximate input-output feedback linearization and cascade-control schemes are applied to the pendubot [101]. The pendubot consists of two rotary joints; the second joint is unactuated. This system represents an interesting example for control, and has been widely used in the literature because it is nonlinear, unstable and nonminimum-phase [108, 109, 110, 111].

This system has suitable properties for the control methodologies presented previously. From the approximate input-output feedback linearization point of view, the pendubot is a nonlinear system, not input-state feedback linearizable. Also, the pendubot is nonminimum phase, which prevents the use of standard input-output feedback linearization. From the cascade-control point of view, the pendubot is an unstable nonlinear system with fast input-output dynamics. Moreover, when input-output feedback linearization is used, the internal dynamics are rather slow, so that model predictive control can be used with a low re-optimization frequency. In addition, no analytical stabilizing feedback law can be formulated for the internal dynamics, therefore justifying the use of predictive control for the stabilization of internal dynamics.

The chapter is organized as follows. The next section describes the pendubot model. The approximate input-output feedback linearization is shown in Section 5.3 together with the experimental results. Section 5.4 presents the cascade-control scheme and shows the corresponding experimental results. Section 5.5 concludes the chapter.

5.2 The Model

The pendubot (Figure 5.1) is a two-degree-of-freedom underactuated mechanical system consisting of an actuated rotating arm controlled via a DC motor, and an unactuated rotating arm.

In order to generate a model of the pendubot, let ψ denote the actuated coordinate, ϕ the unactuated one and τ the motor

Figure 5.1: Pendubot of the Laboratoire d'Automatique

torque (Figure 5.2). Consider the following notations:

$$\begin{aligned} x_1 &= l_1 \sin(\psi) \\ y_1 &= l_1 \cos(\psi) \end{aligned} \tag{5.1}$$

$$\begin{aligned} x_2 &= l \sin(\psi) + l_2 \sin(\phi) \\ y_2 &= l \cos(\psi) + l_2 \cos(\phi) \end{aligned} \tag{5.2}$$

which correspond to the projections of the center of masses of the two arms on the vertical plan. l_1 is the length from the first joint to the center of mass of the first arm, l_2 is the length from the second joint to the center of mass of the second arm and l is the length of the first arm (Figure 5.2). The kinetic and

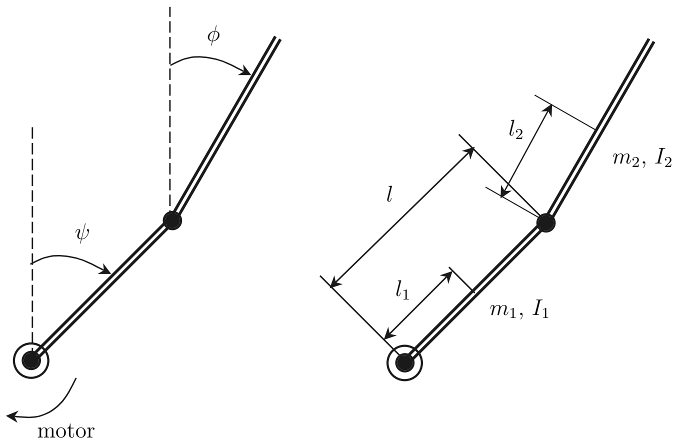


Figure 5.2: Pendubot structure

potential energies are expressed as:

$$\begin{aligned} E_{kinetic} &= \tfrac{1}{2}m_1\dot{x}_1^2 + \tfrac{1}{2}m_1\dot{y}_1^2 + \tfrac{1}{2}m_2\dot{x}_2^2 + \tfrac{1}{2}m_2\dot{y}_2^2 + \tfrac{1}{2}I_2\dot{\phi}^2 + \tfrac{1}{2}I_1\dot{\psi}^2 \\ E_{potential} &= m_1 g y_1 + m_2 g y_2 \end{aligned} \tag{5.3}$$

where m_1, m_2 are the masses of the two arms, I_1, I_2 are the inertias of the two arms computed at their respective centers of mass, and g is the gravitational acceleration. The Lagrangian is given by:

$$\mathcal{L} = E_{kinetic} - E_{potential} \tag{5.4}$$

thus yielding:

$$u = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\psi}} - \frac{\partial \mathcal{L}}{\partial \psi} \tag{5.5}$$

$$0 = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{\partial \mathcal{L}}{\partial \phi} \tag{5.6}$$

where u is the inertial control such that

$$u = \tau - b\dot{\psi} - c_1 sign(\dot{\psi}) \tag{5.7}$$

where τ is the torque produced by the DC motor; $b\dot{\psi}$ and $c_1 sign(\dot{\psi})$ are the viscous and static frictions, respectively, caused by the motor.

The pendubot is then described by the following set of differential equations:

$$J_1\ddot{\psi} + J\cos(\psi - \phi)\ddot{\phi} + J\sin(\psi - \phi)\dot{\phi}^2 - g_1\sin(\psi) = u \tag{5.8}$$

$$J_2\ddot{\phi} + J\cos(\psi - \phi)\ddot{\psi} - J\sin(\psi - \phi)\dot{\psi}^2 - g_2\sin(\phi) = 0 \tag{5.9}$$

where $g_1 = (m_1 l_1 + m_2 l_2)g$, $g_2 = m_2 l_2 g$ are the gravity components, $J = m_2 l_1 l_2$, $J_1 = m_1 l_1^2 + m_2 l_1^2 + I_1$, $J_2 = m_2 l_2^2 + I_2$ the inertia components, c_1 the Coulomb friction coefficient, and b_1 the viscous friction coefficient. Table 5.1 shows the physical values of the parameters obtained from measurements on the experimental setup [112].

parameter	value	unit
g_1	2.20	N
g_2	0.36	N
J	1.85x10^{-2}	kg m^2
J_1	0.22	kg m^2
J_2	1.90x10^{-2}	kg m^2
c_1	1.12	N m
b	6.60x10^{-2}	N s rad^{-1}

Table 5.1: System parameters

5.2.1 State-Space Formulation

The state-space formulation of the pendubot consists of rewriting the equations (5.8)-(5.9) as a set of differential equations of first order. Consider ϕ as the output and let $x = \left[\begin{array}{cccc}\phi, & \dot{\phi}, & \psi, & \dot{\psi}\end{array}\right]^T$. The pendubot equations (5.8)-(5.9) can be rewritten as:

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned} \tag{5.10}$$

where y represents the output, u is the input, x is the state-space vector. Hence, one has:

$$f(x) = \begin{bmatrix} \dot{\phi} \\ -J_1\alpha - J\cos(\psi - \phi)\beta \\ \dot{\psi} \\ J\cos(\psi - \phi)\alpha + J_2\beta \end{bmatrix} \tag{5.11}$$

$$g(x) = \begin{bmatrix} 0 \\ -aJ\cos(\psi - \phi) \\ 0 \\ aJ_2 \end{bmatrix} \tag{5.12}$$

$$h(x) = \phi \tag{5.13}$$

with

$$\begin{cases} a &= \frac{1}{J_1J_2 - J^2\cos^2(\psi-\phi)} \\ \alpha &= a(-g_2\sin\phi - J\sin(\psi - \phi)\dot{\psi}^2) \\ \beta &= a(g_1\sin\psi - J\sin(\psi - \phi)\dot{\phi}^2) \end{cases} \tag{5.14}$$

5.2.2 Relative Degree of the Pendubot

Differentiating the output $\phi = h(x)$ successively with respect to time t yields:

$$\begin{aligned}L_g h(x) &= \frac{\partial h}{\partial x} g(x) \\ &= 0\end{aligned} \tag{5.15}$$

where $L_g h(x)$ is the Lie derivative of $h(x)$ along g. However, the second derivative is given by:

$$\begin{aligned} L_g L_f h(x) &= \frac{\partial L_f h}{\partial x} g(x) \\ &= aJ\cos(\psi - \phi) \\ &\neq 0 \end{aligned} \tag{5.16}$$

indicating that System (5.10) is of relative degree $r = 2$, under the condition $|\psi - \phi| \neq (2k+1)\frac{\pi}{2}$, with k an integer different from 0.

5.2.3 Nonminumun-phase Behavior of the Pendubot

To test the nonminimum-phase property of the pendubot, the output ϕ and its derivative are replaced in Equation (5.9) by their values at equilibrium: $\phi = 0$, $\dot{\phi} = 0$. This yields:

$$\ddot{\psi} = \dot{\psi}^2 \tan\psi \tag{5.17}$$

whose solution is given by:

$$\psi(t) = \arcsin\left(\dot{\psi}(0)\cos\left(\psi(0)\right)t + \sin\left(\psi(0)\right)\right) \tag{5.18}$$

where t is time, $\psi(0)$ and $\dot{\psi}(0)$ are the values of ψ and $\dot{\psi}$, respectively, at time $t = 0$. Noting that $\psi(t) \not\to 0$ as $t \to \infty$, then the zero dynamics (5.18) is not asymptotically stable, and System (5.10) is nonminimum phase (see Chapter 2). Moreover, the linearization of the zero dynamics around the origin (5.18) gives $\ddot{\psi} = 0$, i.e. a double integrator which corresponds to slow internal dynamics.

5.3 Approximate I-O Feedback Linearization of the Pendubot

5.3.1 Description of the Control Scheme

The objective here is to control the unactuated angle ϕ and stabilize the pendulum angle ψ to the upright position. This is achieved according to the following steps:

1. **Observability Normal Form of the Pendubot:** First, System (5.10) is transformed into its observability normal form using the state-space transformation

$$z = T(x) = \begin{bmatrix} h \\ L_f h \\ L_f^2 h \\ L_f^3 h \end{bmatrix} \tag{5.19}$$

where $h(x) = \phi$ and $L_f h(x) = \dot{\phi}$. The lengthly expressions of $L_f^2 h(x)$ and $L_f^3 h(x)$ are included in Appendix A.2.

Then, the observability normal form of System (5.10) has the following expression:

$$\begin{cases} \dot{z}_1 &= z_2 & & \\ \dot{z}_2 &= z_3 & +L_g L_f h & u \\ \dot{z}_3 &= z_4 & +L_g L_f^2 h & u \\ \dot{z}_4 &= L_f^4 h & +L_g L_f^3 h & u, \qquad z(0) = z_0 \end{cases} \tag{5.20}$$

Again, the expressions of $L_g L_f h(x)$, $L_g L_f^2 h(x)$ and $L_g L_f^3 h(x)$ are included in Appendix A.2.

2. **Approximation of the Transformed System:** System

(5.20) can be rewritten as follows:

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ z_4 \\ L_f^4 h \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ L_g L_f^3 h \end{bmatrix} u + \begin{bmatrix} 0 \\ L_g L_f h \\ L_g L_f^2 h \\ 0 \end{bmatrix} u, \quad z(0) = z_0 \tag{5.21}$$

The vector $\begin{bmatrix} 0 \\ L_g L_f h \\ L_g L_f^2 h \\ 0 \end{bmatrix}$ in (5.21) is supposed to be small enough so that its contribution can be neglected. This yields the following approximated system:

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= L_f^4 h + L_g L_f^3 h\, u, \qquad z(0) = z_0 \end{cases} \tag{5.22}$$

which corresponds to a chain of integrators.

3. **Linearizing Feedback Control:** Finally, the residual nonlinearity that contains the approximated transformed system (5.22) is feedback linearized using the following control law:

$$u = \frac{v - L_f^4 h}{L_g L_f^3 h} \tag{5.23}$$

where v is a linear feedback control:

$$v = -Kz \tag{5.24}$$

Then, System (5.22) can be rewritten as:

$$z^{(4)} = -Kz, \qquad z(0) = z_0 \tag{5.25}$$

System (5.25) consists of a linear system, and corresponds to an approximate input-state feedback linearization of System (5.10). The components of the gain vector $K = [k_1\, k_2\, k_3\, k_4]$ are chosen such that the closed-loop system (5.25) is stable.

5.3.2 Stability Analysis

The stability of the control scheme is discussed in this section. The global stability of the control scheme cannot be proved because of the presence of singularities in $L_g L_f^3 h$. However, the local stability of the controller (5.23)-(5.24) can be provided.

Applying the controller (5.23)-(5.24) on the transformed but non approximated system (5.21) leads to:

$$\dot{z} = A_{cl} z + \Delta(z), \qquad z(0) = z_0 \tag{5.26}$$

where

$$A_{cl} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1 & -k_2 & -k_3 & -k_4 \end{bmatrix} \tag{5.27}$$

and

$$\Delta(z) = -\begin{bmatrix} 0 \\ L_g L_f h \\ L_g L_f^2 h \\ 0 \end{bmatrix} \frac{Kz + L_f^4 h}{L_g L_f^3 h} \tag{5.28}$$

Expressions of $L_g L_f h(x)$, $L_g L_f^2 h(x)$ and $L_g L_f^3 h(x)$ are given in Appendix A.2.

Next, using $y = \phi$ as output for the pendubot, it will be shown that the local versions of the assumptions of Theorem 3.1 are not verified.

- As for the observability condition, consider the following matrix

$$M(z) = \begin{bmatrix} \frac{\partial h}{\partial x} & \frac{\partial L_f h}{\partial x} & \frac{\partial L_f^2 h}{\partial x} & \frac{\partial L_f^3 h}{\partial x} \end{bmatrix}$$

  where expressions of $L_f h(x) = \dot{\phi}$, $L_f^2 h(x)$ and $L_f^3 h(x)$ are shown in Appendix A.2.

  Setting $z = 0$, then:

$$M(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 20.64 & 0 & -8.195 & 0 \\ 0 & 20.64 & 0 & -8.195 \end{bmatrix}$$

  whose determinant is non zero and is equal to 67.15. Thus, the distribution $span\{M(0)\}$ has dimension 4, and System (5.10) is locally strongly observable.

- It is impossible to prove analytically that $|L_f^n h|$ is Lipschitz in z because of its complicated and heavy expression. However, this can be done locally and numerically. In fact, setting a small neighborhood of the origin $z = 0$: $\mathcal{N}_0 = \{z \in R^n : \ \|z\| \leq \epsilon, \ \epsilon > 0\}$, and computing $L_f^4 h$, the constant parameter δ_1 can be approximated numerically such that $\forall z \in \mathcal{N}_0 : \ |L_f^4 h| \leq \delta_1 \|z\|$. Setting $\epsilon = 0.004$ yields $\delta_1 = 29.05$.

- Because of the presence of singularities in $L_g L_f^3 h$, it cannot be verified everywhere that its value is limited by below. However, this property can be verified locally. Computing numerically $L_g L_f^3 h$ in the set $\mathcal{N}_0$, the positive constant δ_2 can be approximated such that $\forall z \in \mathcal{N}_0 : \ |L_g L_f^3 h| \geq \delta_2$. Setting $\epsilon = 0.004$ yields $\delta_2 = 140.05$.

- The gains of the linear control (5.24) have been chosen such that the closed-loop poles are set at -1, -2, -10 and -10.
- Then, the sufficient condition for exponentially stability is:

$$\begin{aligned} \|\Delta_1\| &< \frac{\delta_2}{2\lambda_{max}(P)\,(\delta_1 + \|K\|)} \\ &< \frac{140.05}{2 \times 31.08\,(29.05 + 427.05)} \\ &< 4.94 \times 10^{-3} \end{aligned}$$

  where $\lambda_{max}(P)$ is the maximum eigenvalue of the matrix P which is the solution to the Lyapunov equation $PA + A^T P = -I$. I is the identity matrix.

  However, the maximum value of Δ_1, computed numerically in the neighborhood of the origin of length $\epsilon = 0.004$, is 4.82. Therefore, the last assumption of Theorem 3.1 is not verified and the stability of the control scheme cannot be implied.

Although the assumptions of Theorem 3.1 are not verified locally using $y = h(x) = \phi$ as output, it can be shown that they are satisfied if the output is chosen to be $y = \bar{h}(x) = J\psi\cos(\psi - \phi) + J_2\phi$. The assumptions are verified the same way, and the following values are found:

- Consider the matrix

$$\bar{M}(z) = \begin{bmatrix} \frac{\partial \bar{h}}{\partial x} & \frac{\partial L_f \bar{h}}{\partial x} & \frac{\partial L_f^2 \bar{h}}{\partial x} & \frac{\partial L_f^3 \bar{h}}{\partial x} \end{bmatrix}$$

  Setting $z = 0$, then:

$$\bar{M}(0) = \begin{bmatrix} 0.019 & 0 & 0.0185 & 0 \\ 0 & 0.019 & 0 & 0.0185 \\ 0.36 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 \end{bmatrix}$$

whose determinant is non zero and is equal to 4.43×10^{-5}. Thus, the distribution $span\{\bar{M}(0)\}$ has dimension 4, and System (5.10) is locally strongly observable.

- The value of δ_1 is computed numerically similarly to the previous controller. The resulting value is $\delta_1 = 159.63$.
- δ_2 is determined numerically, and its value is $\delta_2 = 1.73$.
- The gains are the same as for the previous controller.
- The condition on the maximum value for Δ_1 is then: $\|\Delta_1\| < 4.75 \times 10^{-5}$. Computing $\|\Delta_1\|$ numerically in the neighborhood of the origin, its maximum value is 3.41×10^{-5}. Thus the assumption is verified locally, and the closed-loop system is locally exponentially stable.

The value of $\epsilon = 0.004$ is small because of the large values for the gain vector K. This value can be increased using smaller gains. For example, if the poles of the closed-loop system are fixed at -1, -1.1, -1.2 and -1.3, corresponding to $K = [1.72\ 6.02\ 7.91\ 4.6]$, then the value of ϵ can be increased ten times to $\epsilon = 0.03$.

5.3.3 Experimental Results

The approximate input-output feedback linearization using the observability normal form has been applied to the pendubot. The experimental results are presented in this section. The implementation environment and the map of the real-time control of the pendubot, including the hardware and software, is described in Appendix A.2. The initial conditions are set as follows: First, the pendubot is set initially at the position $\phi = \psi = \pi$,

$\dot{\phi} = \dot{\psi} = 0$. Then, a swing up based on energy control (see Appendix A.2) is applied to the pendubot. Once the pendubot arms are inside a pre-defined domain, the swing up switches to the controller. Thus, the pendubot is controlled to the up-right position. Finally, the initial conditions for the experiments are set manually, in order to have initial conditions for the derivatives at zero. Here, the initial conditions are $\phi_0 \approx 0$, $\psi_0 = \frac{\pi}{6}$, $\dot{\phi}_0 = \dot{\psi}_0 = 0$.

First, the angle ϕ is considered as output. Although stability cannot be proved when using $y = \phi$ as output, it is seen from the experimental results that the control scheme is stable. This is due to the fact that the stability results here are very conservative. In fact, as mentioned in Chapter 3, the perturbations predicted by the theoretical results are much smaller than the perturbations that could otherwise be accommodated. Secondly, $y = J\psi \cos(\psi - \phi) + J_2\phi$ is considered as output, and the corresponding controller is applied to the pendubot. The vector of gains used for the linear controller for both choices of output is $K = [200\ 340\ 162\ 23]$.

Experimental Results using Output $y = \phi$ The approximate input-output feedback linearization using the observability normal form is applied to the pendubot, considering $y = \phi$. The implementation results are shown in Figures 5.3-5.4. Figure 5.3 shows the evolution of the pendubot angles ϕ and ψ, and Figure 5.4 shows the input u.

Experimental Results using Output $y = J\psi \cos(\psi - \phi) + J_2\phi$ The approximate input-output feedback linearization using the observability normal form is applied to the pendubot, considering

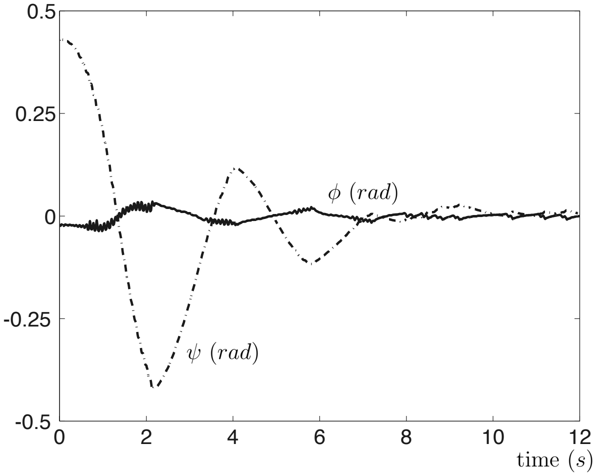


Figure 5.3: Performance of the approximate IOFL scheme with the output $y = \phi$: Angular positions ϕ (-), ψ (- -)

$y = J\psi\cos(\psi - \phi) + J_2\phi$. The implementation results are shown in Figures 5.5-5.6. Figure 5.5 shows the evolution of the pendubot angles ϕ and ψ, and Figure 5.6 shows the input u.

Although the stability cannot be provided for the first controller (choice of output ϕ), both controllers show similar performances. The approximate input-output feedback linearization using the observability normal form gives good results when applied to the pendubot. Indeed, the angles evolution is smooth and the input energy is small.

Although the nonlinear controller stabilizes the system, it does not show a much better performance when compared to that of an LQR. This is essentially due to the presence of singu-

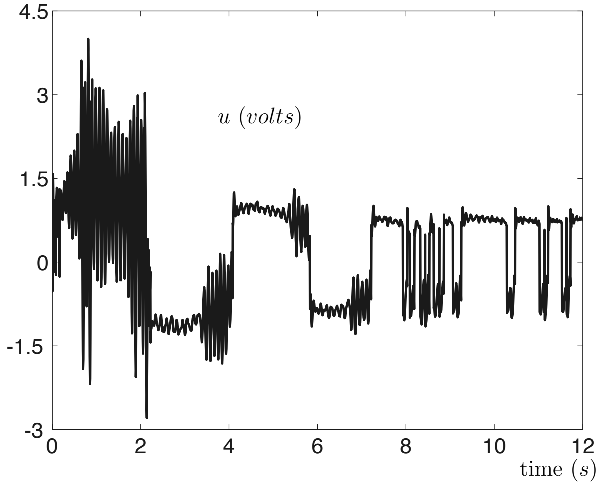


Figure 5.4: Performance of the approximate IOFL scheme with the output $y = \phi$: Input u

larities in $L_g L_f^3 h$, which prevent the use of the controller (5.23) far away from the equilibrium point.

5.4 Cascade Control of the Pendubot

5.4.1 Description of the Control Scheme

The objective is to control the unactuated angle ϕ to the upright position and, at the same time, stabilize the pendulum angle ψ to the upright position. These two tasks are considered separately in the following control structure (Figure 5.7):

1. **Inner Loop: Input-output Feedback Linearization and Linear Feedback.** First, System (5.10) is input-output

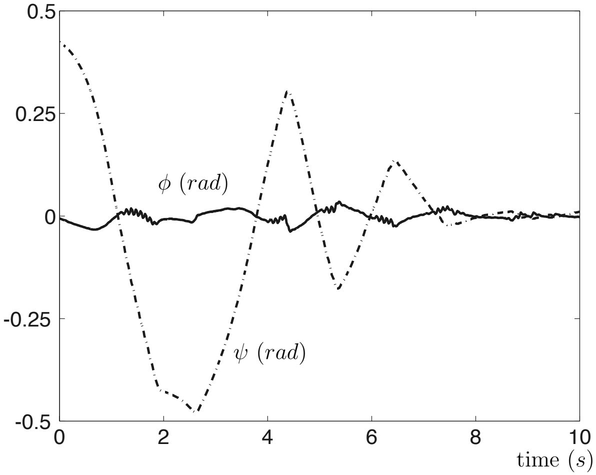


Figure 5.5: Performance of the approximate IOFL scheme with the output $y = J\psi\cos(\psi - \phi) + J_2\phi$: Angular positions ϕ (-), ψ (- -)

feedback linearized into Byrnes-Isidori normal form using the following steps:

- Apply a state feedback law that compensates the nonlinearities in the input-output behavior:

$$u = \frac{v - J_1\alpha - J\cos(\psi - \phi)\beta}{aJ\cos(\psi - \phi)} \tag{5.29}$$

  where a, α and β are given by (5.14).

- Consider the nonlinear transformation:

$$z = T(x) = \begin{bmatrix} \phi \\ \dot{\phi} \\ \eta_1 \\ \eta_2 \end{bmatrix} \tag{5.30}$$

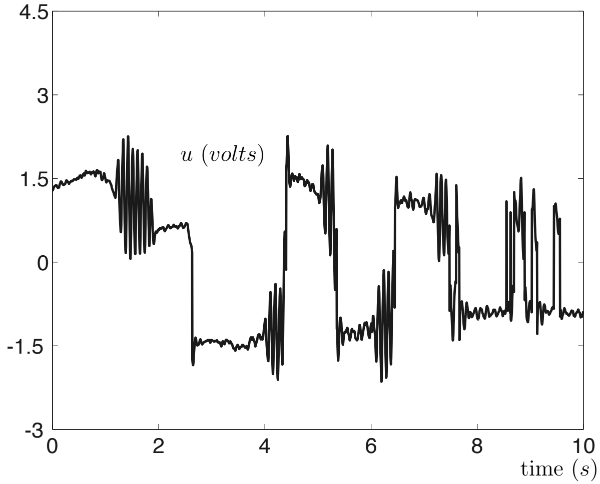


Figure 5.6: Performance of the approximate IOFL scheme with the output $y = J\psi\cos(\psi-\phi) + J_2\phi$: Input u

with

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} J\sin(\psi-\phi) \\ J\dot{\psi}\cos(\psi-\phi) + J_2\dot{\phi} \end{bmatrix} \tag{5.31}$$

Under the above transformation, System (5.10) can be expressed as:

$$\ddot{\phi} = v, \qquad \phi(0) = \phi_0,\ \dot{\phi}(0) = 0 \tag{5.32}$$

$$\dot{\eta} = Q(\eta, \phi, \dot{\phi}), \qquad \eta(0) = \eta_0 \tag{5.33}$$

with

$$Q(\eta, \phi, \dot{\phi}) = \begin{bmatrix} -\dot{\phi}\sqrt{J^2-\eta_1^2} + \eta_2 - J_2\dot{\phi} \\ \frac{\eta_1\dot{\phi}}{\sqrt{J^2-\eta_1^2}}\left(\eta_2 - \dot{\phi}J_2\right) \\ +g_2\sin(\phi) \end{bmatrix} \tag{5.34}$$

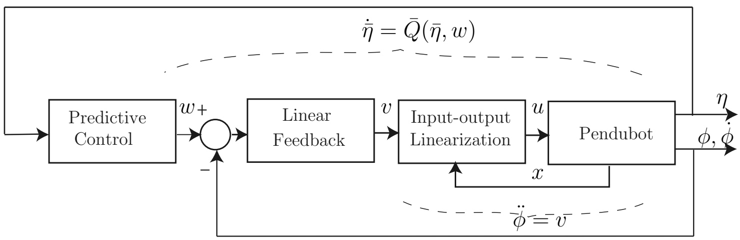


Figure 5.7: Cascade control scheme

where Q is the nonlinear function defining the dynamics of η. The particular choice of η_2 makes the dynamics independent of $\ddot{\phi}$. Also, η_1 is chosen to be the integral of η_2 when $\dot{\phi} = 0$.

The system output ϕ is controlled by a linear feedback control that computes the new input v introduced in (5.29):

$$v = -\frac{1}{\epsilon^2}(\phi - \phi_{ref}) - \frac{2}{\epsilon}(\dot{\phi} - \dot{\phi}_{ref}) \qquad (5.35)$$

where ϕ_{ref} and $\dot{\phi}_{ref}$ are references for the output ϕ and its derivative $\dot{\phi}$, respectively, and are determined by the outer loop of the cascade-control. $\epsilon \to 0$ is a small parameter. The closed-loop poles of the linearized subsystem correspond to a double pole at $-\frac{1}{\epsilon}$. The gains are chosen this way since, for any choice of $\epsilon > 0$, the closed-loop subsystem is stable and ϵ can be used as a single tuning parameter.

2. **Outer Loop: Stabilization of the Internal Dynamics using Nonlinear Model Predictive Control**

The internal dynamics of the pendubot (5.33) depend on both ϕ and its derivative $\dot{\phi}$. However, since the parameter ϵ is small, the quasi-steady-state assumption can be made. This leads to $\phi \rightarrow \phi_{ref}$ and $\dot{\phi} \rightarrow \dot{\phi}_{ref}$ in (5.33)-(5.34). Then, the trajectories $(\phi_{ref}, \dot{\phi}_{ref})$ will be used to stabilize the internal dynamics.

The internal dynamics under quasi-steady-state assumption can be written as:

$$\dot{\bar{\eta}} = \bar{Q}(\bar{\eta}, w), \qquad \bar{\eta}(0) = \bar{\eta}_0 \tag{5.36}$$

where

$$\bar{Q}(\bar{\eta}, w) = \begin{bmatrix} \bar{\eta}_2 - w\left(\sqrt{J^2 - \bar{\eta}_1^2} + J_2\right) \\ g_2 \sin(\bar{\eta}_3) + \frac{\bar{\eta}_1 w}{\sqrt{J^2 - \bar{\eta}_1^2}} (\bar{\eta}_2 - w J_2) \\ w \end{bmatrix} \tag{5.37}$$

and

$$w = \dot{\phi}_{ref} \tag{5.38}$$

$$\bar{\eta} = \begin{bmatrix} \bar{\eta}_1 \\ \bar{\eta}_2 \\ \bar{\eta}_3 \end{bmatrix} = \begin{bmatrix} J \sin(\psi - \phi_{ref}) \\ J\dot{\psi} \cos(\psi - \phi_{ref}) + J_2 \dot{\phi}_{ref} \\ \phi_{ref} \end{bmatrix} \tag{5.39}$$

Note that it is important to add an additional state ϕ_{ref} since it is considered an independent variable. Its derivative w is the manipulated variable for stabilization.

An analytical solution for w that stabilizes the internal dynamics under quasi-steady-state assumption cannot be computed. Thus, predictive control is used to compute numerically the value of a stabilizing w^*:

$$w^* = \arg \min_{w([t,t+T])} \left\{ \frac{1}{2} \bar{\eta}(t+T)^T P \, \bar{\eta}(t+T) \right.$$

$$+\frac{1}{2}\int_{t}^{t+T}\left(\bar{\eta}(\tau)^{T} S\,\bar{\eta}(\tau) + R\,w^{2}(\tau)\right)d\tau\Big\} \qquad (5.40)$$

$$\text{such that} \quad \dot{\bar{\eta}} = \bar{Q}(\bar{\eta}, w) \qquad \bar{\eta}(t) = \bar{\eta}_t$$

$$w(\cdot) \in \mathcal{Y},\ \bar{\eta}(\cdot) \in \mathcal{N},\ \bar{\eta}(t+T) \in \mathcal{N}_f$$

where $\bar{\eta}_t$ are the measured or estimated states at time t; T is the prediction horizon; $\mathcal{Y}$ and $\mathcal{N}$ are the sets of admissible outputs and internal states, respectively; $\mathcal{N}_f \subset \mathcal{N}$ is a closed set that contains the origin. The input w is updated every δ sec, where time δ is greater than or equal to the sampling time.

5.4.2 Stability Analysis

This section discusses the stability of the cascade-control scheme. The key idea is to introduce a time-scale separation in order to be able to use results from singular-perturbation theory, which is enforced here by the presence of the small parameter ϵ.

The assumptions 1 to 7 of Theorem 4.2 can be easily verified for the pendubot example:

1. System (5.36) is not affine in w. However, it is affine in $\dot{w}$. Therefore, the controllability of (5.36) is shown considering $\dot{w}$ as input.

   System (5.36) can be rewritten as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\bar{\eta}} \\ \dot{w} \end{bmatrix} &= \bar{f}(\bar{\eta}, w) + \bar{g}(\bar{\eta}, w)\dot{w} \\ \text{s.t.} \quad & \bar{\eta}(0) = \bar{\eta}_0,\ w(0) = w_0 \end{aligned} \qquad (5.41)$$

where

$$\bar{f} = \begin{bmatrix} \bar{\eta}_2 - w\left(\sqrt{J^2 - \bar{\eta}_1^2} + J_2\right) \\ g_2 \sin(\bar{\eta}_3) + \frac{\bar{\eta}_1 w}{\sqrt{J^2 - \bar{\eta}_1^2}} (\bar{\eta}_2 - w J_2) \\ w \\ 0 \end{bmatrix}$$

$$\bar{g} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

System (5.41) is controllable if the distribution $\Delta(\bar{\eta}, w) = span\{g, ad_f g,\ ad_{ff} g, ad_{fff} g\}$, with $ad_f g = L_g f - L_f g$, has dimension 4, for all $(\bar{\eta}, w)$. It is locally controllable since $\Delta(0,0)$ has dimension 4 [99]. Computing Δ yields a nonlinear function of $\bar{\eta}$ and w, which is not of dimension 4 everywhere. However $\Delta(0,0)$ has dimension 4, and (5.41) is locally controllable using $\dot{w}$ as input. Hence, System (5.36) is locally controllable using w as input.

2. By replacing $(\bar{\eta}, w)$ with $(0,0)$ in (5.37), it can be easily verified that $(\bar{\eta}, w) = (0,0)$ is an equilibrium point.

3. Replacing $w = 0$ in (5.37), and computing the norm of $\bar{Q}(\bar{\eta}, 0)$ gives $\|\bar{Q}(\bar{\eta}, 0)\| < L\|\bar{\eta}\|$, with $L = 1$.

4. P, Q and R are chosen positive definite.

5. and 6. (conditions on the final control w^k and the final set $\mathcal{N}_f$ respectively) depend on item 7. (condition on the prediction horizon T). Here, the prediction horizon $T = 0.6$ sec, for which the pendubot is stable. Therefore, there exists $\mathcal{N}_f$ and w^k which satisfy points 5 and 6.

It is important to note that only local stability of the cascade control of the pendubot can be verified. In fact, an important drawback of the methodology used here is that input-output feedback linearization of the pendubot causes a singularity when $\cos(\psi - \phi) = 0$. At the singularity, the feedback linearizing input (5.29) is infinite!

5.4.3 Experimental Results

Experimental Results with the Cascade Scheme Experimental results of applying the cascade-control scheme to the pendubot are discussed in this section. The parameter of the inner-loop controller (5.35) is $\epsilon = 0.05$. The parameters of the outer-loop controller (5.40) are

$$R = 1, Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } P = \begin{bmatrix} 4.67 & 5.91 & 1.17 \\ 5.91 & 24.32 & 4.89 \\ 1.17 & 4.89 & 2.17 \end{bmatrix}$$

.

The matrix P is computed by solving the algebraic Ricatti equation of the linear quadratic regulation (LQR) problem of the linearized version of Subsystem (5.36)-(5.37) [75]. The choice of the re-optimization time δ is chosen to be $\delta = 0.3$ sec, corresponding to 60 times the sampling period h $= 0.005$ sec.

The initial conditions of the pendubot arms are set the same way that for the approximate input-output feedback control of the pendubot. Here, the initial conditions are $\phi_0 \approx 0$, $\psi_0 = \frac{\pi}{3}$, $\dot{\phi}_0 = \dot{\psi}_0 = 0$.

The experimental results for the cascade-control scheme are presented in Figures 5.8-5.9, where Figure 5.8 shows the evolution of the pendubot angles ϕ and ψ, and Figure 5.9 shows the input u. Although the pendubot is stable, the angle ψ oscillates considerably. This is due to the fact that model predictive control is applied open loop between two re-optimizations. In particular, ψ is fed back only every 0.3 sec. This is the main disadvantage of using a low re-optimization frequency in the presence of disturbances. The measurements are filtered using Butterworth digital filters of the fourth order. Thus, the disturbances here come essentially from the modeling errors, in particular from the values of the viscous and static frictions parameters.

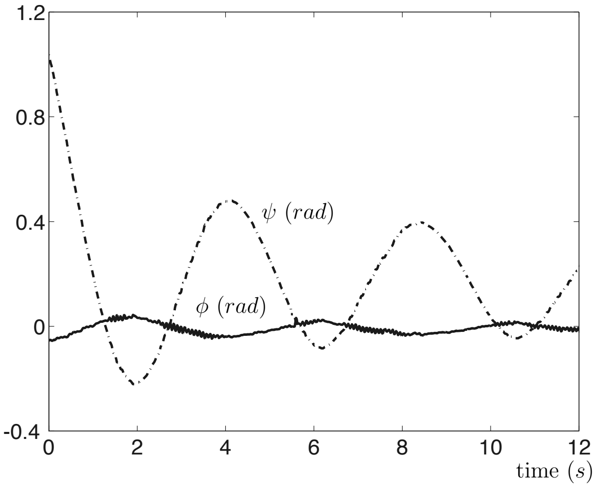


Figure 5.8: Performance of the cascade-control scheme: Angular positions ϕ (-), ψ (- -)

Experimental Results with the Cascade Scheme and Neighboring Extremals The results of neighboring-extremal theory are ex-

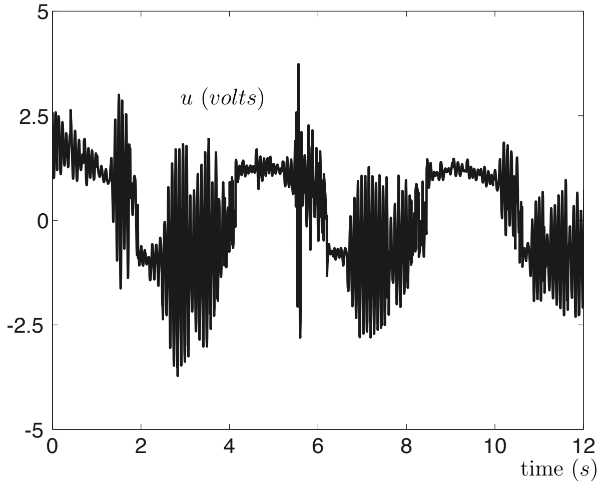


Figure 5.9: Performance of the cascade-control scheme: Input u

ploited in order to solve the problem of oscillations caused by the low re-optimization frequency of the predictive control. An additional linear feedback is used whenever the numerical optimizer (nonlinear model predictive control) is unable to compute the optimal input (Figure 5.10). The feedback is computed based on the analytical law derived from the linearized problem:

- Linear model predictive control is applied on the linearized internal dynamics under quasi-steady-state assumption (5.36)-(5.37), which leads to the linear state feedback $\bar{w}$:

$$\begin{aligned} \bar{w} \quad &= -\,\bar{\eta}_1 - 4.67\,\bar{\eta}_2 - 2.12\,\phi_{ref} \\ \dot{\phi}_{ref} \; &= \bar{w}, \qquad \phi_{ref}(0) = \phi_0 \end{aligned} \tag{5.42}$$

- This linear state feedback is used in combination with the nonlinear model predictive control (w^* is given by (5.40)), yielding the new reference $w = \bar{w} + w^*$ for the inner loop of the cascade-control scheme. Although w^* is updated every 0.3 sec, the reference $\bar{w}$ is updated at each sampling period.

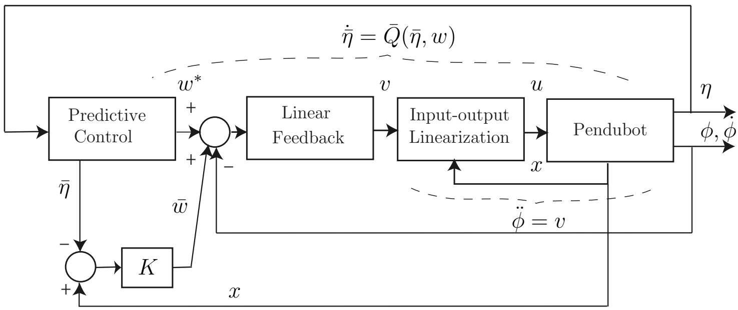


Figure 5.10: Cascade-control scheme using neighboring extremals

Figures 5.11-5.12 present the experimental results of the application of the cascade-control scheme, using the neighboring-extremal theory, to the pendubot. Figure 5.11 shows the evolution of the pendubot angles ϕ and ψ, and Figure 5.12 shows the input u. With the neighboring-extremal approach, the angles are much smoother than when using nonlinear model predictive control on its own. Moreover, the angles converge faster to the origin and the input energy is smaller. The presence of the feedback of ψ at every sampling time helps reject the effect of measurement noise.

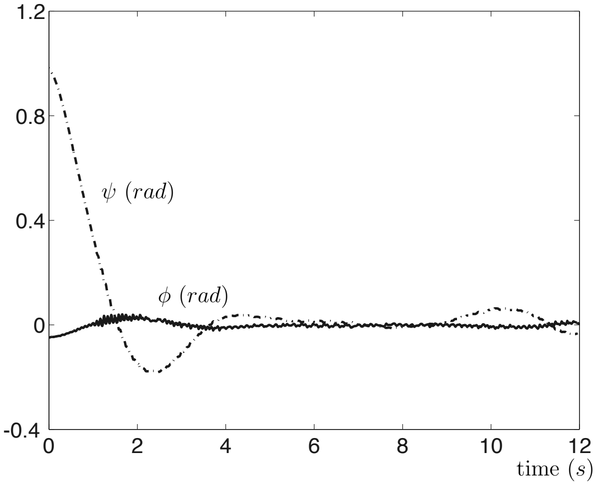


Figure 5.11: Performance of the cascade-control scheme using neighboring extremals: Angular positions ϕ (-), ψ (- -)

5.5 Conclusion

In this chapter, the two control methodologies that have been developed in this work have been applied to the pendubot, with satisfactory results.

The first methodology approximates the system to a input-state feedback linearizable one. Therefore, a input-state linearization on the approximate system can be made and a linear feedback is used for control. Although the experimental results are satisfactory, only local stability of the control scheme has been provided.

The second control methodology takes all the dynamics into account. Input-output feedback linearization is first applied to

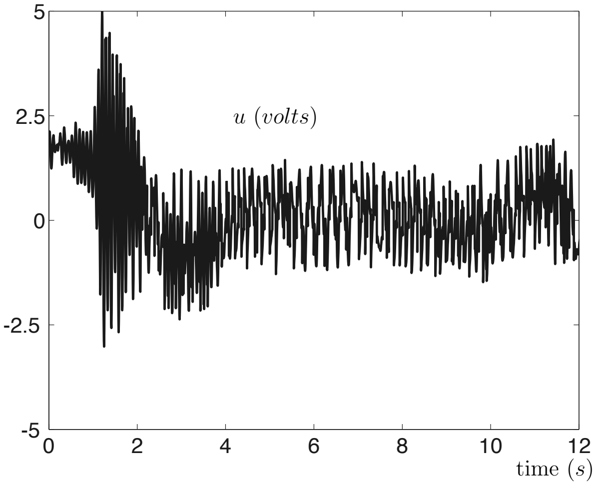


Figure 5.12: Performance of the cascade-control scheme using neighboring extremals: Input u

separate the fast input-output system dynamics of the pendubot from the slow internal dynamics. Model predictive control is then used to stabilize the internal dynamics and is implemented at a lower frequency. The application of this methodology to the pendubot has shown excellent experimental results. However, only the local stability proof has been provided.

Chapter 6

Conclusions

6.1 Summary

The main objective of this work was the control of affine-in-input SISO nonlinear systems, using input-output feedback linearization techniques. This thesis has focused on the case of not input-state feedback linearizable systems. Standard input-output feedback linearization is not effective to control such systems because uncontrolled residual dynamics, called internal dynamics, arise. Since these dynamics can be unstable, their issue needs to be considered.

The main contribution of this thesis for the control of not input-state feedback linearizable systems using input-output feedback linearization techniques has taken two directions:

6.1.1 Neglecting the Internal Dynamics

Approximate input-output feedback linearization using the observability normal form

The proposed controller is computed in three steps. First, the system is transformed into its observability normal form. Then the transformed system is approximated by a chain of integrators, and the neglected part is considered as a perturbation. Finally, a linearizing feedback controller is applied to the approximate system, yielding a linear system that can be controlled using linear feedback.

Stability results

The main contribution of the proposed method is the stability results. In Soroush work, the stability analysis is based on the *small gain theorem*, which requires the open-loop system behavior to be stable. In this work, the objective of the approximate input-output feedback linearization is that no precondition on the stability of the open-loop system behavior is needed. Thus, vanishing perturbation theory for the stability analysis is used instead of the small gain theorem, in order to deal with unstable systems.

However, the main limitation of the developed approximate input-output feedback linearization is essentially due to the use of vanishing perturbation theory for the stability analysis, such that the stability results presented here can be very conservative. In reality, a much larger perturbation could have been accommodated than that predicted by the theoretical results.

Application

The approximate input-output feedback linearization using the observability normal form has been applied to the pendubot with satisfactory results. Although the stability results are only local, the pendubot shows an interesting problem for the approximate input-output feedback linearization, since the neglected dynamics are unstable.

6.1.2 Stabilizing the Internal Dynamics

Cascade-control scheme using IOFL and nonlinear predictive control

The second methodology developed in this work takes the internal dynamics into account instead of neglecting them. The proposed methodology is based on input-output feedback linearization, model predictive control and singular-perturbation theory. The system is first input-output feedback linearized, separating the input-output system behavior from the internal dynamics. Predictive control is then used to stabilize the internal dynamics, using a reference trajectory of the system output (considered as the manipulated variable). This results in a cascade-control scheme, where the outer loop consists of a model predictive control of the internal dynamics, and the inner loop is the input-output feedback linearization.

The main contributions of the cascade-control scheme are: from an input-output feedback linearization point of view, predictive control is a systematic way to stabilize internal dynamics; from a predictive control point view, the re-optimization frequency is determined by the internal dynamics rather than the system dynamics. This can be advantageous if the unstable internal dynamics are slower than the unstable system dynamics.

Stability results

Stability analysis of the cascade-control scheme is provided using results of singular-perturbation theory. The key idea is to introduce a time-scale separation, enforced by the introduction of a small parameter in the controller. Therefore, the quasi-steady-state assumption is made, allowing the input-output behavior of the system to be decoupled from its internal dynamics. Thus, each subsystem is analyzed separately, providing a stability proof of the overall system.

The main contribution of the stability analysis is the global exponential stability of the predictive control scheme. The use of singular perturbation theory requires exponential stability results. However, exponential stability results of the model predictive control of unstable systems is a challenging task. Here, a non restrictive condition (Lipschitz condition) is added to standard conditions for asymptotic stability of predictive control, providing the global exponential stability of the predictive control scheme.

As for the robustness of the predictive control, an extension of the proposed cascade has been developed, such that an additional linear state feedback (based on the neighboring extremal theory) is combined with the predictive control problem, leading to a new reference for the inner loop of the cascade-control scheme. This mainly allows reducing the re-optimization frequency of the MPC problem. In fact, the additional feedback performs optimization implicitly, thereby acting as a standby whenever the numerical optimizer (nonlinear model predictive control) is unable to compute the optimal input. Therefore, model predictive control can be re-optimized at a lower frequency.

Application

Both the cascade-control scheme and its robust extension have been applied to the pendubot. The cascade-control scheme presented good results. However, the angles oscillate a lot due to the low re-optimization frequency of the predictive control. This is due to the presence of disturbances that are not handled by the predictive control, as the latter is applied open loop between two re-optimizations.

The robust extension shows much better results. Therein, the predictive control is updated at a low frequency, while the linear controller is updated at each sampling period.

6.2 Perspectives and Remarks

Several topics related to the proposed control methodologies can be considered for future study.

6.2.1 Approximate Input-output Feedback Linearization Control Scheme

The stability results of the approximate input-output feedback linearization presented in this work can be very conservative. This is due to the use of the vanishing perturbation theory for the stability analysis. It may be possible to widen the domain of stability by including the perturbation into the control law instead of just neglecting it. This could be investigated in future work.

6.2.2 Cascade-Control Scheme

- Although the cascade-control scheme was developed in this work for the control of single-input single-output systems, the obtained results can be easily extended to the MIMO case.

- Although constraints are one of the selling points of predictive control techniques, they have not been addressed in this thesis. The presence of constraints prevents the separation between the input-output dynamics and the internal dynamics, which is crucial to this work. Thus, the proposed method cannot be directly extended to the constrained case.

- Although the developed extension of the proposed cascade using the neighboring extremal theory, serves essentially to avoid problems of robustness, a full and rigorous robustness analysis has not been done. This issue is interesting to address.

- When applying the cascade-control scheme to the pendubot, input-output feedback linearization causes a singularity, thus decreasing the attraction domain of the controller. This singularity can be interpreted as a change in the relative degree of the system. Moreover, one wonders whether it is appropriate to use input-output feedback linearization in the inner loop or whether another pre-compensator should be used. An important issue would be the extension of the cascade scheme to the case of variable relative degree of the system.

- When the extension of the cascade scheme using the neighboring extremal theory is applied to the pendubot, the controller is not global because of the singularity in the inner-loop. One way to widen the attraction domain could be to remove the input-output linearization. In this case, only predictive control is applied to the overall system dynamics, in conjunction with a linear controller resulting from the first-order approximation of the system dynamics.

Appendix A

The Pendubot

A.1 Hardware and Software

The hardware and software which are used for the control of the pendubot are illustrated in this section. A hardware map is illustrated in Figure A.1. The control was done using a Macintosh computer, with operating system OS 9.

The software used was: `LabView 6i` and `CodeWarrior 1.7.4` (for the c code). `LabView 6i` provided the I/O between the plant and the controller(C-code), graphical interface and the real-time kernel, and `Codewarrior 1.7.4` was the C-compiler.

The map of the real-time program is illustrated in Figure A.2

States explanation:

- **State 0** : Configuration of the input/output (I/O) ports A, B and C.

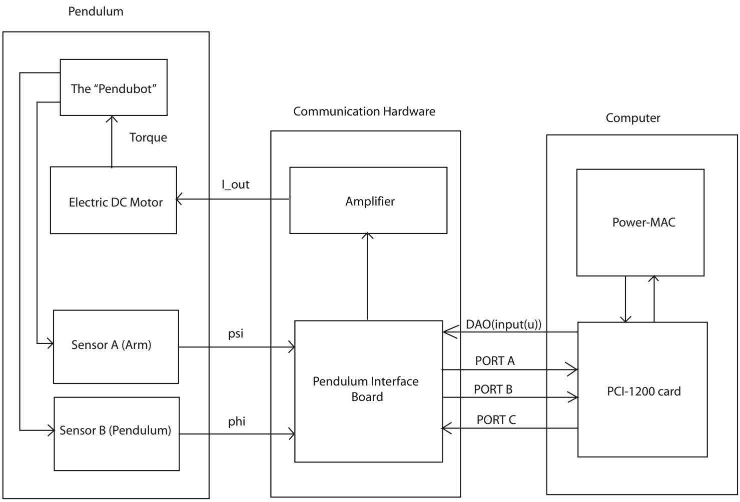


Figure A.1: Hardware map

- **State 1**: Wait one sampling time, which is necessary when calculating the angular velocity, such that in State 0 the first measurement is not available due to the fact that the I/O ports are not defined yet.

- **State 2**: Move fast clockwise. The speed of the arm is controlled by a PI-controller.

- **State 3**: Move slowly clockwise. The speed is PI-controlled as in State 2, but slower. States 2 and 3 are used for calibrations of the sensors.

- **State 4**: Wait 3 seconds. Manually resetting the sensors here.

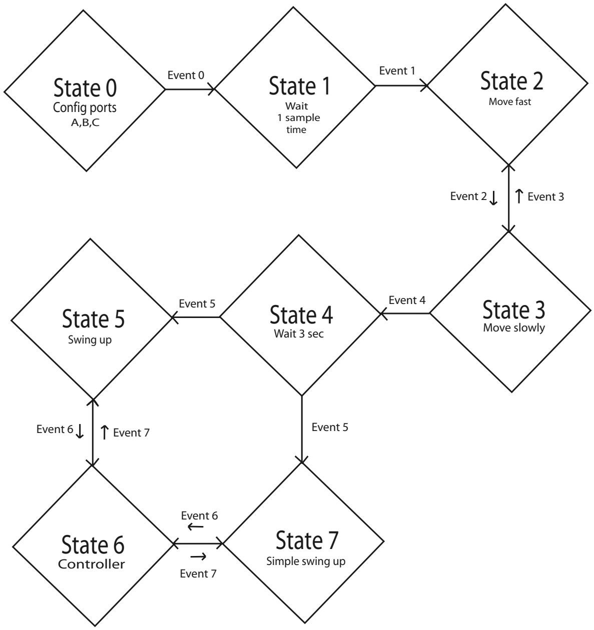


Figure A.2: Real-time chain

- **State 5**: Swing up control. Since neither approximate input-output feedback linearization nor cascade controls are global, because of the singularity at $|\phi - \psi| = \frac{\pi}{2}$, a swing up based on energy control [113] is computed, in order to push the pendubot dynamics to the respective attraction domains. A force is applied to the angle ψ angle, which will make the pendubot oscillate, destabilizing it. Once the ϕ angle passes through $\frac{\pi}{2}$ the signal input is switched to the actual controller. The input for swing up is as follows:

$$u = \frac{1}{a_2}\big((J_1 J_2 - J^2\cos(\psi-\phi))\psi_{ref} - a_1\big) \tag{A.1}$$

with

$$\begin{aligned}
a_1 &= J\cos(\psi-\phi)(-g_2\sin(\phi) + J\sin(\psi-\phi)^2 \\
&\quad \dot{\psi}^2) + J_2(g_1\sin(\psi) + J\sin(\psi-\phi)\dot{\phi}^2 \\
&\quad -b\dot{\psi} - c_1 sign(\dot{\psi}) \\
a_2 &= J\cos(\psi-\phi) + J_2 \\
\psi_{ref} &= -k_2\dot{\psi} - k_1\left(\psi - k_3\arctan(\dot{\phi}-\dot{\psi})\right) \\
and &\quad k_1 = 3.2,\ k_2 = 6.4,\ k_3 = 0.5
\end{aligned}$$

where parameter values of J, J_1, J_2, g_1, g_2, c_1 and b are given in Table 5.1.

- **State 6**: The arms are inside the attraction domain. Thus, the swing up switches to the controller. In the cascade-control scheme, for predictive control of the internal dynamics, the optimization was made using the "Quasi-newton method" method [114, 115], which is a gradient method, where instead of computing the hessian, an algorithm is there to approximate the inverse of the hessian. The integration for prediction is computed using runge-kutta fourth order method [116, 117].

- **State 7**: The arms are outside the attraction domain. Thus, the controller switches to the swing up, until the controller catches it again.

Event explanation:

- **Event 0** : Next sample period.
- **Event 1**: Next sample period.
- **Event 2**: Arm pointing almost straight down.
- **Event 3**: Arm pointing passed: arm pointing down-area,
- **Event 4**: Arm pointing downwards pendulums angular velocity is zero, $\dot{\phi} = 0$.
- **Event 5**: After 3 sec go to swing up.
- **Event 6**: ψ is in upper half($|\psi| < \frac{\pi}{2}$) and $|\phi| < \frac{\pi}{2}$.
- **Event 7**: $|\phi - \psi| \geq \frac{\pi}{2}$. This choice was made, since it represents the singularity caused by the input-output feedback linearization of the pendubot.

A.2 Approximate Input-output Feedback Linearization

Denoting $t_s = \sin(\psi - \phi)$ and $t_c = \cos(\psi - \phi)$, the structures of the functions used in (5.19) are given below:

$$\begin{aligned} L_f^2 h(x) &= \frac{1}{J_1 J_2 - J^2 t_c^2}[J_1 g_2 \sin(\phi) + J_1 J t_s \dot{\psi}^2 \\ &\quad -J t_c g_1 \sin(\psi) + J^2 t_c t_s \dot{\phi}^2] \end{aligned}$$

$$\begin{aligned} L_f^3 h(x) &= \frac{1}{J_1^2 J_2^2 - 2 J_1 J_2 J^2 t_c^2 + J^4 t_c^4}[-2 J^4 t_c^4 \dot{\phi}^3 + 3 J^4 \dot{\phi}^3 t_c^2 \\ &\quad +3 J^3 \dot{\psi}^3 J_1 t_c^3 + J^3 \dot{\psi} t_c^3 g_1 \cos(\psi) + J \dot{\psi}^3 J_1^2 t_c J_2 \\ &\quad -3 \dot{\phi} J_1 J^3 t_c^3 \dot{\psi}^2 + \dot{\phi} J_1^2 g_2 \cos(\phi) J_2 - 4 J^3 \dot{\psi}^3 t_c J_1 \end{aligned}$$

$$
\begin{aligned}
& -2J^2t_c^2\dot{\phi}^3J_1J_2 + J^3\dot{\psi}t_sg_1\sin(\psi)t_c^2 - 3\dot{\phi}J^3 \\
& t_sg_1\sin(\psi)t_c^2 + J^2\dot{\phi}^3J_1J_2 - \dot{\phi}J_1^2Jt_c\dot{\psi}^2J_2 - \dot{\phi}J_1g_2\cos(\phi)J^2 \\
& t_c^2 + 3J\dot{\psi}t_sg_1\sin(\psi)J_1J_2 + 4\dot{\phi}J^2t_ct_sJ_1g_2\sin(\phi) \\
& -\dot{\phi}Jt_sg_1\sin(\psi)J_1J_2 + 4\dot{\phi}J^3t_cJ_1\dot{\psi}^2 + 4J^2\dot{\psi}t_c^2\dot{\phi}^2 \\
& J_1J_2 - J^4\dot{\psi}\dot{\phi}^2t_c^2 - 3J^2\dot{\psi}\dot{\phi}^2J_1J_2 - J\dot{\psi}t_cg_1 \\
& \cos(\psi)J_1J_2 - 4J^2\dot{\psi}t_ct_sJ_1g_2\sin(\phi)]
\end{aligned}
$$

$$
\begin{aligned}
L_f^4h \;=\; & \frac{1}{J_1^3J_2^3 - 3J_1^2J_2^2J^2t_c^2 + 3J_1J_2J^4t_c^4 - J^6t_c^6} \\
& [-J^2g_1\sin(\psi)J_1^2J_2\dot{\psi}^2 + J^2t_sg_1^2J_1J_2t_c - 2J_2J^4\dot{\phi}^2 \\
& g_1\sin(\psi)t_c^2 + J_2t_c^3g_1^2\cos(\psi)\sin(\psi)J^3 \\
& -J_2t_c^3g_1\cos(\psi)J^4t_s\dot{\phi}^2 - 4\dot{\phi}J^5\dot{\psi}t_c^5g_1\sin(\psi) \\
& +J^5\dot{\phi}^2t_c^3g_2\sin(\phi) + 4J^5\dot{\psi}\dot{\phi}t_sg_1\cos(\psi)t_c^4 + 3J^6 \\
& \dot{\psi}^2\dot{\phi}^2t_c^3t_s - 3J^5\dot{\psi}^2t_sg_1\cos(\psi)t_c^4 - t_c^4g_1\cos(\psi)J^4g_2\sin(\phi) \\
& -J^4t_c^3t_sg_2\sin(\phi)g_1\sin(\psi) - 6J_1J_2^2J^2\dot{\phi}^2g_1\sin(\psi) \\
& +J_1J_2^2t_cg_1\cos(\psi)J^2t_s\dot{\phi}^2 - 21J_1J_2J^4\dot{\psi}^2t_c^3\dot{\phi}^2t_s \\
& +J_2J^5\dot{\phi}^4t_st_c^2 - 4J^2t_ct_sJ_1^2g_2^2\cos(\phi)^2 \\
& +4J^2t_ct_sJ_1^2g_2^2 + J_1^3g_2\cos(\phi)J_2Jt_s\dot{\psi}^2 - J_1^2 \\
& g_2^2\cos(\phi)J^2t_c^2\sin(\phi) - J_1^2g_2\cos(\phi)J_2Jt_cg_1\sin(\psi) \\
& +34J^4\dot{\psi}t_c^3\dot{\phi}^3J_1J_2t_s + 25J^5\dot{\phi}^2t_c^2J_1t_s\dot{\psi}^2 \\
& -18J^5\dot{\phi}^2t_c^3g_1\sin(\psi) - Jt_sg_1\sin(\psi)J_1^2J_2g_2\sin(\phi) - 7J^3t_c^2 \\
& t_sJ_1g_2\sin(\phi)g_1\sin(\psi) - 12J^2t_c^2\dot{\phi}^2J_1^2J_2g_2\sin(\phi) \\
& -19J^3t_c^2\dot{\phi}^2J_1^2J_2t_s\dot{\psi}^2 + 19J^3t_c^3\dot{\phi}^2J_1J_2g_1\sin(\psi) \\
& -18J^4t_c^3\dot{\phi}^4J_1J_2t_s - 7J_1^2J^3t_c^3\dot{\psi}^2 \\
& g_2\sin(\phi) - 3J_1^2J^4t_c^3\dot{\psi}^4t_s + 6J_1J^4t_c^4\dot{\psi}^2g_1\sin(\psi) \\
& +3J^4t_sg_1^2t_c^3 - 3J^4t_sg_1^2t_c^3\cos(\psi)^2 + 8J^3t_cJ_1^2\dot{\psi}^2g_2\sin(\phi)
\end{aligned}
$$

$+4J^4t_cJ_1^2\dot{\psi}^4t_s - 7J^4t_c^2J_1\dot{\psi}^2g_1\sin(\psi) - 14J^2\dot{\psi}\dot{\phi}J_1^2$
$J_2g_2\sin(\phi) - 14J^3\dot{\psi}^3\dot{\phi}J_1^2J_2t_s + 38J^3\dot{\psi}\dot{\phi}J_1J_2t_cg_1\sin(\psi)$
$-38J^4\dot{\psi}\dot{\phi}^3J_1J_2t_ct_s + 26J^2\dot{\psi}t_c^2\dot{\phi}$
$J_1^2J_2g_2\sin(\phi) + 34J^3\dot{\psi}^3t_c^2\dot{\phi}J_1^2J_2t_s - 38J^3\dot{\psi}t_c^3\dot{\phi}J_1J_2g_1\sin(\psi)$
$-10J^6\dot{\psi}\dot{\phi}^3t_c^3t_s - 19J^4t_c^4\dot{\phi}^2J_1g_2\sin(\phi) - 12J^5t_c^4\dot{\phi}^2J_1t_s\dot{\psi}^2$
$+12J^5t_c^5\dot{\phi}^2g_1\sin(\psi) - 6J^6t_c^5\dot{\phi}^4t_s - J_1J_2J^3\dot{\psi}^2t_sg_1\cos(\psi)t_c^2$
$-34J^4\dot{\psi}\dot{\phi}t_c^2J_1g_2\sin(\phi) - 34J^5\dot{\psi}^3\dot{\phi}t_c^2J_1t_s$
$+10J^5\dot{\psi}\dot{\phi}t_c^3g_1\sin(\psi) + 7J_1^2g_2\cos(\phi)J_2J^2t_ct_s\dot{\phi}^2$
$-J_1^2g_2\cos(\phi)J^3t_c^2t_s\dot{\psi}^2 + J_1g_2\cos(\phi)J^3t_c^3g_1\sin(\psi)$
$-7J_1g_2\cos(\phi)J^4t_c^3t_s\dot{\phi}^2 + 7J^2\dot{\phi}^2J_1^2J_2g_2\sin(\phi)$
$+7J^3\dot{\phi}^2J_1^2J_2t_s\dot{\psi}^2 - 14J^3\dot{\phi}^2J_1J_2t_cg_1\sin(\psi) + 13J^4\dot{\phi}^4$
$J_1J_2t_ct_s - J_1^3Jt_c\dot{\psi}^2J_2g_2\sin(\phi) - J_1^3J^2t_c\dot{\psi}^4J_2t_s$
$+15J^6\dot{\phi}^4t_c^3t_s + 22J_1J_2J^3\dot{\psi}^2t_c^3g_1\sin(\psi) + 29J_1J_2J^4\dot{\psi}^2\dot{\phi}^2$
$t_ct_s - 2J_1J_2\dot{\phi}J^3\dot{\psi}t_sg_1\cos(\psi)t_c^2 - 29J_1J_2J^3t_c\dot{\psi}^2g_1\sin(\psi)$
$-8J_1J_2J^3t_c^3\dot{\phi}^2g_2\sin(\phi) - 4J_1J_2^2J^3t_c^2\dot{\phi}^4t_s - J_1J_2^2Jt_c$
$g_1^2\cos(\psi)\sin(\psi) + 7J_1J_2J^3\dot{\phi}^2t_cg_2\sin(\phi) + 7J_1J_2^2J^2t_c^2\dot{\phi}^2$
$g_1\sin(\psi) - 15J_1^2J_2J^3\dot{\psi}^4t_st_c^2 - 11J_1^2J_2^2J^2\dot{\psi}^2t_c\dot{\phi}^2t_s$
$-J_1^3J_2^2\dot{\phi}^2Jt_s\dot{\psi}^2 - J_1^3J_2^2J\dot{\psi}^4t_s + 2J_1^3J_2^2\dot{\phi}J\dot{\psi}^3t_s$
$+6J_1\dot{\phi}J^4\dot{\psi}t_c^3t_sg_2\cos(\phi) - J_1^3J_2^2\dot{\phi}^2g_2\sin(\phi)$
$+22J_1\dot{\phi}J^4\dot{\psi}t_c^4g_2\sin(\phi) + 12J_1\dot{\phi}J^5\dot{\psi}^3t_st_c^4 - 6J_1^2$
$J_2^2\dot{\phi}J\dot{\psi}t_cg_1\sin(\psi) - 4J_1^2J_2^2J^2t_c\dot{\phi}^4t_s + J_1J_2t_c^2g_1$
$\cos(\psi)J^2g_2\sin(\phi) - 21J_1J^4\dot{\psi}^2t_c^4g_2\sin(\phi) + 28J_1J^4t_c^2$
$\dot{\psi}^2g_2\sin(\phi) + 3J_1J_2^2J^3\dot{\phi}^4t_s - 12J_1J^5\dot{\psi}^4t_c^4t_s$
$+24J_1J^5\dot{\psi}^4t_st_c^2 + J_1^2J_2^2\dot{\phi}^2Jt_cg_1\sin(\psi) + 4J_1^2J_2^2J$
$\dot{\psi}^2t_sg_1\cos(\psi) - 2J_1^2J_2^2\dot{\phi}J\dot{\psi}t_sg_1\cos(\psi)$
$-6J_1^2J_2\dot{\phi}J^2\dot{\psi}t_ct_sg_2\cos(\phi) + 14J_1^2J_2^2J^2\dot{\psi}t_c$

$$\dot\phi^3 t_s + 7J_1^2 J_2^2 J\dot\psi^2 t_c g_1 \sin(\psi) + 4J_1^2 J_2 J^3 \dot\psi^4 t_s$$
$$-J^2 t_s g_1^2 J_1 J_2 t_c \cos(\psi)^2 + 3J_1 J_2^2 J t_s g_1^2 - 3J_1 J_2^2 J t_s g_1^2 \cos(\psi)^2$$
$$+4J_1^2 J_2 J^2 \dot\psi^2 g_2 \sin(\phi) + 4J_1 J^3 t_c^2 t_s g_2^2 - 4J_1 J^3 t_c^2 t_s g_2^2 \cos(\phi)^2$$
$$-11J_1^2 J_2 J^2 \dot\psi^2 t_c^2 g_2 \sin(\phi) - 7J_1 J_2 J^2 t_c t_s g_2 \sin(\phi) g_1 \sin(\psi)$$
$$+2J_1^2 J^2 t_c^2 \dot\psi^2 J_2 g_1 \sin(\psi) + 25J^4 \dot\phi^2 t_c^2 J_1 g_2 \sin(\phi) + J_1^3$$
$$g_2^2 \cos(\phi) J_2 \sin(\phi) + J_2 J^4 \dot\phi^2 g_1 \sin(\psi) t_c^4 - 3J^5 \dot\psi^2$$
$$g_1 \sin(\psi) t_c^3 + 3J^5 \dot\psi^2 g_1 \sin(\psi) t_c^5$$
$$+J_2 t_s g_1^2 J^3 t_c^2 - J_2 t_s g_1^2 J^3 t_c^2 \cos(\psi)^2]$$

The structures of the functions used in (5.20) are given below:

$$L_g L_f h = -\frac{J t_c}{J_1 J_2 - J^2 t_c^2}$$

$$L_g L_f^2 h = \frac{2J t_c \left(-J^2 t_c^2 \dot\phi + J_1 \dot\psi J_2\right)}{J_1^2 J_2^2 - 2J_1 J_2 J^2 t_c^2 + J^4 t_c^4}$$

$$L_g L_f^3 h = \frac{J}{J_1^3 J_2^3 - 3J_1^2 J_2^2 J^2 t_c^2 + 3J_1 J_2 J^4 t_c^4 - J^6 t_c^6}[6J^4 t_c^5 \dot\phi^2$$
$$-9J^4 \dot\phi^2 t_c^3 - 3J\dot\phi^2 J_1 J_2^2 + 14t_c J^2 \dot\psi \dot\phi J_1 J_2 - J_2 J^3 \dot\phi^2 t_c^2$$
$$+J_2 J^2 t_s g_1 \sin(\psi) t_c^2 + 3J_1^2 t_c \dot\psi^2 J_2^2 + 3J_1 J^3 t_c^4 \dot\psi^2$$
$$+2J^4 \dot\psi \dot\phi t_c^3 - 4J^3 t_c^2 J_1 \dot\psi^2 - 12J_2 J^2 t_c J_1 \dot\psi^2 + J_2 J^2$$
$$t_c^3 g_1 \cos(\psi) - 4J^2 t_c^2 t_s J_1 g_2 \sin(\phi) + 6J^2 t_c^3 \dot\phi^2 J_1 J_2$$
$$+J_1 g_2 \cos(\phi) J^2 t_c^3 + 3J^3 t_s g_1 \sin(\psi) t_c^3 + J_1^2 J t_c^2 \dot\psi^2 J_2$$
$$-t_c J_1^2 g_2 \cos(\phi) J_2 - 3t_c J^2 \dot\phi^2 J_1 J_2 - 14J^2 \dot\psi$$
$$t_c^3 \dot\phi J_1 J_2 - t_c g_1 \cos(\psi) J_1 J_2^2 + 3t_s g_1 \sin(\psi) J_1 J_2^2 -$$
$$2\dot\phi J_1^2 t_c \dot\psi J_2^2 + 4J t_c^2 \dot\phi^2 J_1 J_2^2 + 9J_2 J_1 J^2 t_c^3$$
$$\dot\psi^2 + t_c J t_s g_1 \sin(\psi) J_1 J_2 - 4J_2 J t_c t_s J_1 g_2 \sin(\phi)]$$

Bibliography

[1] H. S. Black. Stabilized feedback amplifiers. *Bell System Technical Journal*, 13:1–18, 1934.

[2] H. S. Black. Stabilized feedback amplifiers. In *IEEE Spectrum*, pages 55–60, 1997.

[3] A. J. Krener. On the equivalence of control systems and the linearization of nonlinear systems. *SIAM Journal of Control*, 11:670–676, 1973.

[4] A. Isidori. *Nonlinear Control Systems*. Springer-Verlag, 1990.

[5] L. R. Hunt, R. Su, and G. Meyer. Global transformations of nonlinear systems. *IEEE Transactions on Automatic Control*, 28:24–31, 1983.

[6] R. Su. On the linear equivalents of nonlinear systems. *Systems and Control Letters*, 2:48–52, 1982.

[7] H. K. Khalil. *Nonlinear Systems*. Prentice Hall, New Jersey, 1996.

[8] J. J. Slotine and W. Li. *Applied Nonlinear Control*. Prentice-Hall International, 1991.

[9] G. O. Guardabassi and S. M. Savaresi. Approximate linearization via feedback - An overview. *Automatica*, 37:1–15, 2001.

[10] A. J. Krener. Approximate linearization by state feedback and coordinate change. *Systems and Control Letters*, 5:181–185, 1973.

[11] A. J. Krener, S. Karahan, M. Hubbard, and F. Frezza. Higher order linear approximations to nonlinear control systems. In *27th Control and Decision Conference*, pages 519–523, Los Angeles, USA, 1987.

[12] A. J. Krener, S. Karahan, and M. Hubbard. Approximate normal forms of nonlinear systems. In *26th Control and Decision Conference*, pages 1223–1229, USA, 1988.

[13] W. Kang. Approximate linearization of nonlinear control systems. *Systems and Control Letters*, 23(5):43–52, 1994.

[14] J. P. Barbot, S. Monaco, and D. Normand-Cyrot. Quadratic forms and approximate feedback linearization in discrete time. *International Journal of Control*, 67:567–586, 1997.

[15] J. P. Barbot, S. Monaco, and D. Normand-Cyrot. Discrete-time approximated linearization of SISO systems under output feedback. *IEEE Transactions on Automatic Control*, 44(9):1729–1733, 1999.

[16] H. G. Lee and S. J. Marcus. Approximate and local linearizability of nonlinear discrete-time systems. *International Journal of Control*, 44(4):1103–1124, 1986.

[17] K. Nam, A. Arapostathis, and S. Lee. Some numerical aspects of approximate linearization of single-input nonlinear systems. *International Journal of Control*, 57(2):463–472, 1993.

[18] K. Nam, S. Lee, and S. Won. A local stabilizing control scheme using an approximate feedback linearization. In *32nd Control and Decision Conference*, pages 2783–2784, San Antonio, USA, 1993.

[19] K. Nam, S. Lee, and S. Won. A local stabilizing control scheme using an approximate feedback linearization. *IEEE Transactions on Automatic Control*, 39(11):2311–2314, 1994.

[20] C. Champetier, P. Mouyon, and C. Reboulet. Pseudo-linearization of multi-input nonlinear systems. In *23th Control and Decision Conference*, pages 96–97, Las Vegas, USA, 1984.

[21] C. Champetier, P. Mouyon, and J. F. Magni. Pseudolinearization of nonlinear systems by dynamic precompensation. In *24th Control and Decision Conference*, pages 1371–1372, Lauderdale, USA, 1985.

[22] C. Reboulet and C. Champetier. A new method for linearizing nonlinear systems: The pseudolinearization. *International Journal of Control*, 40:631–638, 1984.

[23] S. A. Bortoff and M. W. Spong. Pseudolinearization of the acrobot using spline functions. In *31st Control and Decision Conference*, pages 593–598, Tucson, USA, 1992.

[24] S. A. Bortoff. Approximate state-feedback linearization using spline functions. *Automatica*, 33(8):1449–1458, 1997.

[25] D. E. Davison and S. A. Bortoff. Enlarge your region of attraction using high-gain feedback. In *33rd Control and Decision Conference*, pages 634–639, Lake Buena Vista, USA, 1994.

[26] D. A. Lawrence. A general approach to input-output pseudo-linearization for nonlinear systems. In *34th Control and Decision Conference*, pages 613–618, New Orleans, USA, 1995.

[27] D. A. Lawrence. Approximate model matching for nonlinear systems. In *38th Control and Decision Conference*, pages 720–725, Phoenix, USA, 1999.

[28] J. Wang and W. J. Rugh. On the pseudo-linearization problem for nonlinear systems. *Systems and Control Letters*, 12:161–167, 1989.

[29] W. J. Rugh. Design of nonlinear compensators for nonlinear systems by an extended linearization technique. In *23rd Control and Decision Conference*, pages 69–73, Las Vegas, USA, 1984.

[30] W. J. Rugh. An extended linearization approach to nonlinear system inversion. *IEEE Transactions on Automatic Control*, 31(8):725–733, 1986.

[31] W. T. Baumann and W. J. Rugh. Feedback control of nonlinear systems by extended linearization. *IEEE Transactions on Automatic Control*, 31(1):40–46, 1986.

[32] J. Wang and W. J. Rugh. Feedback linearization families for nonlinear systems. *IEEE Transactions on Automatic Control*, 32(10):935–940, 1987.

[33] J. Wang and W. J. Rugh. Parameterized linear systems and linearization families for nonlinear systems. *IEEE Transactions on Circuits and Systems*, 34(6):650–657, 1987.

[34] F. Allgower, A. Rehm, and E. D. Gilles. An engineering perspective on nonlinear H_∞ control. In *33rd Control and Decision Conference*, pages 2537–2542, Lake Buena Vista, USA, 1994.

[35] C. Bonivento, R. Zanasi, and M. Sandri. Discrete variable structure integral controllers. *Automatica*, 34(3):355–361, 1998.

[36] J. Y. Hung, W. Gao, and J. C. Hung. Variable structure control: A survey. *IEEE Transactions on Industrial Electronics*, 40(1):2–21, 1993.

[37] V. I. Utkin. *Sliding Modes in Control Optimization*. Springer, Berlin, 1992.

[38] A. U. Levin and K. S. Narendra. Control of nonlinear dynamical systems using neural networks. Part ii: Observability, identification, and control. *IEEE Transactions on Neural Networks*, 7(1):30–42, 1996.

[39] K. S. Narendra and X. Parthasarathy. Identification and control of dynamical systems using neural networks. *IEEE Transactions on Neural Networks*, 1(1):4–27, 1990.

[40] K. S. Narendra and K. Mukhopadhyay. Adaptive control using neural networks and approximate models. In *American Control Conference*, pages 355–359, Seattle, USA, 1995.

[41] S. Bittanti, C. Brasca, S. Corsi, G. Guardabassi, M. Pozzi, and S. M. Savaresi. Approximate linearization of a nonlinear plant: The case of a synchronous generator working in underexcitation condition. In *13th IFAC World Congress*, pages 175–180, San Francisco, USA, 1993.

[42] S. M. Savaresi, H. Nijmeijer, and G. O. Guardabassi. On the design of approximate nonlinear parametric controllers. *International Journal of Robust and Nonlinear Control*, 10:137–155, 2000.

[43] G. O. Guardabassi and S. M. Savaresi. Virtual reference direct design method: An off-line approach to data-based control system design. *IEEE Transactions on Automatic Control*, 45(5):954–959, 2000.

[44] Y. Ichikawa and T. Sawa. Neural network application for direct feedback controllers. *IEEE Transactions on Neural Networks*, 3(2):224–231, 1992.

[45] J. Hauser. Nonlinear control via uniform system approximation. In *29th Control and Decision Conference*, pages 792–797, Honolulu, Hawai, 1990.

[46] J. Hauser. Nonlinear control via uniform system approximation. *Systems and Control Letters*, 17:145–154, 1991.

[47] A. Banaszuk and J. Hauser. Least squares approximate feedback linearization: A variational approach. In *32nd*

Control and Decision Conference, pages 2760–2765, San Antonio, USA, 1993.

[48] A. Banaszuk, A. Swiech, and J. Hauser. Approximate feedback linearization: Least squares approximate integrating factors. In *33rd Control and Decision Conference*, pages 1621–1626, Lake Buena Vista, USA, 1994.

[49] A. Banaszuk and J. Hauser. Approximate feedback linearization: A homotopy operator approach. *SIAM Journal on Control and Optimization*, 34(5):1533–1554, 1996.

[50] E. C. Gwo and J. Hauser. A numerical approach for approximate feedback linearization. In *American control conference*, pages 1495–1499, San Francisco, 1993.

[51] E. C. Gwo and J. Hauser. Approximate feedback linearization: An L2 numerical approach. In *32nd Control and Decision Conference*, pages 2772–2777, San Antonio, USA, 1993.

[52] D. Chen and B. Paden. Stable inversion of nonlinear nonminimum-phase systems. *International Journal of Control*, 64(1):81–97, 1996.

[53] M. Niemiec and C. Kravaris. Nonlinear model-state feedback control for nonminimum-phase processes. *Automatica*, 39(7):1294–1302, 2003.

[54] C. Kravaris, P. Daoutidis, and R. A. Wright. Output feedback control of nonminimum-phase nonlinear processes. *Chemical Engineering Science*, 49(13):2107–2122, 1994.

[55] J. M. Kanter, M. Soroush, and W. D. Seider. Continuous-time, nonlinear feedback control of stable processes. *Ind. Eng. Chem. Res.*, 40:2069–2078, 2001.

[56] C. Kravaris and P. Daoutidis. Output feedback controller realizations for open-loop stable nonlinear processes. In *American control conference*, page 2576, Chicago, 1992.

[57] M. Soroush and C. Kravaris. A continuous-time formulation of nonlinear model predictive control. *International Journal of Control*, 63(1):121–146, 1996.

[58] J. R. Kanter, M. Soroush, and W. D. Seider. Nonlinear feedback control of multivariable nonminimum-phase processes. *Journal of Process Control*, 12:667–686, 2002.

[59] C. Kravaris and P. Daoutidis. Nonlinear state feedback control of second-order nonminimum-phase nonlinear systems. *Computers and Chemical Engineering*, 14:439, 1990.

[60] C. Panjapornpon, M. Soroush, and W. D. Seider. Model-based control of unstable nonminimum-phase nonlinear processes. In *IEEE Conference on Decision and Control*, pages 6151–6156, 2003.

[61] F. J. Doyle III, F. Allgower, and M. Morari. A normal form approach to approximate input-output linearization for maximum phase nonlinear systems. *IEEE Transactions on Automatic Control*, 41(2):305–309, 1995.

[62] F. Allgower. Approximate input-output linearization of nonminimum phase nonlinear systems. In *European Control Conference*, page 604, Brussels, Belgium, 1997.

[63] R. Gurumoorthy. *Output Tracking of Nonminimum-Phase Nonlinear Systems*. PhD thesis, University of California at Berkeley, 1992.

[64] A. F. Okou, O. Akhrif, and L.A Dessaint. Nonlinear control of nonminimum-phase systems: Application to the voltage and speed regulation of power systems. In *IEEE Conference on Control Applications*, pages 609–615, 1999.

[65] A. P. Hu and N. Sadegh. Nonlinear nonminimum-phase output tracking via output redefinition and learning control. In *American Control Conference*, pages 4264–4269, 2001.

[66] B. Srinivasan, K. Guemghar, P. Huguenin, and D. Bonvin. A global stabilization strategy for an inverted pendulum. In *IFAC World Congress*, page 1224, Barcelona, Spain, 2002.

[67] C. S. Huang and K. Yuan. Output tracking of a nonlinear nonminimum-phase PVTOL aircraft based on nonlinear state feedback control. *International Journal of Control*, 75(6):466–473, 2002.

[68] H. Sun, Y. G. Xi, S. J. Shi, and Z. J. Zhang. Predictive control of nonlinear systems based on extended linearization. In *American Control Conference*, pages 1718–1722, USA, 1990.

[69] M. J. Kurtz and M. A. Henson. Input-output linearizin control of constrained nonlinear processes. *Journal of Process Control*, 7(1):3–17, 1997.

[70] M. K. Maaziz, P. Boucher, and D. Dumur. New control strategy for induction motor based on nonlinear predictive control and feedback linearization. *International Journal of Adaptive Control and Signal Processing*, 14(2):313–329, March 2000.

[71] V. Nevistić and M. Morari. Constrained control of feedback-linearizable systems. In *European Control Conference*, pages 1726–1731, Rome, 1995.

[72] T. J. J. Van den Boom. Robust nonlinear predictive control using feedback linearization and linear matrix inequalities. In *American Control Conference*, pages 3068–3072, USA, 1997.

[73] P. G. Adams and A. T. Schooley. Adaptive predictive control of a batch reaction. *Rubber Chemistry and Technology*, 43(6):1523–1524, December 1970.

[74] D. A. Mellichamp. Predictive time-optimal controller for second-order systems with time delay. *Simulation*, 14(1):27–35, January 1970.

[75] M. Morari and J. H. Lee. Model predictive control: Past, present, and future. *Computers and Chemical Engineering*, 23:667–682, 1999.

[76] S. J. Qin and T. A. Badgwell. An overview of industrial model predictive technology. In *Chemical Process Control V*, pages 232–256, Tahoe City, CA, 1997.

[77] J. S. Qin and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7):733–764, July 2003.

[78] J. B. Rawlings. Tutorial: Model predictive control technology. In *American control conference*, pages 662–676, San Diego, California,USA, 1999.

[79] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36:789–814, 2000.

[80] F. Allgower, R. Findeisen, and Z.K. Nagy. Nonlinear model predictive control: From theory to application. *Journal of the Chinese Institute of Chemical Engineers*, 35(3):299–315, May 2004.

[81] A. Bemporad and M. Morari. Robust model predictive control: A survey. *Lecture Notes in Control and Information Sciences*, 245:207–226, 1999.

[82] C. E. Garcia, D. M. Prett, and M. Morari. Model predictive control: Theory and practice: A survey. *Automatica*, 25(3):335–348, 1989.

[83] M. Morari and J. H. Lee. Model predictive control: the good, the bad, and the ugly. In *Fourth international conference on chemical process control*, pages 419–444, 1991.

[84] F. Allgower and A. Zheng. *Nonlinear Model Predictive Control, Progress in Systems and Control Theory*. Birkhauser Verlag, 2000.

[85] E. F. Camacho and Bordons C. *Model Predictive Control*. Springer, 1999.

[86] J.M. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, 2002.

[87] J. B. Rawlings, E. S. Meadows, and K. R. Muske. Nonlinear model predictive control: A tutorial and survey. In *IFAC ADCHEM'94*, pages 185–197, Kyoto, Japan, 1994.

[88] W. J. Rugh. *Linear System Theory.* Prentice Hall, New Jersey, 1993.

[89] P. S. Shcherbakov. Alexander mikhailovitch lyapunov: on the centenary of his doctoral dissertation on stability of motion. *Automatica*, 28(5):865–871, 1992.

[90] D. W. Clarke and R. Scattolini. Constrained receding horizon predictive control. *IEE Proceedings D: Control Theory and Applications*, 138(4):347–354, 1991.

[91] E. Moska and J. Zhang. Stable redesign of predictive control. *Automatica*, 28(6):1229–1233, 1992.

[92] R. R. Bitmead, M. Gevers, and V. Wertz. *Adaptive Optimal Control. The thinking Man's GPC.* Prentice Hall, New Jersey, 1989.

[93] J. B. Rawlings and K. R. Muske. The stability of constrained receding horizon control. *IEEE Transactions on Automatic Control*, 38:1512–1516, 1993.

[94] E. Zafiriou. Robust model predictive control of processes with hard constraints. *Computers and Chemical Engineering*, 14(4/5):359–371, 1990.

[95] A. Zheng and M. Morari. Robust stability of constrained model predictive control. In *American Control Conference*, pages 379–383, San Francisco, California, 1993.

[96] A. Jadbabaie and J. Yu. Unconstrained receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control*, 46(5):776–783, 2001.

[97] A. Jadbabaie, J. Yu, and J. Hauser. Stabilizing receding horizon control of nonlinear systems: A control lyapunov function approach. In *American Control Conference*, pages 1535–1539, San Diego, California, 1999.

[98] H. Nijmeijer and A. J. Van Der Schaft. *Nonlinear Dynamical Control Systems.* Springer-Verlag, 1990.

[99] M. Vidyasagar. *Nonlinear Systems Analysis.* Society of Industrial and Applied Mathematics, SIAM, 2002.

[100] S. M. Shinners. *Advanced Modern Control System Theory and Design.* Wiley and sons, New York, 1998.

[101] M. W. Spong, D. J. Block, and K. J. Astrom. The mechatronics control kit for education and research. In *Proceedings of the IEEE Conference on Control Applications*, pages 105–110, 2001.

[102] M. A. McClure and R. N. Paschall. Applying variations of the quantitative feedback technique (qft) to unstable nonminimum phase aircraft dynamics models. In *Proceedings of the IEEE 1992 National Aerospace and Electronics Conference NAECON*, pages 334–341, 1992.

[103] D. Hahs and J. Sorrels. Dynamic vehicle control (problem). In *American Control Conference*, pages 2967–3968, Boston, 1991.

[104] I. Rusnak. Control for unstable nonminimum phase uncertain dynamic vehicle. *IEEE Transactions on Aerospace and Electronic Systems*, 32(3):945–951, 1996.

[105] J. Yu, A. Jadbabaie, J. Primbs, and Y. Huang. Comparison of nonlinear control design techniques on a model of the cal tech ducted fan. *Automatica*, 37(5):1971–1978, 2001.

[106] E. Ronco, B. Srinivasan, J. Y. Favez, and D. Bonvin. Predictive control with added feedback for fast nonlinear systems. In *European Control Conference*, pages 3167–3172, Porto, Portugal, 2001.

[107] A. E. Bryson. *Dynamic Optimization*. Addison-Wesley, Menlo Park, California, 1999.

[108] I. Fantoni, R. Lozano, and M. W. Spong. Energy based control of the pendubot. *IEEE Transactions on Automatic Control*, 45(4):725–729, 2000.

[109] K. Yamada, K. Nitta, and A. Yuzawa. A nonlinear control design for the pendubot. In *IASTED International Conference on Modelling Identification and Control*, pages 208–213, 2001.

[110] M. Zhang and T. J. Tarn. Hybrid control of the pendubot. *IEEE/ASME Transactions on Mechatronics*, 7(1):79–86, 2002.

[111] X. Q. Ma and C. Y. Su. A new fuzzy approach for swing up control of pendubot. In *Proceedings of the American Control Conference*, pages 1001–1006, 2002.

[112] J. Y. Favez. Commande d' un pendule polaire. Technical Report EL 089, Laboratoire d' Automatique, EPFL, Switzerland, 1997.

[113] M.W. Spong. The swing up control problem for the acrobot. *IEEE Control Systems Magazine*, 15(1):49–55, 1995.

[114] R. Fletcher. *Practical Methods of Optimization.* Wiley and sons, New York, 1987.

[115] C. T. Kelly. *Iterative Methods for Optimization.* Society of Industrial and Applied Mathematics, SIAM, 1999.

[116] L. W. Johnson and R. Dean. *Numerical Analysis.* Addison-Wesley Publishing Company, 1982.

[117] W. T. Vetterling, S. A. Teukolsky, W. H. Press, and B. P. Flannery. *Numerical Recipes.* Cambridge University Press, 1994.

Curriculum Vitae

Formations

2000-2004 Laboratoire d'automatique, EPF-Lausanne, Suisse. Docteur ès sciences, en mécanique, option Automatique : On the Use of Input-output Linearization Techniques for the Control of Nonminimum-phase Systems.

09/2002 INP-Grenoble, France. Ecole doctorale d'automatique de Grenoble: modélisation et commande de véhicules automobiles.

04/2001 ETH-Zurich, Suisse. Cours sur la commande prédictive, par Pr. Morari.

1998-1999 Supelec, Université de Paris XI-Orsay, France. DEA (Diplôme d'Etudes Approfondies) en Automatique et Traitement du Signal.

1993-1998 Ecole Nationale Polytechnique d'Alger (ENP), Algérie. Diplôme d'Ingénieur en génie électrique, option Contrôle des systèmes (Automatique).

Expérience professionnelle

1999-2004 Laboratoire d'automatique, EPF-Lausanne, Suisse. Assistante d'enseignement et de recherche.

- Encadrer des travaux pratiques et séances d'exercices,
- Superviser des projets de semestre et de diplôme,
- Collaborer avec des industriels: EDF, Serpentine SA, Espace des Inventions,
- Développer des manipulations de laboratoire pour travaux pratiques.

2000-2001 Espace des Inventions (EDI), Lausanne, Suisse. Animatrice, chargée de l'accueil et responsable de la caisse et de la boutique du musée, souvent bénévolement.

1999 Institut de Recherche en Informatique et Automatique (INRIA), Sophia-Antipolis (France). Stage de DEA, commande des véhicules à roues.

1995 Entreprise Nationale des Systèmes Informatiques, Alger, Algérie. Stage de maintenance de systèmes informatiques.

1995-1998 Alger, Algérie. Cours particuliers pour lycéens: mathématiques, physique.

Connaissances linguistiques

Français, Arabe: Langues maternelles

Anglais: Très bonnes connaissances

Connaissances informatiques

Systèmes d'exploitation: Mac OS, Windows, Unix, MS DOS.

Programmation/logiciels: C, Fortran, Matlab, Simulink, Mathematica, Maple, Labview, LaTeX, MS Office, Adobe illustrator, Adobe photoshop, Adobe GoLive.

Centres d'interêt

Voyages, basket-ball (7 ans en club), musique (2 ans dans une chorale polyphonique), organisation de soirées (responsable des soirées et manifestations du Laboratoire d'Automatique durant 2 ans), photographie.

Distinctions reçues

2001 Récompense pour prestations au sein du Laboratoire d'Automatique, EPF-Lausanne.

1995-1998 Quatre fois récompense pour major de promotion Ecole Polytechnique d'Alger.

Publications

Guemghar K., B. Srinivasan, P. Mullhaupt and D. Bonvin. Predictive Control of Fast Unstable and Nonminimum-phase Nonlinear Systems. American Control Conference, Anchorage, Alaska (May 2002), 4764-4769.

B. Srinivasan, K. Guemghar, P. Huguenin and D. Bonvin. A Global Stabilization Strategy for an Inverted Pendulum. IFAC

World Congress 2002, Barcelona, Spain (July 2002), 1224.

Guemghar K., B. Srinivasan and D. Bonvin. Approximate Input-Output Linearization of Nonlinear Systems Using the Observability Normal Form.European Control Conference, Cambridge, UK (September 2003).

Guemghar K., B. Srinivasan, Ph. Mullhaupt and D. Bonvin. A Cascade Structure with Predictive Control and Feedback Linearization for the Control of Nonlinear Systems. IEE Proceedings on Control Theory and Applications (accepted).

Guemghar K., B. Srinivasan, and D. Bonvin. Cascade Control of Pendubot using Input-output Feedback Linearization and Nonlinear Predictive Control. IFAC World Congress 2005, Prague, Czech Republic, (July 2005) (accepted).