COMMUNICATIONS AND COMMENTS

Repairing Gamut Problems in CIECAM02: A Progress Report

Abstract: The color-appearance model CIECAM02 has several problems, which can result in mathematical instabilities, due to the position of the chromatic-adaptation primaries relative to the spectrum locus and to the presumed physiological cone primaries. To keep a corresponding (adapted) color within the positive gamut given by the chromatic adaptation primaries, the gamut must lie within the cone primary octant. To contain adapted colors within the positive cone-primary octant, it suffices to truncate the action of adaptation at the boundary of that octant. Such modifications may be needed to avoid the mathematical problems in CIECAM02. Col Res Appl, 33, 424–426, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.20432

Key words: CIECAM02; color appearance; model; chromatic adaptation; gamut; primary

INTRODUCTION

CIECAM02[1] has enjoyed recent popularity as a gamut mapping or profile-connection space in color management, but has some mathematical problems arising from the sequential application of a chromatic-adaptation transform using sharpened primaries (CAT02), a transformation to cone-primary coordinates (Hunt-Pointer-Estévez or HPE), and a nonlinearity involving a noninteger power function. In this article, we describe the problems and suggest an approach to solving them. What we propose here is a mathematical solution to solving CIECAM02 implementation issues; we do not claim that the proposed improvements to the model accurately predict color appearance in such extreme situations.

MATHEMATICS OF CIECAM02

The steps in the CIECAM02 algorithm (with the option of full chromatic adaptation) are as follows:

1. A von Kries type chromatic adaptation transform in a "sharpened-sensor" coordinate system adapts the tristimulus values of a test color to a different illuminant, using the corresponding tristimulus values for white:

$$\mathbf{R}_\mathrm{c} = \mathbf{D}\ \mathbf{M}_\mathrm{CAT02}\ \mathbf{X} \tag{1}$$

   where $\mathbf{X}$ is the column 3-vector of the test-color tristimulus values X, Y, Z; $\mathbf{M}_\mathrm{CAT02}$ is the 3×3 matrix transforming $\mathbf{X}$ to the sharpened-primary basis; $\mathbf{D}$ is the diagonal matrix containing the von Kries scaling factors; and $\mathbf{R}_\mathrm{c}$ is the tristimulus vector of the adapted test color in the sharpened-sensor basis. Note that we use here the term "von Kries" loosely, because CIECAM02 uses an additional multiplicative factor Y_W (Y for a paper white) in the scaling factor calculation. For caveats about the use of Y_W, see Ref. 2, p. 269. For the rest of this article, as we will be talking about adapted chromaticities, factors Y_W will cancel.
2. Transform $\mathbf{R}_\mathrm{c}$ to HPE "cone-space" basis $\mathbf{R}'$:

$$\mathbf{R}' = \mathbf{M}_\mathrm{HPE}\ \mathbf{M}_\mathrm{CAT02}{}^{-1}\ \mathbf{R}_\mathrm{c} \tag{2}$$

   where $\mathbf{M}_\mathrm{HPE}$ is the transformation from XYZ to the HPE basis.
3. Nonlinearly map each component of $\mathbf{R}'$ to the corresponding component of a vector $\mathbf{R}'_\mathrm{a}$, using a power function with the (noninteger) exponent 0.42. The vector $\mathbf{R}'_\mathrm{a}$ has components R'_a, G'_a, B'_a.

In CIECAM02, it is important for the adapted test color to lie in the positive octant of the HPE space, which in chromaticity space means the test color must lie inside or on the triangle whose vertices are the HPE primaries. Various difficulties arise when this containment fails, as it does in CIECAM02.

THE YELLOW-BLUE PROBLEM

Part of the difficulty, which we refer to here as the "yellow-blue problem," arises because the red and green CAT02 primaries lie outside the HPE triangle.[3] As a result, extreme blue adaptation can drive a corresponding CAT02 color outside the HPE triangle, where it acquires a negative HPE blue coordinate. To avoid complex values, the nonlinearity must detach the negative sign and reattach it. The sign-reattachment patch does not avoid an infinite slope of the nonlinearity at the HPE triangle edge. The continuous form of the nonlinearity is strictly invertible at zero even with an infinite slope there. However, in digital representations, the infinite slope can produce a jump in the mapping, which has been inconvenient to users.

Therefore, CIE Division 8 asked Süsstrunk to investigate new CAT02 primaries that lie within the HPE triangle. She devised new primaries by changing the third row of the chromatic-adaptation matrix $\mathbf{M}_\mathrm{CAT02}$ from [0.0030, 0.0136, 0.9834] to [0, 0, 1]. This moves the RG side of the CAT02 triangle so it lies on the $z = 0$ line, which at long wavelengths corresponds with the spectrum locus. The RG side of the new $\mathrm{CAT02_{mod}}$ triangle is thereby placed partly outside and partly on the spectrum locus, but always inside the HPE triangle (see Fig. 1). The problem noted in Ref. 3 is thereby repaired. This slight modification of the blue primary does not statistically significantly change asymmetric match predictions.[4]

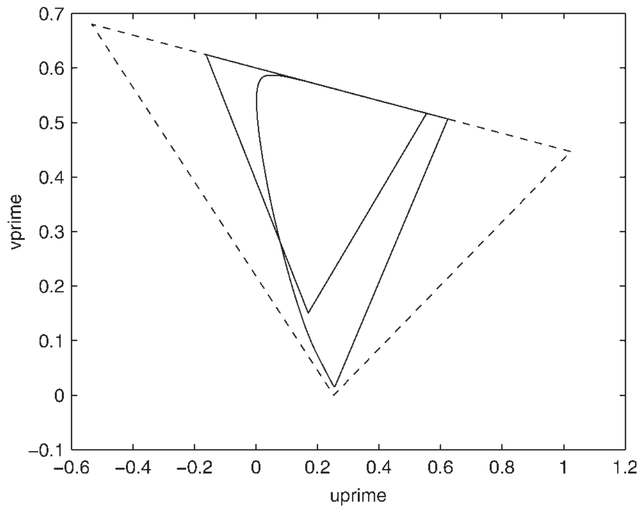

FIG. 1. Nonnested gamuts of CIECAM02: spectrum-locus (black curve), $\mathrm{CAT02_{mod}}$ (solid triangle), and HPE (dashed triangle). $\mathrm{CAT02_{mod}}$ nests within HPE, but neither CAT02 nor $\mathrm{CAT02_{mod}}$ contains the spectrum locus.

THE PURPLE PROBLEM

A bigger difficulty, which we refer to here as the "purple problem," is that the CAT02 triangle lies partly inside the spectrum locus. From Fig. 1, note that the solid $\mathrm{CAT02_{mod}}$ triangle excludes the saturated purples, and the B vertex is inside the spectrum locus. The effect is large in $u'v'$ space, but smaller in xy space. The same is true for the original CAT02 triangle.

To illustrate why this is bad, consider a saturated purple. Because it is on the opposite side of the $\mathrm{CAT02_{mod}}$ RB line from the G primary, it will have a negative $\mathrm{CAT02_{mod}}$ G component. Such a saturated-purple test light can adapt to a purplish adapting light within the CAT02 triangle by moving well outside the spectrum locus, and outside the HPE triangle. For example: $\mathrm{CAT02_{mod}}$ with light $(x_\mathrm{w}, y_\mathrm{w}) = (0.4, 0.2)$ maps the test color $(x, y) = (0.4, 0.12)$ to the chromaticity $(x_\mathrm{p}, y_\mathrm{p}) = (0.5788, -0.0444)$, which is well outside the HPE triangle (see Appendix for Matlab source code). Exiting the HPE triangle produces the infinite-slope anomaly, as noted by Brill,[3] and additionally leads to an inherently uncontained locus of corresponding colors. But the main impact of the "purple problem" is the following paradoxical behavior: If a saturated-purple test light starts outside the CAT02 triangle, then moving an adapting light toward purple (but still within the CAT02 triangle) makes the test light *more* saturated purple—the opposite of what would happen if the test light were inside the CAT02 triangle. No relocation of the HPE triangle can save the situation.

For all corresponding colors to lie inside the HPE triangle, three chromaticity sets must be nested: The HPE triangle must encompass the $\mathrm{CAT02_{mod}}$ triangle, which in turn must encompass the spectrum locus.

THE BRIGHTNESS PROBLEM

The most model-disabling consequence of the HPE triangle's failure to contain adapted chromaticities is the "not a number" evaluation that can happen in computing the brightness J. The quantity J is computed as

$$J = 100\ (A/A_\mathrm{W})^{cz} \tag{3}$$

where cz is a noninteger quantity,

$$A = [2\,R'_\mathrm{a} + G'_\mathrm{a} + 0.05\,B'_\mathrm{a} - 0.305]\,N_\mathrm{bb} \tag{4}$$

is the adapted test-color's achromatic coordinate, A_W is the value of A corresponding to white, and N_bb is a defined positive number. Li and Luo,[5] who first described the problem, showed that A_W cannot be negative but that A can be negative if the adapted chromaticity of the test color lies outside the HPE triangle. A negative quantity A leads to J being evaluated as "not a number." [Note: the term -0.305 in Eq. (4) merely subtracts off an inhomogeneous term 0.1 that was added to each of the power-function expressions defining R'_a, G'_a, B'_a; thus $A = 0$ when $X = Y = Z = 0$.] Because Li and Luo showed that this "brightness problem" disappears if the adapted test-color has a chromaticity that lies within the HPE triangle, their investigation confirms the desirability of subjecting the adapted test color to the constraint suggested here.

POSSIBILITIES OF REPAIR

To repair CIECAM02 with minimum consequences, we can consider the following solutions: To solve the yellow-blue problem, change the CAT02 primaries to the ones reported here. To solve the purple and brightness problems, truncate the negative values. Thus, after R', G', B' are computed but before R'_a, G'_a, B'_a are computed, set $R' \leftarrow \max(R', 0)$, $G' \leftarrow \max(G', 0)$, and $B' \leftarrow \max(B', 0)$. For consistency, even though the white coordinates R'_W, G'_W and B'_W are unlikely to be negative, the same truncation should be applied, if needed, to those coordinates.

These repairs effectively keep all adapted colors within the positive HPE octant (i.e., inside the HPE triangle in chromaticity space). Because HPE encloses the spectrum locus and also the (here-revised) $\mathrm{CAT02_{mod}}$ triangle, it should keep all color values reasonable. All the experimental fits that were originally used as the basis for CIECAM02 will be unchanged.

One remaining issue should be noted. Although still a continuous mapping, the model so modified is not invertible for the "rare but uncomfortable" colors that start outside the CAT02 triangle and migrate farther afield due to adaptation. When one inverts CIECAM02, those colors return to legal, manageable ones on the HPE boundary. If the noninvertibility is unacceptable, a more comprehensive repair of CIECAM02 is needed.

APPENDIX: MATLAB SOURCE CODE FOR PURPLE PROBLEM EXAMPLE

```
MCAT02mod = [0.7328 0.4296 -0.1624; -0.7036 1.6975 0.0061; 0. 0. 1.];
ma = MCAT02mod^(-1);
Xw = [0.4 0.2 0.4]';
X = [0.4 0.12 0.48]';
R = MCAT02mod*X;
Rw = MCAT02mod*Xw;
Rc = R./Rw;
Xc = ma*Rc;
xp=Xc./(Xc(1) + Xc(2) + Xc(3))
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