Experimental and numerical study of stochastic branching noise in nuclear reactors

Présentée le 12 juin 2020
à la Faculté des sciences de base
Laboratoire de physique des réacteurs et de comportement des systèmes
Programme doctoral en physique
pour l'obtention du grade de Docteur ès Sciences
par

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Acknowledgments

No you haven't.

Obi-Wan to Anakin

It is with a sense of humility that I finish the last touches of this thesis. After handing it in, I am expected to be able to conduct independent scientific work – yet especially in the creation of this manuscript was I reminded of how great things are achieved together. This page is dedicated to thank the people who made the past years ever so special.

An initial acknowledgment goes to Prof. Andreas Pautz. His fair, social, and predictable management style has been a great inspiration for how to treat responsibility.

For his energetic dedication I thank my supervisor and friend Dr. Vincent Lamirand. It a cliche, but I am sure you understand when I say: Words cannot do my gratitude justice. I am happy that our paths crossed the way they did. We listened, learned, lamented, but always laughed along the way - thank you.

I extend a specific thank you to the academic advice and reliable skeptical voices of Dr. Grégory Perret and Dr. Mathieu Hursin. For a highly enjoyable time in Paris I further thank Dr. Andrea Zoia, Dr. Davide Mancusi, Dr. Wilfried Monange, and Dr. Eric Dumonteil.

As an experimentalist I am indebted to the CROCUS facility team - thank you Dr. Pavel Frajttag, Daniel Godat, and Laurent Braun. Despite the challenges we faced, we always managed to emerge with a smile. I still think the control room needs a proper sound system though :)

I feel very strongly about the importance of my friends who were along for the ride, many of whom also aided in the review of the manuscript. I wish to thank Clara Abegg, Dr. Carlo Fiorina, Alice Gillen, Alexander Holloway, Andreas Hug, Adrian Liepelt, Moritz Rabe, Stefan Radman, Andreas Rowald, Alessandro Scolaro, Dr. Daniel Siefman, and Fanny Vitullo. Each of you has a special place in my heart.
Abstract

Noise analysis applied to nuclear reactor physics is a powerful tool to investigate a reactor's kinetic parameters, and more generally underlying physical processes determining core behavior. The kinetic parameters are the coefficients of simplified time-dependent neutron population equations, the so-called point kinetics equations. Experimentally determined kinetic parameters aid to validate codes and to potentially evaluate nuclear data. This thesis focuses on improvements to the kinetic parameter and uncertainty quantification, the experimental techniques, and the direct simulation of noise experiments. In particular, the notion of spatial effects, i.e. effects that render point kinetics assumptions inaccurate for noise measurements, was investigated. This was achieved by drawing on experiments conducted in the zero power reactor CROCUS.

New detection instrumentation for noise experiments was developed for CROCUS, namely current mode amplifiers for neutron detectors to allow for higher detection efficiency and thus precision; and a scintillation-based gamma detection array, called LEAF, to enable the study of gamma noise. A set of reference neutron and gamma noise experiments were conducted to exemplify an improved method of parameter and uncertainty estimation based on bootstrapping. It lead to a full uncertainty budget showing a relative uncertainty of 3.6% and 1.3% on the prompt decay constant with neutron and gamma noise, respectively. Moreover, reporting of full distributions of kinetic parameters was shown to be required in order to provide an accurate representation of the experimental results. Gamma noise was shown to be superior in terms of precision by a factor of at least two compared to neutron noise when determining the prompt decay constant.

To study spatial effects and the spatial extent of the noise field of CROCUS, a set of experiments with varying detector locations, reactivity, and particle type were conducted. The neutron noise field study showed that measurements are limited to the immediate proximity of the core, and that CROCUS is reliably modeled as a point kinetic reactor for static configurations. Nonetheless, a systematic trend was shown when comparing to code predictions, pointing to a weak spatial effect biasing neutron noise measurements at a distance. The observable gamma noise field using high efficiency detectors was shown to extend beyond neutron noise limits, enabling ex-vessel measurements. The prompt decay constant was determined at about 0.9 meters to the core center with comparable accuracy to that of gamma in-core reference experiments using less-efficient detectors. Gamma correlations were be observed outside of the reactor cavity in front of an experimental channel, enabling a prompt decay constant determination at 7 meters to the core center.
Abstract

To complement the experimental investigation and to study the underlying physics, a simulation methodology to estimate noise responses was developed. Specifically, the explicit simulation of noise experiments using analog Monte Carlo transport coupled to a fission model was studied. An established code with an existing fission library coupling, TRIPOLI-4, was used as reference. In addition, a newly developed coupling of a fission library to Serpent 2 allowed for code-to-code comparison. The full methodology, verification, and validation to the aforementioned CROCUS experiments is presented.

Keywords: Reactor physics, Integral experiments, Kinetic parameters, Neutron noise, Gamma noise
Résumé


De nouvelles instrumentation de détection dédiées aux expériences de bruit ont été développées pour leur utilisation dans CROCUS, à savoir un amplificateur en mode courant pour des détecteurs de neutrons de haute sensibilité, afin d’améliorer l’efficacité de détection et par conséquent la précision de mesure ; et un système de détection gamma par scintillation, appelé LEAF, pour l’étude du bruit gamma. Un ensemble d’expériences de référence de bruits neutronique et gamma a été réalisé pour illustrer une nouvelle méthode d’estimation des paramètres et des incertitudes basée sur la technique du bootstrap. Celle-ci a permis une quantification extensive du bilan d’incertitudes, permettant des incertitudes réduites à 3,6% et 1,3% sur l’estimation de la constante de décroissance prompte par mesures neutron et gamma, respectivement. Par ailleurs, il est démontré la nécessité de rendre compte de la distribution complète des paramètres cinétiques afin de représenter avec exactitude les résultats expérimentaux. Le bruit gamma s’est avéré supérieur en termes de précision, d’un facteur d’au moins deux par rapport au bruit neutronique pour la détermination de la constante de décroissance prompte.

Pour l’étude des effets spatiaux et de l’étendue spatiale d’observation, ou champ, du bruit de CROCUS, une série d’expériences avec variation de la position des détecteurs, de la réactivité et du type de particules détectées ont été menées. L’étude du champ de bruit neutronique a montré que sa mesure est limitée à la proximité immédiate du cœur, et que CROCUS est modélisé avec justesse par la cinétique ponctuelle en configuration statique. Cependant, une tendance a néanmoins été mise en évidence par la comparaison aux calculs, indiquant un faible effet spatial baissant les mesures de bruit neutronique à distance. Il a
Résumé

été également démontré que le champ de bruit gamma observable au moyen de détecteurs de haute sensibilité s'étend au-delà des limites du bruit neutronique, permettant ainsi des mesures hors-cuve. La constante de décroissance prompte a été déterminée à environ 0,9 mètre du centre du cœur, avec une précision comparable à celle des expériences de référence gamma en cœur au moyen de détecteurs moins sensibles. Les corrélations gamma ont été observées hors de la cavité dans l’axe d’un canal expérimental, permettant une détermination de la constante de décroissance prompte jusqu’à 7 mètres du centre du cœur, limite fixée par l’installation.

Pour compléter l’investigation expérimentale et étudier la physique sous-jacente, une méthode de calcul a été développée pour estimer les réponses de bruit. Plus précisément, des simulations explicites d’expériences de bruit ont été menées par transport Monte Carlo analogue couplé à un modèle de fission. Le code TRIPOLI-4, précédemment couplé à une bibliothèque de fission, a été utilisé comme référence. De plus, le développement du couplage du code Serpent 2 avec la même bibliothèque de fission a permis de comparer les codes entre eux. La méthodologie complète, sa vérification, et sa validation au moyen des expériences CROCUS mentionnées précédemment sont présentées.

Mots-clés : Physique des réacteurs, Expérience intégrale, Paramètres cinétiques, Bruit neutronique, Bruit gamma
Contents

Abstract iii

Résumé v

Table of contents vii

List of Figures xiii

List of Tables xviii

1 Introduction 1
   1.1 Of randomness and nuclear reactors .......................... 1
   1.2 Kinetic parameters of nuclear reactors ...................... 3
      1.2.1 Kinetic parameter prediction & uncertainty .............. 3
      1.2.2 Noise measurements to determine kinetic parameters ....... 4
   1.3 Aim and structure ........................................... 6

2 Noise measurements in nuclear reactors 9
   2.1 Nuclear reactor kinetics ...................................... 9
      2.1.1 Time dependent Boltzmann transport equation ............... 9
      2.1.1.1 The diffusion approximation ........................... 11
      2.1.1.2 Adjoint transport equations ........................... 12
      2.1.2 Point kinetics equations ................................ 12
      2.1.3 Reactor design regarding neutron kinetics .................. 14
         2.1.3.1 The importance of delayed neutrons ................. 14
         2.1.3.2 Kinetic parameter prediction ......................... 14
      2.1.4 Reactor kinetics measurements ............................. 15
   2.2 Temporal fluctuation: Noise .................................. 17
   2.3 Reactor noise .................................................. 18
Contents

2.3.1 Zero power or branching noise .................................. 18
2.3.2 Power reactor or external noise ................................ 19
2.3.3 State space analysis of a zero power system ...................... 19
2.4 Intuitive zero power reactor noise analysis methods .............. 22
  2.4.1 Auto-correlation or Rossi-\(\alpha\) method .................. 22
  2.4.2 Variance over mean method ................................ 23
  2.4.3 Power spectral density method .............................. 24
2.5 Experimental realization of noise measurements .................... 25
  2.5.1 Rossi-\(\alpha\) .................................. 25
  2.5.2 Feynman-\(\alpha\) ................................ 27
  2.5.3 Power spectral density ................................ 28
  2.5.4 Review of noise measurement experiments .................... 29
2.6 Beyond point kinetics: Spatial description of radiation noise ........ 31
  2.6.1 Motivation for spatial and spectral description of reactor noise . 31
  2.6.2 Two point model ........................................ 32
  2.6.3 Explicit spatial treatment of neutron fluctuations ............. 33
  2.6.4 Summary of spatial effects on neutron noise measurements .... 33
2.7 Gamma noise ................................................................ 34
  2.7.1 Review of theory ........................................... 35
  2.7.2 Review of experiments ...................................... 35
  2.7.3 Prediction of a power spectral density of gamma noise .......... 36
2.8 Chapter's salient elements ............................................ 37

3 Reference noise experiments in CROCUS ............................... 39
  3.1 Noise measurement methodology .................................. 39
    3.1.1 Non linear least squares (NLLS) for noise analysis .......... 40
    3.1.2 Model choice .............................................. 41
    3.1.3 Uncertainty estimation of NLLS parameters .................. 42
    3.1.4 Limits to noise measurement ................................ 43
    3.1.5 Summary of uncertainty quantification methodology in noise measurements .............. 46
  3.2 Experimental setup for noise experiments in CROCUS ................ 47
    3.2.1 The CROCUS zero power research reactor .................... 47
      3.2.1.1 Reactor characteristics ............................... 47
      3.2.1.2 Operation and associated constraints .................. 50
      3.2.1.3 Detection instrumentation .............................. 50
3.2 Development of noise detection instrumentation ........................................... 53
  3.2.1 Neutron detection: Current mode amplifiers ........................................ 53
  3.2.2 Development of gamma detection instrumentation:
      LEAF ........................................... 54
3.3 Reference neutron and gamma noise measurements in CROCUSS .................... 60
  3.3.1 Arrangement of noise instrumentation ............................................. 60
  3.3.2 Neutron noise reference .............................................................. 62
  3.3.3 Gamma noise reference ............................................................... 64
  3.3.4 Optimization of gamma noise lower level threshold ................................ 66
  3.3.5 Comparison of neutron and gamma noise .......................................... 68
3.4 Chapter’s salient elements ................................................................. 71

4 Experimental study of the branching noise field of CROCUSS .......................... 73
  4.1 Neutron noise field experiments .......................................................... 74
    4.1.1 Testing of point kinetics behavior of neutron noise .......................... 74
    4.1.2 Experimental setup ........................................................................ 75
      4.1.2.1 Critical neutron noise field .................................................... 76
      4.1.2.2 Sub-critical neutron noise field ............................................. 76
    4.1.3 Comparison to code: Serpent 2 model of the experiments .................. 78
    4.1.4 Analysis of experiments .................................................................. 79
      4.1.4.1 Calculation of VTM/CTMs and APSD/CPSDs .............................. 79
      4.1.4.2 Discussion of anomalies ....................................................... 79
      4.1.4.3 Model fitting ........................................................................ 81
    4.1.5 Results and discussion ..................................................................... 81
      4.1.5.1 Critical neutron noise field .................................................... 81
      4.1.5.2 Sub-critical neutron noise field ............................................. 83
    4.1.6 Summary of neutron noise field study ............................................. 87
  4.2 Gamma noise field experiments .............................................................. 89
    4.2.1 Experimental setup ........................................................................ 89
    4.2.2 Results and discussion ..................................................................... 89
      4.2.2.1 PSD amplitude of gamma noise with distance .......................... 89
      4.2.2.2 Ex-vessel prompt decay constant reference ............................. 91
      4.2.2.3 Far field correlation validation .............................................. 92
  4.3 Normalization of reaction rate maps: Geometric efficiency as driver of noise
      success ............................................................................................... 94
  4.4 Chapter’s salient elements ...................................................................... 96
5 Monte Carlo simulation of branching noise

5.1 Monte Carlo methods for noise simulation

5.1.1 Monte Carlo for neutronics

5.1.2 Analog Monte Carlo transport

5.1.3 Implicit capture Monte Carlo transport

5.1.4 Fission neutron sampling

5.1.5 Capturing time correlation in Monte Carlo

5.2 Verification: Design of a numerical experiment

5.2.1 Whole core fission tracking

5.2.2 T4 input specifications for noise simulation

5.2.3 S2 coupling to FREYA

5.2.4 Noise from simulation: Rossi-\(\alpha\)

5.2.5 Noise from simulation: Feynman-\(\alpha\)

5.2.6 Noise from simulation: PSD

5.2.7 To \(\beta\) or not to \(\beta\)

5.2.8 Computational requirements

5.2.9 The Source problem

5.2.10 Is analog or FREYA necessary?

5.2.11 What does the noise method measure?

5.3 Validation: Comparison to IFP and experiments

5.3.1 \(\Delta\alpha\) for different configurations

5.3.2 Explicit modeling of He-3 experiments

5.3.3 Location of first and last fission of an observed fission chain

5.4 Extrapolation: Noise simulations for physics interrogation

5.4.1 Decoupled kinetics: Split core CROCUS

5.5 Chapter's salient elements

6 Conclusion and Outlook

6.1 General summary and conclusion

6.2 Outlook and recommendations

A Glossary of terms

B Probability generating function method

C Kolmogorov forward approach to derive point kinetics

C.1 Derivation of Feynman-\(\alpha\) from the forward approach
List of Figures

1.1 The logistic map for different growth rates. .......................... 2
1.2 Illustration of the fission chain reaction. ........................... 3
1.3 The three branches of physics. ........................................... 4
1.4 Schematic of how nuclear data is created for particle transport codes. . . 5
1.5 Illustration of the application range of noise measurements. ................ 6

2.1 Schematic of the relation between reactor kinetics measurements and prediction codes. .................................................. 16
2.2 Schematic graph of the measured power of a nuclear reactor at steady state. 17
2.3 Illustration of the neutron branching process in time for a multiplying medium due to fission. ........................................... 18
2.4 Block diagram of a state space representation of a point kinetic reactor. . . 20
2.5 Graph of the inhour equation and the prompt kinetics approximation curve. 21
2.6 Frequency vs. impulse response plot of the neutron population in a subnuclear reactor system. ........................................... 21
2.7 Illustration of measured Rossi-α and PSD curves of a nuclear reactor. .... 23
2.8 Comparison of noise methods in dependence of ε and $F_0$ .................. 26
2.9 Measured auto correlation for the CROCUS reactor using a 1 g $^{235}$U fission chamber at 40 mW reactor power. .......................... 27
2.10 Measured Y-variance and covariance functions using 1 g $^{235}$U fission chambers of the MINERVE reactor. ............................... 28
2.11 Measured auto and cross power spectra of the MINERVE reactor using 1 g $^{235}$U fission chambers. ........................................... 29
2.12 Linear scale depiction of the auto-correlation function of the space independent point model (standard Rossi-α) compared to the space dependent infinite medium point detector model. ............................. 34
2.13 Analytic examples of PSD neutron, neutron gamma, and gamma PSDs. .... 36

3.1 Calculated APSD using the analytic expression with different levels of AWGN characterized by a SNR in dB. ............................... 44
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>Kinetic parameters uncertainty when fitting the analytic APSD expression with AWGN of different SNR and different detector efficiencies $\epsilon$.</td>
</tr>
<tr>
<td>3.3</td>
<td>Isometric view of the CROCUS reactor vessel and internals and the top grid.</td>
</tr>
<tr>
<td>3.4</td>
<td>Schematic and dimensions of the radial fuel arrangement of CROCUS.</td>
</tr>
<tr>
<td>3.5</td>
<td>Schematic and dimensions of the axial fuel arrangement of CROCUS.</td>
</tr>
<tr>
<td>3.6</td>
<td>Overview of the CROCUS top grid.</td>
</tr>
<tr>
<td>3.7</td>
<td>Schematic of the board logic of the new current amplifier for noise measurements.</td>
</tr>
<tr>
<td>3.8</td>
<td>Technical drawing of the CeBr$_3$ detector.</td>
</tr>
<tr>
<td>3.9</td>
<td>Technical drawing of the BGO detector.</td>
</tr>
<tr>
<td>3.10</td>
<td>Examples of gamma ray spectra acquired in different reactor states using the LEAF CeBr$_3$ (top) and BGO (bottom) detectors.</td>
</tr>
<tr>
<td>3.11</td>
<td>Evolution of count rates in LEAF and the reactor power monitor with reactor configuration.</td>
</tr>
<tr>
<td>3.12</td>
<td>Top view of the critical kinetics benchmark core configurations for neutron and gamma noise.</td>
</tr>
<tr>
<td>3.13</td>
<td>Schematic of a neutron noise measurement experimental setup in CROCUS.</td>
</tr>
<tr>
<td>3.14</td>
<td>Schematic of a gamma neutron noise measurement experimental setup in CROCUS.</td>
</tr>
<tr>
<td>3.15</td>
<td>Signals and PSDs of the qualification measurement of the CFUL01 detectors and current mode amplifiers.</td>
</tr>
<tr>
<td>3.16</td>
<td>Probability densities of the bootstrapped kinetic parameters estimated for the neutron noise benchmark measurement.</td>
</tr>
<tr>
<td>3.17</td>
<td>Signals and PSDs of the qualification measurement of the CeBr$_3$ detectors.</td>
</tr>
<tr>
<td>3.18</td>
<td>Comparison of gamma PSDs with increasing lower level threshold compared to the analytical neutron PSD expression.</td>
</tr>
<tr>
<td>3.19</td>
<td>Probability densities of the bootstrapped kinetic parameters estimated for the gamma noise benchmark measurement.</td>
</tr>
<tr>
<td>4.1</td>
<td>Top view of the critical kinetics benchmark core configurations for neutron and gamma noise.</td>
</tr>
<tr>
<td>4.2</td>
<td>Illustration of the He-3 detector setup for sub-critical measurements.</td>
</tr>
<tr>
<td>4.3</td>
<td>Side view of the sub-critical reactor configurations at 950 mm and 800 mm.</td>
</tr>
<tr>
<td>4.4</td>
<td>Examples of measured CPSD and CTM curves for different water levels and distances in CROCUS.</td>
</tr>
<tr>
<td>4.5</td>
<td>Comparison of APSD/CPSD and Feynman-$\alpha$ VTM/CTM curves at different water levels at 4 cm distance for anomaly illustration.</td>
</tr>
<tr>
<td>4.6</td>
<td>Overview of the BGO detector locations for the gamma noise field characterization.</td>
</tr>
</tbody>
</table>
List of Figures

4.7 Example of curve fit results on measurement data with residuals for SE ad
ME fits. ........................................ 83
4.8 CTM and CPSD analysis (SE and ME) results for the different experimental
water levels compared to Serpent 2 IFP JEFF 3.3. .......................... 84
4.9 C/E-1, parameter standard deviation, and z score for $\alpha$ yielded by Serpent
2 IFP using JEFF 3.3 relative to CPSD or CTM analysis for ME fits. ..... 85
4.10 C/E-1, parameter standard deviation, and z score for $\alpha$ yielded by Serpent
2 IFP using JEFF 3.3 relative to CPSD or CTM analysis for ME fits. ..... 86
4.11 C/E-1 results comparing the prompt decay constant from all 25 sub-critical
experiments to Serpent 2 IFP predictions using JEFF3.3, sorted by reac-
tivity or distance). ................................ 87
4.12 E/E-1 results comparing the prompt decay constant from all 25 sub-critical
experiments to in-core gamma noise sorted by distance of the He-3 experi-
ment to the core. .................................. 88
4.13 Top view of the BGO detector locations for the gamma noise field charac-
terization. ....................................... 90
4.14 Overview of the measurement duration for the gamma noise field character-
ization. .......................................... 90
4.15 Comparison of the measured BGO APSD and CPSD amplitudes with dis-
tance to the core. .................................. 91
4.16 Overview of the BGO detector locations for the gamma noise field charac-
terization. ....................................... 92
4.17 Probability distribution of the bootstrapped kinetics parameters estimated
for the neutron noise benchmark measurement. ......................... 92
4.18 Comparison of $\gamma$ CTM curves after 2hours of measurement of a critical
configuration of CROCUS at 20 mW, at the reactor vessel and at the far
end of the reactor hall. ................................ 93
4.19 Normalized total efficiency (Counts per fission) for different detection ma-
terials in the CROCUS reactor. ................................ 95

5.1 Example of fission neutron yield for an analog calculation using TRIPOLI4. 101
5.2 Flowchart of the S2 particle transport coupled with FREYA for fission event
treatment. ......................................... 104
5.3 Basic principle to create a Rossi-$\alpha$ distribution from Monte Carlo transport
calculations. ...................................... 105
5.4 Effect of changing the amount of bins ($10^2$ to $10^6$ bins) used for creating a
Rossi-histogram of fission time stamps. ................................ 105
5.5 Difference in two-exponent model fit results for different bin amounts of a
created Rossi-distribution from T4 using the CROCUS model at 800mm
water level. ........................................ 106
5.6 Fit results for two exponent models on the $10^6$ bins distribution using the
binning test data set. .................................. 107
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.7</td>
<td>Comparison of Feynman-α curves obtained from noise simulations with differing maximum time $T$.</td>
</tr>
<tr>
<td>5.8</td>
<td>Illustration of how to treat time stamp data from MC simulations to create the observable noise data.</td>
</tr>
<tr>
<td>5.9</td>
<td>Rossi distributions and fit results for single exponent models for different simulation modes: Delayed neutrons on and off.</td>
</tr>
<tr>
<td>5.10</td>
<td>Rossi distributions and fit results for single exponent models for increasing single CPU time use.</td>
</tr>
<tr>
<td>5.11</td>
<td>T4 model of CROCU S with sketched location of the PuBe start-up source with neutron energy spectrum of the source.</td>
</tr>
<tr>
<td>5.12</td>
<td>Fission source distributions comparison: T4 k-static vs. explicitly modeled source.</td>
</tr>
<tr>
<td>5.13</td>
<td>Rossi and PSD fission distributions and fits in black for single exponent models for different water levels. The calculations used analog transport and FREY A fission.</td>
</tr>
<tr>
<td>5.14</td>
<td>Rossi distributions and fit results for single exponent models for different simulation modes: Analog, analog with FREY A and standard implicit capture.</td>
</tr>
<tr>
<td>5.15</td>
<td>Rossi fission distributions and fits in black for single exponent models for different water levels. The calculations used analog transport and FREY A fission.</td>
</tr>
<tr>
<td>5.16</td>
<td>Prompt decay constant for the different experimental configurations in CROCU S, as predicted by S2 using IFP, T4 using IFP, and T4 using noise methods.</td>
</tr>
<tr>
<td>5.17</td>
<td>Prompt decay constant for the different experimental configurations in CROCU S, as predicted by S2 using IFP, T4 using IFP, and T4 using noise methods.</td>
</tr>
<tr>
<td>5.18</td>
<td>Comparison of Rossi distributions for all five water levels and distances resulting from (n,p) tracking in the Helium-3 detectors.</td>
</tr>
<tr>
<td>5.19</td>
<td>Top view of a scatter plot of coordinates for last fission locations of a chain that results in a (n,p) detection.</td>
</tr>
<tr>
<td>5.20</td>
<td>Top view of a scatter plot of coordinates for first fission locations of a chain that results in a (n,p) detection.</td>
</tr>
<tr>
<td>5.21</td>
<td>Top view of the T4 model of CROCU S in split core configuration at 4 U$_{\text{met}}$ pitches distance with added fuel.</td>
</tr>
<tr>
<td>5.22</td>
<td>Rossi distributions with single exponent fits for the CROCU S split core configuration.</td>
</tr>
<tr>
<td>5.23</td>
<td>Rossi distribution with single and two exponent fits for the CROCU S split core at 4 U$_{\text{met}}$ pitches with added fuel.</td>
</tr>
<tr>
<td>6.1</td>
<td>Power spectral density obtained from BGO gamma detectors set at approximately 7 meters from the core center.</td>
</tr>
<tr>
<td>D.1</td>
<td>View of the current mode amplifier board geometry.</td>
</tr>
</tbody>
</table>
D.2 Detailed electronic schematic of the current mode amplifier 2. 140
D.3 Measured transfer function of the current amplifier box 1 using a pulse generator. 141
D.4 Linearity tests of the current mode amplifiers boxes 1 and 2 in CROCUS. 141
F.1 Schematic describing the bootstrap method for PSD analysis. 145
G.1 Periodogram versus 200 Bartlett averaged spectra for PSD estimation of a neutron noise signal acquired in CROCUS. 149
G.2 Comparison of PSD estimates for different number of averaged spectra on the total signal. 151
G.3 Southern APSD $a + \tilde{c}$ and $\tilde{c}$ parameter distributions for the number of averaged spectra. 152
G.4 Southern CPSD $a + \tilde{c}$ and $\tilde{c}$ parameter distributions for the number of averaged spectra. 152
G.5 Mean $\mu_a$ and standard deviation $\sigma_a$ of PSD plateau "a" from 1 to 5 Hz. $\sigma_a$ is divided by 10 for illustrative purposes. 154
G.6 Display of the $a + \tilde{c}$ and $\tilde{c}$ PSD parameters for robust segment adding to the signal of which the PSD is estimated. 155
G.7 Southern CPSD $a$ and $\tilde{c}$ parameter distributions for the number of averaged spectra. 156
H.1 Schematic describing the SNR estimation method. 158
H.2 SNR over time for the CFUL01 and the current amplifiers. 159
H.3 Kinetic parameters and their first order uncertainties with measurement duration. 159
I.1 Pictures of chosen CFUL01 detection setups for the neutron noise field experiments. 162
I.2 Pictures of chosen BGO detection setups for the gamma noise field experiments. 164
I.3 Pictures of chosen BGO detection setups for the gamma noise field experiments. 165


## List of Tables

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Overview of noise instrumentation SNR and efficiency in typical measure-</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>ment locations.</td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>Overview of the CROCUS reactor operational states.</td>
<td>50</td>
</tr>
<tr>
<td>3.3</td>
<td>Overview of commonly used neutron detectors in CROCUS.</td>
<td>52</td>
</tr>
<tr>
<td>3.4</td>
<td>Overview of kinetic parameters obtained from the neutron and gamma noise benchmarks.</td>
<td>70</td>
</tr>
<tr>
<td>4.1</td>
<td>Overview of parameters of each of the critical neutron noise field experiments using CFUL01 fission chambers</td>
<td>76</td>
</tr>
<tr>
<td>4.2</td>
<td>Measurement time in hours for each position and water level $H$ with estimated reactivity using Serpent 2 JEFF 3.1.1 in dollar</td>
<td>78</td>
</tr>
<tr>
<td>4.3</td>
<td>Critical kinetics parameters determined with CFUL01 fission chambers at different distances to the fuel</td>
<td>82</td>
</tr>
<tr>
<td>4.4</td>
<td>Critical kinetics parameters determined with LEAF scintillators at different distances to the reactor</td>
<td>93</td>
</tr>
<tr>
<td>5.1</td>
<td>Summary of methods and their free parameters for determining noise curves from Monte Carlo transport</td>
<td>109</td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison of the prompt decay constant $\alpha$ for different water levels of CROCUS for IFP predicted values and fit results for the noise method using both T4 and S2</td>
<td>116</td>
</tr>
<tr>
<td>5.3</td>
<td>Comparison of the prompt decay constant $\alpha$ for different water levels of CROCUS for experiments, IFP predicted values for S2 and T4, and noise method predictions.</td>
<td>118</td>
</tr>
<tr>
<td>5.4</td>
<td>Overview of number of time stamps and results for $\alpha$ from $^3\text{He}(n,p)$ tracking simulations</td>
<td>119</td>
</tr>
<tr>
<td>G.1</td>
<td>Relation between window length, number of averaged spectra and the spectral resolution for the reference measurement</td>
<td>153</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Of randomness and nuclear reactors

The concepts of randomness and random events have existed since the dawn of humanity, albeit used to discover fate by flipping coins or, later, gambling [1]. Yet with the birth of Newtonian mechanics and the study of dynamics and differential equations, philosophers had a strong case to argue that a coin flip could be predicted if the initial conditions were known – Laplace’s demon and scientifically formulated determinism were born [2]. It remains arguably one of the most groundbreaking discoveries, when in the late 19th century scientists, such as Becquerel, Curie, and Rutherford, observed that the newly discovered radioactive atoms decayed spontaneously, pointing to the inherent random fundamental nature of reality\(^1\). Ultimately, this lead to the formulation of quantum mechanics and the revolutionized view on physics of the 20th century. At the onset of the 21st century this fundamental randomness has yet to be reconciled with gravity, but physicists are met with the next headache [4]: Complex systems and chaos that emerge from seemingly simple non-linear equations, such as the logistic map [5] to describe populations over discrete times. Given different growth rates, the equation can yield a steady population, bifurcate into oscillations, or pseudo-random chaotic behavior, as depicted in Figure 1.1\(^2\). Random noisy behavior of a seemingly simple system apparently emerges as a feature, rather than a bug\(^3\).

Nuclear reactors are, generally speaking, purposeful arrangements of fissile material designed to maintain a controlled chain reaction – to produce heat for electricity generation, radioisotopes, or serve as research facilities to study their behavior. At the heart of this chain reaction is neutron-induced fission in a heavy nucleus such as uranium. This nucleus, having absorbed the neutron and transitioned into an almost immediate unstable configuration, disintegrates into smaller constituents. These mostly include the heavy daughter

\(^1\)Hidden variables still loom, however, waiting to prove our cosmic subjugation to causality – or as Einstein put it: "he does not play dice" [3].

\(^2\)Incidentally, these types of deterministic equations that can exhibit chaos were used as the first sources for pseudo-random numbers on computers.

\(^3\)"It's not a bug, it's a feature", the result of a study showing that about a third of reported bugs are mis-classified and are actually useful [6].
Chapter 1. Introduction

Figure 1.1 The logistic map for different growth rates $r$. Given an initial condition for $r$, the final population can be asymptotically stable ($r \in [2,3]$), bifurcate into oscillations ($r \in [3,3.56]$), or exhibit chaos.

nuclei (fission products), gamma rays, neutrinos, and notably, free neutrons. These newly released neutrons can then induce the next fission, allowing for a chain reaction to occur, as depicted in Figure 1.2.

In studying the dynamics of nuclear reactors, a blend of all the aforementioned physics is found: Individual nuclear interactions are described by probability distributions of spontaneous quantum events and large scale neutron populations require sets of coupled non-linear equations to describe their transport. Even bifurcations and chaotic dynamics can emerge in boiling water reactors, prompting the use of all the discussed tools to understand their origin [7].

Unlike human populations, one does not require logistic growth models to describe neutron population dynamics, using a few educated assumptions. A common approximation is to use a branching process to describe the effects of fission as a generator of a random amount of offspring [8], also noticeable from Figure 1.2. Branching processes are a type of stochastic process used to model varying types of simplified population phenomena, such as the propagation and extinction of family names.

Typically, reactors are operated at a critical state, a state in which the average production rate of neutrons equals the average loss rate. Deviations from this critical state are characterized by the so-called kinetic parameters of a nuclear reactor. These parameters are the coefficients of the time dependent neutron balance equations that can be derived, among others, from a branching process formulation of neutron populations. The resulting equations are called point kinetics equations, and are perhaps the most cost effective yet still accurate way to model reactor dynamics [9].
1.2 Kinetic parameters of nuclear reactors

Reactor dynamics modeling is an important step in the design process and operation of a reactor, in particular to enable cost effective simulations to assess the impact of a change on operational requirements or safety. The point kinetics equations require the kinetic parameters as inputs, which in turn need to be predicted by calculation or determined experimentally. This ties into the more general circular process of theoretically describing compound physical phenomena, their computational implementation and experimental validation, as illustrated in Figure 1.3.

1.2.1 Kinetic parameter prediction & uncertainty

Reactor kinetics can be predicted by a variety of computational methods, often classified as deterministic or stochastic (Monte Carlo\(^4\)) methods. These codes require geometry, material and nuclear data to produce a prediction. Nuclear data hereby refers to a library containing interaction probabilities and other quantities pertinent to a calculation, e.g. cross sections and fission neutron yields.

Nuclear data is provided to the codes in the shape of evaluated data libraries, such as the Joint Evaluated Fission and Fusion File (JEFF) [10] or the Evaluated Nuclear Data File (ENDF) [11, 12]. For example, cross sections are to date not predictable by theory due to the complexity of multi-body quantum interactions; evaluated data is hence created by using available differential experiments and nuclear models to create inter- and extrapolation rules to have a continuous energy file with respective evaluated uncertainties. When predicting a kinetic parameter, these uncertainties are propagated. Thus a prediction will carry biases and uncertainty stemming directly from nuclear data [13].

This is however not the only source of bias and uncertainty, as the transport codes used for the computational part have their own contributions. A common source is in the approximation of theoretical models, such as discretization of space, energy and angle for

\(^4\)The now widely used name for stochastic simulation was coined by Los Alamos scientist Nicholas Metropolis in the 50s when working on the first sampling calculation schemes with John von Neumann and Stanislaw Ulam.
deterministic codes, or the scoring method to infer macroscopic quantities in continuous Monte Carlo codes. Nuclear data choices, model approximations and methods thus require validation against experimental data. Experiments used for transport code validation are typically called integral measurements, as they yield some compound quantity such as kinetic parameters. The overall scheme is also sketched in Figure 1.4. On top of validation, integral data can be used for data assimilation (DA), a type of statistical learning, to back-track the propagated uncertainties in terms of their relative impact [13]. Given a diversity of experimental data that is accurate enough, DA can provide means to constrain nuclear data distributions and thus aid in the evaluation.

1.2.2 Noise measurements to determine kinetic parameters

A self-evident way to measure kinetic parameters would be by explicit dynamic experiments: A certain non-critical configuration in a reactor is imposed and the result observed. Branching processes, however, exhibit correlations that can be exploited in steady state experiments, considered as non-invasive in comparison. Based on the analysis of the temporal fluctuations of a signal, they are also called noise analysis methods. Noise commonly refers to unknown or unwanted temporal fluctuations in a signal, but acquires a new meaning if the noise sources are correlated — such as in a neutron-multiplying medium such as a nuclear reactor. Subsequent neutron detector counts can originate from the same fission chain, yielding information about the system kinetics without needing active perturbation. Practically this means that kinetic parameters can be measured in systems where dynamic experiments are expensive, impractical, or unsafe.

Neutron noise measurements using point kinetic models have been successfully used for zero power research reactor characterization [16] and to validate kinetic parameter prediction codes [17], but have shown promise as well for nuclear security applications [18, 19], and
1.2. Kinetic parameters of nuclear reactors

- Evaluated Nuclear Data: ENDF, JEFF, JENDL

- Particle Transport Code: e.g. Monte Carlo: MCNP, Serpent, TRIPOLI

**Figure 1.4** Schematic of how nuclear data is created for particle transport codes and how integral experiments are used for validation and possibly evaluation. Largely adapted from [15].

Integral experiments require both a detailed analysis to remove sources of experimental biases, and uncertainties that are small enough to give statistical weight to a validation or even DA [21]. The uncertainties for delayed neutron fraction integral measurements in particular are required to be smaller than 3% in order to meaningfully contribute to nuclear data evaluation [22]. However, several experiments [23, 24] showed uncertainties too small to explain observed discrepancies to code predictions – pointing to a problem in the standard approach on how uncertainties are estimated or to the models chosen. As we will elucidate in Chapter 2, there are several potentially important sources of bias and uncertainty in noise measurements, notably:

- Bias arising from the model choice used to analyze noise experiments. The standard approach is based on point kinetics [25], although both in theory [26] and experiment [27] an impact of so called spatial effects has been observed in certain reactor configurations.

- Bias from the methods to extract the relevant correlation information out of a detector’s signal, e.g. found in numerical parameters for frequency domain calculations [28], or in the calibration of fission rates [29].

- Inconsistencies in the uncertainty quantification and the resulting reporting of uncertainties from one experiment to another. These differences stem from the variety of optimization schemes pertaining to noise analysis and the varying adequacy of non-linear optimization schemes – often tied to the common assumption of normally distributed parameters. Monte Carlo based DA, for example, can incorporate more complex distributions and enable a more nuanced incorporation of experimental data. This calls for a standardized approach optimized for noise experiments to report results with full distributions.
Chapter 1. Introduction

Figure 1.5 – Illustration of the application range of noise measurements. Photo credits: Left: Dean Calma/IAEA. Center left: Rod Adams/ANS.

1.3 Aim and structure

Hitherto, noise measurements are conducted to 1) characterize a zero power reactor’s kinetic parameters experimentally, 2) validate codes that predict kinetic parameters, and 3) explore applications of branching physics to probe the limits of point kinetics and new applications of the noise method.

We identified a general possibility of nuclear data improvement by providing accurate integral data. We answer this need first by attempting to standardize the uncertainty and bias estimation of noise experiments, second by conducting a directed set of experiments to improve accuracy and study spatial effects, third by providing a simulation methodology to study noise experiments explicitly.

The CROCUS core, despite its long history, has only recently been characterized with regards to its kinetic parameters by dynamic [30, 31, 32] and neutron noise measurements [33, 24]. In addition, current and future experimental programs [34] will employ noise methods for analysis and require an accurate knowledge of the noise field in the core. Efforts to quantify active perturbation reactor noise caused by fuel rod oscillations and bubbles have been initiated in the CROCUS facility [35]. The underlying branching noise requires a characterization to allow for meaningful references to enable distinction. Therefore, the interest of investigating spatial effects on noise, specifically in the CROCUS reactor, is a further driving motivation of this thesis.

The goals of this thesis can thus be formulated as follows:

• **Provide a standardized approach to parameter and uncertainty estimation in noise analysis:** In order to contribute to validation or even DA to improve nuclear data, the biases and uncertainties of an individual measurement need to be well quantified. We seek to define the uncertainty quantification procedure for noise measurements that allows for consistent and accurate representation of the experimental results.

• **Study the validity of point kinetics in noise experiments:** The CROCUS reactor is a flexible facility to provide validation data. Given the potential biases, how true can a result produced by point kinetic model be? Are they accurate enough,
1.3. Aim and structure

or are more sophisticated models required to provide feedback to predictions or even nuclear data evaluation? Is the noise data produced in CROCUS biased by spatial effects and – if so – to what extent? The focus is hereby set on the potential limits of point kinetics in the ex-core region of the reactor: The detectors are to be placed in the water reflector surrounding the core and ex-vessel.

- **Reduce bias and uncertainties using optimized and new measurements:** The noise detection capabilities of CROCUS are limited to neutron detectors used at close to critical states and low powers. We thus specifically intend to extend the noise detection instrumentation of CROCUS by two systems: Firstly, a high efficiency, flexible neutron detection system in current mode to enable the study of the reactor noise field. Secondly, a gamma detection system to enable gamma noise measurements. Gamma noise could be an alternative to neutron noise as a kinetics measurement method, and is as an exploratory topic investigated in this thesis. Gamma radiation emitted during fission carries the same correlation information as neutrons, yet gamma noise research has halted after initial studies in the 60s and 70s [36]. Due to the difference in propagation compared to neutrons, gamma radiation is expected to be less prone to spatial effects and carry the correlation information farther, enabling more flexible detector placement. Gamma noise could thus provide means to identify biases using this perspective, and possibly reduce overall uncertainties.

- **Simulate noise:** Spatial effects and other biases to point kinetics models exist, yet cannot be predicted by a universal tool to be used for arbitrary geometries. Monte Carlo methods are potentially the best tool to simulate noise experiments, as they are agnostic to most transport models. This direct modeling would serve as a tool for further analysis and understanding of the physical processes at hand. In addition, given the experimental data gathered in this thesis, we propose to validate a methodology based on explicit analog Monte Carlo simulations of noise experiments to provide a new way to assess the physics of a noise experiment.

A summary of the individual chapters is given below.

**Chapter 2: Noise measurements in nuclear reactors** Noise measurements is a field of study with its origins in the first nuclear boiler studies in the 50s, mainly developed in the 60s and 70s, and only recently re-gaining attention as a tool to measure kinetic parameters. This chapter discusses the theoretical groundwork behind nuclear reactor dynamics and derives the point kinetics approximation. By introducing the notion of reactor noise, we discuss how noise measurements can be motivated from a linearized reactor system, and discuss an intuitive description of common noise analysis methods. Then, an overview of experimental realizations of the respective noise methods and their limitations is presented. This is followed by a discussion on beyond point kinetic models and how they have been used in the past, displaying how spatial effects are described in theory and how they potentially influence a noise measurement. The chapter is closed by introducing gamma noise and providing a prediction of different gamma noise scenarios in preparation of our experimental investigation.
Chapter 3: Reference noise experiments in CROCUS  

Noise measurements, despite having a long history, have an inconsistent record when it comes to the reporting of results and uncertainty estimation. A particular focus is laid on presenting the general methodology for uncertainty estimation used for the experiments in this thesis. Then, the experimental stage upon which the experiments stand is presented: the CROCUS zero power research reactor. Before this thesis, CROCUS had standard capabilities for neutron noise measurements, yet had none for beyond low-power and close-to-criticality neutron noise measurements or gamma detection. In order to investigate spatial effects and gamma noise, the instrumentation of CROCUS needed to be extended by new developments, namely current amplifiers for high efficiency fission chambers and a gamma detection system comprising modern scintillators. Using the newly extended capabilities for noise measurements, reference experiments at criticality for CROCUS’ kinetic parameters using neutron and gamma noise measurements are presented.

Chapter 4: Experimental study of the branching noise field of CROCUS  

The CROCUS reactor has a unique design that allows for relatively flexible detector placement to study the noise field. In particular, we examined the critical and sub-critical neutron noise field using two different detector setups to reveal the extent of the measurable noise field and identify, if any, spatial effects. By conducting sub-critical experiments, we attempted to maximize the manifestation of observable spatial effects. We also studied the measurable extent of the gamma noise field, and compare the overall performance to neutron noise.

Chapter 5: Monte Carlo simulation of branching noise  

Explicit modeling of branching processes has become increasingly viable with modern computing power and Monte Carlo methods. A methodology for noise simulations is a research interest to account for the biases found in deterministic prediction tools, such as biases and limitations arising from point kinetics assumptions. In this chapter we present the full methodology, numerical experimentation, and validation to CROCUS experiments of noise simulations using Monte Carlo methods. The methodology is insofar expected to be less biased, as it requires only transport physics and is agnostic to approximations such as point kinetics. We specifically test a new methodology using two established Monte Carlo particle transport codes, TRIPOLI-4 and Serpent 2. We further investigate the general behavior of fission chains in CROCUS to deduce a qualitative explanation as to why point kinetics is a valid approximation, as compared to the aforementioned experiments in order to provide additional insights. The validated methodology is then used on a split core model of CROCUS to investigate its capabilities to capture beyond point kinetic behavior.
Chapter 2

Noise measurements in nuclear reactors

This chapter is aimed at summarizing the framework of neutron reactor kinetics to provide an understanding of noise measurements in theory and application. The structure is as follows: In Section 2.1 we discuss the notion of reactor kinetics, derive the so-called point kinetics equations, and discuss kinetics measurements. Then, in Section 2.2 through 2.3, the concept of noise in general and with regards to reactor kinetics is elucidated. Noise measurements are typically analyzed using three established methods, which we discuss in the context of a simplified reactor description in Section 2.4. A review of previous experiments in the field is presented thereafter in Section 2.5. In Section 2.6, we introduce the peculiarities of spatial effects on noise measurements, reviewing theory and summarizing potential impacts on experiments. Before summarizing and concluding in Section 2.8, we review theory and experiments of gamma noise systems and discuss their prospects in Section 2.7.

2.1 Nuclear reactor kinetics

We will refer to time dependent phenomena by kinetics as opposed to dynamics, which refer to time dependent phenomena with feedback, following the nomenclature of [9]. These types of systems are also called zero power systems, and are the primary object of study for noise experiments. Retracing the steps in [9], we are able to derive differential equations with constant coefficients to describe the time dependent behavior of nuclear reactors. These coefficients are often referred to as kinetic parameters; the sections thereafter are aimed at introducing the kinetic parameters and their measurements.

2.1.1 Time dependent Boltzmann transport equation

The neutron transport equation is primarily motivated by the need to predict the power generated in a reactor at a given time in order to adjust cooling parameters, estimate fuel performance and depletion as well as time dependent behavior for accident analysis. A
neutron flux distribution $\phi$ in a reactor with no feedback in dependence of time $t$, direction $\Omega$, energy $E$ and position $r$ can be described by the linear Boltzmann equation, a balance equation between losses and sources $Q$

$$\left[\frac{1}{v(E)} \frac{\partial}{\partial t} + \Omega \nabla + \Sigma_t(r,E,t)\right] \phi(r,\Omega,E,t) = Q(r,\Omega,E,t) = Q_{\text{scattering}}(r,\Omega,E,t) + Q_{\text{fission}}(r,\Omega,E,t) + Q_{\text{external}}(r,\Omega,E,t) \tag{2.1}$$

where $v$ is the neutron velocity and $\Sigma$ is a macroscopic cross section. The terms on the left hand side are respectively the time derivative term to account for changes in the neutron distribution, the losses due to leakage, and total losses due to absorption, fission, etc. The terms on the right hand side describe scattering, fission and external sources of neutrons and are in explicit form found to be

$$\left[\frac{1}{v(E)} \frac{\partial}{\partial t} + \Omega \nabla + \Sigma_t(r,E,t)\right] \phi(r,\Omega,E,t) =$$

$$\int_0^\infty dE' \int_0^{4\pi} d\Omega \Sigma_s(r,\Omega' \rightarrow \Omega,E' \rightarrow E) \phi(r,\Omega',E',t) +$$

$$\int_0^\infty dE' \int_0^{4\pi} d\Omega \nu(E') \Sigma_f(r,E',t) \phi(r,\Omega,E',t) + Q_{\text{external}}(r,\Omega,E,t),$$

with $\chi$ being the emission spectrum of neutrons from fission, $\nu$ being the amount of neutrons emitted per fission, and the subscript $s$ referring to scattering. This integro-differential equation needs both initial conditions and boundary conditions in order to be solved. The fission source is further split up to specifically account for prompt neutrons and $i$ delayed neutron precursor groups, with a representative subscripts $p,d$:

$$Q_p = \frac{\chi_p(E)}{4\pi} \int dE' \int d\Omega \nu_p(E') \Sigma_f(r,E',t) \phi(r,\Omega,E',t) \tag{2.3}$$

and

$$Q_d = \sum_i \frac{\chi_{d,i}(E)}{4\pi} \lambda_i C_i(r,t). \tag{2.4}$$

The added delayed neutron precursor density $C_i$ with respective decay constants $\lambda_i$ from precursor group $i$ require additional equations to be solved with the neutron transport:

$$\frac{\partial C_i(r,t)}{\partial t} = -\lambda_i C_i(r,t) + \int dE' \int d\Omega \nu_{d,i}(E') \Sigma_f(r,E',t) \phi(r,\Omega,E',t). \tag{2.5}$$

Commonly a structure of 6 or 8 precursor groups are used to limit computational and experimental database needs on the calculations. For practical reasons the following notation is introduced:

$$M\phi = L\phi + R\phi - S\phi \tag{2.6}$$

with

$$L\phi = \Omega \nabla \phi, \tag{2.7}$$

$$R\phi = \Sigma_t(r,E,t) \phi, \tag{2.8}$$
2.1. Nuclear reactor kinetics

\[ S \phi = \int_0^\infty dE' \int_0^{4\pi} d\Omega \Sigma_s(r,\Omega' \rightarrow \Omega, E' \rightarrow E) \phi(r, \Omega', E', t), \]  
\[(2.9)\]

\[ F \phi = \frac{\chi(E)}{4\pi} \int_0^\infty dE' \int_0^{4\pi} d\Omega \nu(E') \Sigma_f(r, E', t) \phi(r, \Omega, E', t), \]  
\[(2.10)\]

which again can be split up to account for delayed neutrons

\[ F_p \phi = \frac{\chi_p(E)}{4\pi} \int_0^\infty dE' \int_0^{4\pi} d\Omega (1 - \beta) \nu(E') \Sigma_f(r, E', t) \phi(r, \Omega, E', t), \]  
\[(2.11)\]

\[ F_{d,i} \phi = \frac{\chi_{d,i}(E)}{4\pi} \int_0^\infty dE' \int_0^{4\pi} d\Omega \beta_i \nu(E') \Sigma_f(r, E', t) \phi(r, \Omega, E', t), \]  
\[(2.12)\]

with \( \beta_i \) being the fraction of delayed neutrons in group \( i \). The sum gives the total delayed neutron fraction

\[ \beta = \sum_i \beta_i. \]  
\[(2.13)\]

The final, more compact, form of the transport equation is hence

\[ \left[ \frac{1}{\nu(E)} \frac{\partial}{\partial t} + \mathbf{M} \right] \phi = F_p \phi + Q_{\text{fission}} + Q_{\text{external}}. \]  
\[(2.14)\]

The application of Equation 2.2 for criticality or safety calculations is not always practical, as the system parameters, such as the cross sections and the fission neutron yields \( \nu \), will have to be fine tuned to find a non-trivial solution [37]. Solving these equations in non-equilibrium is nonetheless possible given modern computers and memory availability. For larger problem geometries and computational efficiency a simplification, so called diffusion approximation, is used instead.

### 2.1.1.1 The diffusion approximation

To reduce computation and modeling resources the transport equation is often solved in the diffusion approximation. By introducing the net current \( J(r, E, t) \) and by assuming isotropic angular dependency, allowing for integration over \( d\Omega \), we can write [9]

\[ \left[ \frac{1}{\nu(E)} \frac{\partial}{\partial t} + \Sigma_t(r, E, t) \right] \phi(r, E, t) + \nabla J(r, E, t) = \]  
\[ \int dE' \Sigma_a(r, E' \rightarrow E) \phi(r, E', t) + Q_{\text{fission}}(r, E, t) + Q_{\text{external}}(r, E, t). \]  
\[(2.15)\]

The current \( J(r, E, t) \) can be brought into relation with the flux using Fick’s law in the shape of

\[ J(r, E, t) = -D(r, E, t) \nabla \phi(r, E, t) \]  
\[(2.16)\]
with $D(r,E,t)$ being the diffusion coefficient. The final expression for the diffusion equation reads

$$\left[ \frac{1}{v(E)} \frac{\partial}{\partial t} - \nabla D(r,E,t) \nabla + \Sigma_t(r,E,t) \right] \phi(r,E,t) = \int dE' \Sigma_a(r,E' \rightarrow E) \phi(r,E',t) + Q_{\text{fission}}(r,E,t) + Q_{\text{external}}(r,E,t).$$

which can analogously be written as

$$\left[ \frac{1}{v(E)} \frac{\partial}{\partial t} + M_D - F \right] \phi(r,E,t) = Q_d + Q_{\text{external}}$$

with $M_D$ now in diffusion approximation. The exact time dependent behavior is often not desired or too expensive to calculate, as most calculations are concerned with steady-state conditions of a reactor and whether a given geometry can reach criticality. Common methods to again simplify the task at hand is to assume the reactor will evolve in its fundamental mode, and allowing the re-statement of the equations as eigenvalue problems. Common examples are the $\alpha$ and $k$ eigenvalue methods [38]. The arguably most effective method is the so-called point kinetics approximation [9], that relies on an adjoint weighted perturbed transport equations, presented next.

### 2.1.1.2 Adjoint transport equations

For perturbation theory applications the adjoint to Equation 2.14 is used in the form:

$$\left[ \frac{1}{v(E)} \frac{\partial}{\partial t} + M_D - F \right] \phi^\dagger(r,\Omega,E,t) = F^\dagger \phi^\dagger + Q_d^\dagger + Q_{\text{external}}^\dagger.$$

Adjoint scalar products with an operator $A$ will be denoted using brackets

$$(\phi^\dagger, A \phi).$$

The adjoint operators are not shown explicitly, the reader is referred to [9]. The adjoint flux is commonly physically interpreted as the importance function associated to the flux, i.e. it describes how likely a given neutron will productively contribute to the total balance by inducing fission.

### 2.1.2 Point kinetics equations

Full space, energy, angle and time dependent solutions are mostly not necessary nor feasible for applied calculations [9]. To study transient phenomena, the time dependence of the flux often suffices. The so-called point kinetics approach assumes the independence of the time behavior of the neutron density with respect to space, angle and energy, allowing for separation of the time dependent functions. We begin with the transport equation in compact form

$$\frac{1}{v} \frac{\partial}{\partial t} \phi = (F - M) \phi.$$
Here, we introduce the separability of the flux into a time dependent amplitude and space and energy dependent shape function
\[ \phi(r,E,t) = n(t)\phi_0(r,E) \]  
with \( \phi_0 \) being the solution to the initial static equation without perturbation, i.e. critical without external source.

\[ F_0\phi_0 = M_0\phi_0 \]  
By introducing the adjoint weighted multiplication and by integrating over space and energy we get
\[ \frac{\partial}{\partial t}(\phi_0^\dagger, \frac{1}{v}\phi) = (\phi_0^\dagger, [F-M]\phi) \]  
With \( \phi_0^\dagger \) being the solution to the initial static adjoint equation without perturbation

\[ M^\dagger\phi_0^\dagger = F^\dagger\phi_0^\dagger \]  
We insert the separation of Equation 2.22, yielding the so called point kinetics equations:

\[ \frac{dn(t)}{dt} = \frac{\rho(t) - \beta_{\text{eff}}}{\Lambda}n(t) + \sum_i \lambda_ic_i + S. \]  
The precursor equations are found using a Bateman equation
\[ \frac{dc_i}{dt} = -\lambda_ic_i + \frac{\beta_{\text{eff}}}{\Lambda}n(t) \]  
with
\[ c_i = \frac{(\phi_0^\dagger, \chi_{d,i}C_i)}{\Lambda(\phi_0^\dagger, F_0\phi_0)}. \]

The so called kinetic parameters of a reactor are the introduced prompt generation time, the time between birth of a neutron in fission and it’s subsequent capture that results in fission

\[ \Lambda = \frac{(\phi_0^\dagger, \phi_0)}{(\phi_0^\dagger, F_0\phi_0)} \]  
the effective delayed neutron fraction, the adjoint weighted delayed neutron fraction

\[ \beta_{\text{eff}} = \sum_i \frac{(\phi_0^\dagger, F_{d,i}\phi_0)}{(\phi_0^\dagger, F_0\phi_0)} \]  
and the reactivity, a measure of magnitude of the reactor state compared to equilibrium

\[ \rho = \frac{(\phi_0^\dagger, (F - M)\phi_0)}{(\phi_0^\dagger, F_0\phi_0)} \]  
These parameters are, within the given approximations, sufficient to describe a reactor’s temporal behavior and are thus reactor characteristics. Direct solutions to the point kinetic equations can now be calculated with prior knowledge of the kinetic parameters, e.g. to predict a reactor’s behavior to a rod ejection event. These equations can also be derived from a transition rate model for a branching process, shown in Appendix C. The reactivity is also often expressed in terms of the effective multiplication \( k_{\text{eff}} \)

\[ k_{\text{eff}} = \frac{(\phi_0^\dagger, F\phi_0)}{(\phi_0^\dagger, M\phi_0)} = \frac{1}{1 - \rho} \]
2.1.3 Reactor design regarding neutron kinetics

2.1.3.1 The importance of delayed neutrons

The reactor power is directly proportional to the average fission rate, which can be determined from the neutron population $n(t)$ present in the core. To illustrate the neutron population time dependency we consider point kinetics without delayed neutrons. In this case, the point kinetics equation reduces to

$$\frac{dn(t)}{dt} = n(t) \frac{\rho}{\Lambda}$$

Integration gives

$$n(t) = e^{\rho \Lambda t}.$$  \hfill (2.34)

Assuming all neutrons born to be prompt with a generation time of $\Lambda = 10^{-5}$ s and a reactivity of $\rho = 100$ pcm, the population and thus power increases exponentially and reaches an amplification of $2 \cdot 10^4$ after 0.1 s. In a realistic case such a fast reactor behavior would not be controllable.

The fact that a fraction of the neutrons are not emitted promptly but delayed result in the aforementioned delayed source and the reactor becomes controllable. A reactor without external sources is considered to be critical if the neutron population has no time dependence. If the environment, however, does not allow for fission chains to propagate in a self-sustaining manner, the neutron population will die out and the system is called sub-critical. On the contrary, if fission chains create more neutrons on average than are lost, the system is super-critical and the neutron density will diverge in time. We define the reactor states using the reactivity in dependence of $\beta_{\text{eff}}$ as:

- $\rho < 0$ sub-critical
- $\rho = 0$ (delayed) critical
- $0 < \rho < \beta_{\text{eff}}$ (delayed) supercritical
- $\rho > \beta_{\text{eff}}$ prompt supercritical

The reactivity is traditionally also expressed as a fraction of $\beta_{\text{eff}}$ with the unit of dollars $\$ and cents $\$. Deliberate and accidental reactivity insertions should be below $\beta_{\text{eff}} = 1\$ to avoid prompt super criticality. Thus $\beta_{\text{eff}}$ is a measure of the controllability and is a safety characteristic of a reactor, while $\rho$ is a dynamic parameter that needs to be imposed through reactivity controls such as control rods, and requires monitoring during operation.

2.1.3.2 Kinetic parameter prediction

In the process of designing a new reactor, a new fuel loading pattern, or a new experiment, we are generally interested in understanding the impact of a design in terms of reactor
dynamics – be it to minimize post-hoc corrections to the design to reduce costs, or to ensure safety.

A widely used method of estimating \( k_{\text{eff}} \) is the so called k-static method. By restating the time dependent equations as time invariant and forcing equilibrium by re-scaling \( \nu \) by the namesake eigenvalue \( k \), we are able to effectively estimate a systems tendency away from the steady state. To predict \( \alpha, \beta \) or \( \Lambda \), we are required to add an adjoint calculation scheme. In deterministic codes the adjoint flux is thus often calculated alongside the flux to explicitly evaluate the point kinetic expression from Equation 2.26. In neutronics Monte Carlo codes an explicit backwards or adjoint approach has been shown to be complex to implement [39]. Commonly used implementations to avoid an adjoint calculation are based on the perturbation [40], Next-Fission Probability/Meulekamp [41], or, the arguably most successful candidate, iterated fission probability (IFP) method [42, 43].

### 2.1.4 Reactor kinetics measurements

Predicted kinetic parameters are subject to bias, at best only due to the inaccuracy of the imposed physical model. Therefore, integral measurements are necessary in order to validate calculation methods, qualify models and, most importantly, establish a physical reference. A schematic of the measurement methods and their relation to predicted values is depicted in Figure 2.1. Generally, kinetics measurements can be categorized as follows [44], [45]:

- **Static methods** based on the premise of comparing a perturbed state to a reference. This includes methods of source multiplication, compensation and variation.

- **Dynamic methods** based on actively perturbing the reactor by introducing a non-zero reactivity. This includes the methods of rod drop, rod ejection, source pulsing, and pile oscillation.

- **Fluctuation or noise analysis methods** based on exploiting the modified statistical behavior of a multiplying medium at steady state. Prominent methods include the Rossi-\( \alpha \), Feynman-\( \alpha \) and the PSD methods (see Section 2.4).

Within the framework of this work the focus rests on noise methods. The next sections are dedicated to describing the general phenomena behind noise and the deduced kinetic parameter measurement methods. The problem of space and energy dependence of noise is discussed thereafter, giving an introduction to the experimental investigation at hand.
Figure 2.1 – Schematic of the relation between reactor kinetics measurements and prediction codes. Based on the chosen measurement method, inputs from an accompanying calculation are necessary. Measured parameters then aid in the validation of calculations, forming an input/validation feedback loop for kinetics parameters.
2.2 Temporal fluctuation: Noise

Noise commonly refers to the phenomenon of the temporal fluctuation of a given measure around its mean value. The measurement and investigation of said noise is called noise analysis. In a stationary reactor the power $P$ predicted by a deterministic calculation is a scalar

$$P_0 = \Sigma_f \Phi E_f V$$

(2.35)

with the inputs of macroscopic fission cross section $\Sigma_f$ and the reactor volume $V$ convoluted with the calculated flux $\Phi$ and the energy released per fission $E_f$. A measurement, however, will reveal a non-static noisy behavior, as illustrated in Figure 2.2. The origin of noise can be of the following kind (non-exhaustive list):

- Thermal noise caused by random motion of particles at a certain temperature, described by the Nyquist formula.
- Pressure or temperature variations and the resulting mechanical vibrations.
- $1/f$ noise, or flicker noise, appearing mostly in direct current applications due to e.g. conductor impurities, usually following a normal distribution.
- Interference, i.e. the change or cancellation of a signal caused by other electronic systems such as WiFi.
- Spontaneous processes, such as radioactive decay, that exhibit a natural fluctuation around the mean.

In radiation detection measurements, for example in the case of a single photon decay sample, the resulting fluctuations in radiation flux and thus detector signal originate from the constant independent decay probability of the isotope. If other noise sources can be neglected or are filtered out, the resulting time intervals between subsequent signals follow a Poisson distribution. Noise analysis allows here to predict the uncertainty of one

\[\text{Figure 2.2 – Schematic graph of the measured power of a nuclear reactor at steady state. A deterministic calculation will yield a static flux and thus a constant power. A measurement would be subject to fluctuations on the signal, also referred to as noise.}\]
measurement by assuming the underlying statistical model. Deviations from Poisson distributed values can then indicate added noise or correlation sources.

In a neutron-multiplying medium such as a nuclear reactor or a fissile sample, the measured statistical behavior is modified due to events generating several particles at the same time, such as $(n,xn)$ reactions and fission. This altered statistical behavior can be exploited by means of noise analysis, yielding for example the reactivity or kinetic parameters of a given sample or system. In the following sections we will introduce the notion of reactor noise and its analysis and display the standard equations.

2.3 Reactor noise

Classically, for noise of a signal originating from a nuclear reactor, a distinction is made between so called zero power noise (Section 2.3.1) and power reactor noise (Section 2.3.2), following [46].

2.3.1 Zero power or branching noise

In systems where thermo-hydraulic and mechanical noise effects can be neglected, the observed fluctuations are referred to as zero power noise. This type of noise is entirely dependent on the underlying nuclear processes of fission chains, and arises namely from the following sources:

- The amount of neutrons emitted per fission $\nu$ is, as opposed to what is assumed in deterministic calculations in the form of $\bar{\nu}$, a statistical parameter which can lead to local neutron emission spikes or dips.

- The time between nuclear interactions $\tau$ is the result of an average of mean interaction paths over mean velocities of neutrons. A random, more rapid or slower succession of events therefore act as fluctuation source.

- The interaction of a specific neutron with a nucleus, whether it being absorption, fission, etc., can only be expressed through probabilistic measures such as the cross section. Hence a succession of fission or absorption analogously influence the noise of a static neutron flux.

These effects can also be summarized by the kinetics of the branching process of fission chains propagating in a multiplying medium (see Figure 2.3 and [8]). Subsequent
detector counts can originate from the same fission chain and thus exhibit temporal correlation. This temporal correlation can be used to infer reactor characteristics, e.g. the kinetic parameters.

2.3.2 Power reactor or external noise

This type of reactor noise is typically superimposed on the branching noise and originates from mechanical vibrations causing local macroscopic cross sections to fluctuate. These vibrations have various causes such as coolant flow, coolant boiling, or local temperature fluctuations. These effects become more dominant the higher a reactor power is, and are thus the noise source encountered in full scale nuclear power plants. In zero power facilities these noise source can potentially be measured independently by actively induced perturbations, such as by fuel rod [35] or sample oscillations [32] or bubble creation [47, 48] experiments.

2.3.3 State space analysis of a zero power system

A formal description of zero power noise can be approached from multiple angles, such as differential methods using a forward Kolmogorov equation for a stationary diffusion process [49], integral methods through a backward Kolmogorov equation [50], or more general stochastic branching formulations [8]. A less prominent, but arguably more intuitive method is based on linear system theory – and will be the angle of introducing noise measurements for this thesis.

Dynamic systems that can be described by linear differential equations with constant coefficients – such as the point kinetic equations applicable to a sub-critical zero power reactor – are also called a linear time invariant (LTI) system [51]. The point kinetics equations in a LTI state space description are

\[
\dot{x} = Ax(t) + Bu(t). \quad (2.36)
\]

\[
y(t) = Cx(t) \quad (2.37)
\]

Here, \(x(t)\) is a system state variable vector that is sufficient to fully describe the system. These variables are generally not directly observable, yet are influenced by an input \(u(t)\) and observed at the output as \(y(t)\). The matrix \(A\) describes internal kinetics; using the 6 group delayed neutron convention for point kinetics it reads:

\[
A = \begin{pmatrix}
\frac{\rho - \beta}{\lambda} & \lambda_1 & \ldots & \lambda_6 \\
\frac{\beta}{\lambda} & -\lambda_1 & 0 & 0 \\
\vdots & 0 & \ddots & \vdots \\
\frac{\beta_6}{\lambda} & 0 & \ldots & -\lambda_6
\end{pmatrix} \quad (2.38)
\]

with the system states being

\[
x^T = [n(t), c_i(t)]; \quad \dot{x}^T = \left[ \frac{dn(t)}{dt}, \frac{dc_i(t)}{dt} \right] \quad (2.39)
\]
This system is only governed by neutronics and has no feedback mechanism and is hence a so called zero power system. \( B \) is typically a control input matrix such as a control rod or external source, as illustrated in Figure 2.4. The temporal behavior of LTI systems can be fully characterized by examining the system matrix eigenvalues and the impulse/frequency response [52].

**Eigenvalue decomposition** The eigenvalues of this matrix \( \alpha_j \) are the poles of the system and thus exponents governing the time dependent behavior of the system

\[
\begin{align*}
\dot{n}(t) &= \sum_j \Phi_j e^{\alpha_j t} \\
n(t) &= \sum_j \Phi_j e^{\alpha_j t}
\end{align*}
\]

with \( \Phi_j \) being initial condition dependent coefficients. The exponents are also often referred to as inverse periods of the reactor. Taking \( \det(A - \alpha I) = 0 \) yields the characteristic equation of this matrix, the so called *inhour equation*, solved for the reactivity

\[
\rho(\alpha) = \alpha \Lambda + \beta_{\text{eff}} - \sum_i \frac{\beta_i \lambda_i}{\alpha + \lambda_i}
\]

A plot of this function is shown in Figure 2.5 for an example of the kinetic parameters of CROCUS (see Chapter 3), displaying the seven - six delayed and one prompt - solutions for a given reactivity. For illustration, the *prompt kinetics* approximation \( (\lambda_i = 0, \forall i) \) is also displayed:

\[
\rho = \alpha \Lambda + \beta_{\text{eff}}
\]

Therefore, we can define \( \alpha_p \), the *prompt neutron decay constant* or *prompt inverse period* as

\[
\alpha_p = \frac{\rho - \beta_{\text{eff}}}{\Lambda}
\]

As the dominant eigenvalue of \( A \) the subscript \( p \) is often omitted.
2.3. Reactor noise

Figure 2.5 – Graph of the inhour equation (blue) and the prompt kinetics approximation curve (orange). The poles of the inhour equation are close to the precursor decay constants $\lambda_i$.

**Impulse response** The systems impulse response is examined by setting the input to a Dirac delta at $t = 0$. The resulting graph for $A$ is shown in Figure 2.6. In linear scale we observe an exponential decay corresponding to the prompt neutron density decay in a sub-critical system, which gives the introduced name of prompt decay constant to $\alpha$.

**Transfer function** In reactor control theory the zero power transfer function is used to quantify the output given an arbitrary input. The transfer function of a nuclear reactor system is found by Laplace transformation of the point kinetic equations or inverse Laplace

Figure 2.6 – Frequency (left) vs. impulse (right) response plot of the neutron population in a sub-critical nuclear reactor system $A$. Imposing a Dirac delta at $t = 0$, the system is found to respond with an exponential decay in the time domain, and conversely with a Lorentzian shape in the frequency domain. The exponents of this decay (or cut-offs of the Lorentzian) are the eigenvalues of $A$, dominated by the prompt decay constant $\alpha$. The graphs are both in arbitrary units for illustration.
transforming the impulse response, giving

\[ H(\alpha) = \frac{1}{\alpha \Lambda + \beta_{\text{eff}} - \sum_i \frac{\beta_i \lambda_i}{\alpha + \lambda_i} - \rho} \]  

(2.44)

with the transfer function poles being again the eigenvalues of \( A \). The transfer function, evaluated at \( i\omega/2\pi \), gives the frequency response of the system in dependence of an angular frequency \( \omega \), also shown in Figure 2.6. Instead of exponential decays, the graph is now characterized by their Fourier transform: Lorentzian bell curves with cut off frequencies at the respective exponent values.

The impulse and frequency response of the system allows for insight into the measurement of kinetics: We are only able to observe the output \( y(t) \) that is convoluted by the detectors transfer function \( C(s) \), the Laplace transform of \( C \). Detection processes are assumed to add white noise to the final output, i.e. a \( \delta \) at time zero or a constant in the frequency domain. Thus, when analyzing the auto-correlation or PSD of the output, we may infer the shape of the underlying impulse or frequency response functions. The shape is dependent on the eigenvalues of \( A \) and therefore curve regression allows access to measure the systems eigenvalues. This is the principle of so called noise based kinetics measurements, introduced in the next section.

2.4 Intuitive zero power reactor noise analysis methods

The most prominent methods for zero power noise analysis are presented in prompt approximation (i.e. without delayed neutrons), namely the auto correlation (also Rossi-\( \alpha \)), the variance over mean or Feynman-\( \alpha \) and the power spectral density or PSD methods. All three methods are based on the measurement of a train of pulses from a neutron detector close to the reactor in order to examine the statistics of the time between pulses in time or frequency domain. Sections 2.4.1 to 2.4.3 are an overview of the most commonly used methods for noise measurements. Thereafter a more rigorous derivation of all three methods is undertaken. A discussion of measurement limitations regarding the introduced methods and a selection of previous reactor noise experiments is presented thereafter.

2.4.1 Auto-correlation or Rossi-\( \alpha \) method

The state space description of a point kinetics reactor reveals that its impulse response can be observed by calculating the auto-correlation of the detector signal. Historically, this method was called the Rossi-\( \alpha \) method [53]. It relies on measuring the time difference between subsequent pulses of a detector to estimate the inter-arrival interval distribution, which effectively approximates the auto correlation function of the detector output, see Figure 2.7.

Assuming a detection of a neutron at \( t = 0 \), a second detection at time \( t \) can originate independently of the first detection from another unrelated fission chain. Conversely a
2.4. Intuitive zero power reactor noise analysis methods

second detection can also originate from the same fission chain in a prompt process. This is illustrated in Figure 2.3. Neglecting the delayed neutrons in the fission chain and higher alpha mode effects, the total detection probability at time $t$ within a detection time $dt$ can thus be written as

$$p(t)dt = F(\alpha + be^{\alpha t})dt$$

with $\alpha < 0$ describing the exponential decay of probability to detect a neutron from the same prompt chain. The constant $a$ describes the uncorrelated detection of two neutrons and is found to be the count rate in the detector

$$a = Fe,$$

with $F$ being the fission rate of the system and $e$ the detector efficiency in counts per fission in the reactor. $b$ describes the detection of two correlated neutrons, i.e. coming from the same fission chain:

$$b = \epsilon D_{\nu}$$

$D_{\nu} = \nu_p(\nu_p - 1)/\nu_p^2$ is the Diven factor [54] and $\nu_p$ is the prompt neutron yield per fission. This formulation of the Rossi-$\alpha$ method was also referred to as Type I in early literature and is nowadays the most used version. A measurement of this type hence yields the absolute value of $\alpha$ from the shape of the auto-correlation curve, whilst the amplitude yields $b_{\text{eff}}$ and $\Lambda$ in proportionality to $D_{\nu}$ and $e$. Commonly the shape and amplitude are extracted via least squares fitting.

If a second detector is available, the cross correlation between the two can used instead [55]

$$p_{1,2}(t)dt = e_1 F\left(\epsilon_2 F + \frac{\epsilon_2 D_{\nu}}{2\alpha \Lambda^2} e^{\alpha t}\right)dt.$$  \hspace{1cm} (2.48)

2.4.2 Variance over mean method

Also called the Feynman-$\alpha$ method [56], this method relies on the measurement of variance and mean value of counted neutrons for a variable time interval $\Delta t$. The formulation can be motivated by using the Rossi-$\alpha$ probability to find the mean number of counted neutrons
in a detector. Consider the mean number of neutrons counted after an initial count at $t_1$, we next seek to find the average amount of neutrons counted in the time interval until $t_2$ and then $t$:

$$Z(Z-1)/2 = \int_0^t dt_2 \int_0^{t_2} dt_1 p(t_2 - t_1)$$

(2.49)

First integration gives:

$$Z(Z-1)/2 = \int_0^t dt_2 \left( A\epsilon F t_2 + B\epsilon F \frac{1 - e^{-\alpha t_2}}{\alpha} \right)$$

(2.50)

Second integration then yields:

$$Z(Z-1)/2 = At + 2B \frac{1 - e^{-\alpha t}}{\alpha t}$$

(2.51)

The average counts can also be expressed by the fission rate $F$ multiplied by the efficiency in counts per fission rate $Z = \epsilon F t$, hence one gets

$$Z(Z-1)/Z = 2At + 2B \frac{1 - e^{-\alpha t}}{\alpha t}$$

(2.52)

This expression can finally be rearranged to the Feynman-$\alpha$ formulation of variance over mean:

$$\frac{Z^2 - Z^2}{Z} = \frac{\text{Var}(Z)}{\text{Mean}(Z)} = 1 + \frac{\epsilon D_\nu}{(\rho - \beta_{\text{eff}})^2} \left( 1 - \frac{1 - e^{-\alpha t}}{\alpha t} \right)$$

(2.53)

This formulation shows the aspect of modified variance due to branching: Instead of a variance over mean of one as it is for a Poisson distribution, the variance is higher due to the multiplication process. This is in dependence of the measurement time bin $t$. This implies that at very short times the noise appears Poisson like ($\text{Var}(Z) = \text{Mean}(Z)$), whilst showing an increased variance for longer bins. As before, the shape in dependence of the variable time bin gives $\alpha$ with the asymptotic amplitude accessing a proportionality to $\beta_{\text{eff}}$ and $A$ with prior knowledge of $D_\nu$ and $\epsilon$.

Analogously, using two detectors the covariance to mean equation reads, assuming equal efficiency:

$$\frac{Z_1Z_2 - Z_1 Z_3}{\sqrt{Z_1Z_2}} = \frac{\text{Cov}(Z_{1,2})}{\sqrt{Z_1Z_2}} = \frac{\epsilon D_\nu}{(\rho - \beta_{\text{eff}})^2} \left( 1 - \frac{1 - e^{-\alpha t}}{\alpha t} \right)$$

(2.54)

2.4.3 Power spectral density method

This method is sometimes referred to as Cohn-$\alpha$ [57] method and relies on the frequency domain analysis of the output. As shown in Section A and 2.3.3, the PSD is calculated by Fourier transforming the auto correlation function (see also Figure 2.7):

$$G_{xx}(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} p(t) = \epsilon F + \frac{\epsilon^2 F D_\nu}{(\rho - \beta_{\text{eff}})^2} \frac{1}{1 + \omega^2/\alpha^2}$$

(2.55)
The shape of the curve is a Lorentzian bell curve with a cut-off frequency at \( \alpha \). The amplitude of the lower frequency region, also called the PSD plateau, is proportional to \( \beta_{\text{eff}}^2, D_\nu \) and \( \epsilon^2 \). The cross spectrum of two detectors is

\[
G_{xy}(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} p_{1,2}(t) = \frac{\epsilon^2 F D_\nu}{(\rho - \beta_{\text{eff}})^2} \frac{1}{1 + \omega^2/\alpha^2} \quad (2.56)
\]

## 2.5 Experimental realization of noise measurements

Using the presented simplified equations for noise, we next compare their respective shapes when varying experimental input parameters such as the reactor power and the detector efficiency in Figure 2.8. For this example we used kinetic parameters of CROCUSt at criticality. The PSD and Rossi-\( \alpha \) curves were normalized to their respective end regions to illustrate the relative difference in amplitudes. A general observation is that a higher efficiency consistently yields a larger correlation amplitude. Using the PSD as example, we already asserted that an uncorrelated signal (white noise) is a constant in the frequency domain. Any deviation from this shape, i.e. towards the transfer function shape of the reactor, is hereby considered "visible correlation". This is also the experimental approach to assess whether a signal contains the desired correlations or not. Furthermore, the curves show that the relative Rossi-\( \alpha \) amplitude is also dependent on the reactor power, making it challenging to implement in reactors above \( \sim \)mW of power. We discuss this further in Section 3.1.4, when introducing our considerations with regards to noise measurement limitations and their link to observation noise.

Noise measurements require, depending on the choice of analysis method, varying experimental instrumentation with respective limits and requirements. The following Sections introduce a review of recent experimental work using the three main analysis methods and give an outline of application regimes and limitations.

### 2.5.1 Rossi-\( \alpha \)

Two main methods for Rossi-\( \alpha \) measurements are most commonly used:

- Distribution method: The time \( \tau = t_1 - t_0 \) between an initial neutron detected at \( t_0 \) and the next interaction at \( t_1 \) is measured by initiating a trigger at \( t_0 \). This results in an interpretation just as motivated in Section 2.4.1, i.e. the prompt decay of individual fission chains is registered.

- Time tagging method: All counts are registered with their respective time of measurement and a difference histogram between all counts is performed.

This is realized by using neutron detectors connected to pulse discrimination hardware which in turn usually feeds analog data to a digitization unit. The obtained correlation curve is then fitted with least squares methods assuming a single decay exponent, i.e. assuming the prompt approximation decay of \( \alpha \). As an example, we depict Figure 2.9 a measured auto correlation of a neutron detector in the CROCUSt reactor at 40mW reactor
Figure 2.8  Comparison of noise methods in dependence of $\epsilon$ and $F_0$. The Rossi-\(\alpha\) and PSD curves are normalize to their respective ends to illustrate the relative difference in amplitude.
2.5. Experimental realization of noise measurements

Figure 2.9 Measured auto correlation for the CROCUS reactor using a 1 g $^{235}$U fission chamber at 40 mW reactor power [23]. The shape was acquired using the time tagging method on a measurement of $\sim$3 h length.

power. Rossi-α has been the focus of classical reactor noise experiments in the 60s (e.g. as listed in [25]) as well as modern renewed noise measurement efforts e.g. [58]. Delayed decay modes have not been able to be measured consistently due to the necessary auto-correlation length of more than 0.03 s [49]. Higher prompt shape modes in deeply sub-critical assemblies have been successfully measured using multi exponent fitting of the measured auto correlation [27], [44], [59]. Generally, cross correlation yields better results in terms of fit convergence, required measurement time and thus statistical uncertainty as it filters auto-correlated noise and thus improves the signal to noise ratio (see also Section 3.1.4).

The main limitation of these methods is the inability to differentiate between random and correlated counts, inherent to the method. A minimal efficiency of detection is necessary to be able to register neutrons from subsequent fissions in one chain. For high fission chain densities as is the case for critical or close to critical assemblies, the overlap of chains coupled with the need of a highly efficient detector conflict in utility and limit the application of Rossi-α measurements to sub-critical or low power ($<\sim$50 mW) environments.

2.5.2 Feynman-α

Feynman-α measurements require the evaluation of the (co-)variance and respective mean of counts obtained during time bins of variable length. This is typically realized using a multi channel scaler that registers counts in a digital channel with as small as possible dwell time and constructing the longer time bins lengths by summing adjacent bins. Alternatively, the time tagging method may also be used. The analysis in both cases requires a calculation of the $Y$ function for arbitrary length dwell times, beginning at the hardware given bottom limit.

As opposed to the auto correlation, the average behavior of the neutron chains detected
Chapter 2. Noise measurements in nuclear reactors

Figure 2.10 Measured $Y$-variance and covariance functions using 1 g $^{235}$U fission chambers of the MINERVE reactor [60]. The shape was acquired using custom post processing of count data in dwell bins of 1 ms length for a measurement of 4 h length.

is insensitive to overlapping fission chains and thus allows for critical and supercritical measurements, i.e. a generally more flexible and robust method. This can qualitatively be seen as a result of the integration steps to derive the simplified equation in Section 2.4.2. Figure 2.10 depicts a (co-)variance over mean evolution of two detectors set asymmetrically in the MINERVE zero power reactor. The shape is once more dominated by $\alpha$, with the amplitude being proportional to $\beta^2_{\text{eff}}$.

The limitation of Feynman-$\alpha$ is mainly the dead time of a detector, as experimentally investigated by [61]. Although the method is insensitive to overlapping fission chains, the need for a highly efficient detector once again conflicts with the hardware limits at higher power ($> \sim 100 \text{mW} \text{ giving } >10^6 \text{cps}$). Dead time corrections have been developed [62] and remedy this effect and are applied in most modern measurements [24]. Alternatively, the detectors can be operated in current mode. This has been found to reduce the count loss effects [63], but can be challenging to interpret [64] due to added noise components that perturb the variance.

Higher prompt modes, i.e. prompt shapes have been theoretically derived for variance to mean equations [59] and measured in deeply sub-critical ($<-1\%$) assemblies [27]. For uncertainty reduction in kinetic parameter measurements the so-called bunching technique [65] can be employed to inter-correlate adjacent time bins. For improved efficiency limits and reduced measurement times two detector sum or covariance measurements are standard.

2.5.3 Power spectral density

Power spectra of neutron detectors can be estimated in two main ways:

- Direct spectrum analysis: The hardware of the acquisition system is directly outputting a power spectrum through hardwired spectrum analysis. This method gives
2.5. Experimental realization of noise measurements

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**Figure 2.11** Measured auto and cross power spectra of the MINERVE reactor using $1g \text{ } ^{235}\text{U}$ fission chambers [60]. The shape was acquired using custom Fourier analysis post processing of count data in dwell bins of 1 ms length for a measurement of 4h length.

---

the fastest results but is hard to modify once the hardware is constructed.

- Post processing of time domain measurement: The time tagged count train or current over time of a detector can be processed on a computer after the acquisition using dedicated digital Fourier analysis algorithms. This method is more flexible as the spectrum estimation method can be adapted to the measurement i.e. by choosing adequate averaging filters.

Fissile assembly noise measurements using power spectra have been carried out from the very beginning of noise measurements [57] and have steadily been employed as reliable method to measure kinetic parameters [66], [67]. Modern PSD measurements include [58], [68], [69], [16]. The main advantage of the method is the robustness with respect to uncorrelated fission chains even at higher reactor powers. Once the detection rate exceeds standard counting electronics limits ($>10^6$ cps), the detectors can be operated in current mode and still provide noise information.

The requirement is as with all methods a sufficient efficiency of the detector in counts per fission, as we will discuss in more detail in Section 3.1.4. Cross correlation has been shown to improve the efficiency limitations by up to one order of magnitude [70]. The PSD can be estimated for current and count train signals and thus offers them most flexible analysis tool with respect to applicable reactors. However, the discrimination of noise due to gamma and electronic events in the detection system is not possible in current mode. This limits the application range of current mode to detectors with inherently higher charge generation per neutron detected than for gammas such as fission chambers.

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2.5.4 Review of noise measurement experiments

Integral experiments require both a detailed analysis to remove sources of experimental biases, as well as uncertainties that are small enough to give statistical weight to a validation or even DA [21]. The uncertainties for delayed neutron fraction integral measurements
in particular are required to be smaller than 3% in order to meaningfully contribute to nuclear data evaluation [22].

A non-exhaustive list of experiments to determine the kinetic parameters of zero power reactors include programs in the Swiss facility PROTEUS using Feynman-α, e.g. [71], with extensive study of spatial effects [72, 73, 74]. After its closure, many experiments in the field were conducted in the IPEN/MB-01 research reactor in Brazil using Feynman,[75], PSD [76], and Rossi experiments [77]. Measurements in the CEA Cadarache facilities MINERVE and MASURCA include [58, 78], and [64], respectively. The VENUS-F reactor at SCK-CEN in Belgium has also been of recent interest for noise measurements [79, 80, 81]. The EPFL reactor CROCUS has been recently characterized as well [33, 24, 69].

The published results each allow for similar conclusions: The experimental uncertainties on the measurements on α using noise methods can be reduced down to around 1% using sufficient measurement times, with spatial effects being visible only in highly heterogeneous systems. For the experiments in MINERVE and CROCUS the results were obtained via non-linear least squares fitting of the response curves. The uncertainties were estimated based on confidence bounds given by the fitting algorithm, which is often the first order approximation of the covariance matrix. The experiments however disagreed among each other, and led to the conclusion that uncertainties or biases are likely underestimated.

This is equally true for the measurements for β and Λ or ρ. We note in particular that the published literature is inconsistent in the estimation of the uncertainties related to the kinetic parameter determination. A particular interest lay in the fact that the systematic difference between calculated reactivities with different nuclear libraries (e.g. JEFF [10] or ENDF [11]), can be up to 16% [82]. This discrepancy is among the biggest drivers to provide more reliable measured data in order to assist in the (re-)evaluation of cross section and decay constant data via integral measurements.

**Delayed decay estimation with noise methods** Delayed neutron emitters have been historically grouped into precursor groups according to their half lives. The first experiments conducted on Godiva by Keepin in the late 50s lead to the six group structure of delayed neutron precursor decay constants and yields [83]. This is still the used structure in the American evaluation ENDF, whilst a NEA sponsored work-group concluded that an eight group structure is necessary [84, 85]. Despite newer evaluations of these multi-purpose libraries, studies still show significant discrepancies arising from the mean half life differences [82, 85]. From here, the 6- or 8-group delayed neutron group conventions are a focus of current research and modern evaluations [85, 86]. Measurements of high accuracy are therefore potentially useful to aid in the effort of optimizing delayed neutron data. It should be noted at this point, that delayed neutron decay constant estimation is, theoretically, extremely time consuming using noise methods – as shown by Dragt [49], even the shortest decay constant requires a measurement time of 0.7 y in a stable reactor system to achieve the uncertainty of Keepin. It is therefore concluded, that delayed decay constants can be verified by noise measurements but not improved.
In the effort to determine $\beta_{\text{eff}}$, a method to minimize the amount of input parameters (such as $D_\nu$, $F_0$ and $\epsilon$), measurements using several subcritical states using a two region model have been conducted in the IPEN/MB-0 reactor [87]. The utilized model indeed allows for absolute experimental values of $\beta_{\text{eff}}$ with reported uncertainty of 1.5%. The authors, along with the NEA/WPEC subgroup 6 [88], suggest the overall expansion of noise measurement efforts to add to $\beta_{\text{eff}}$ databases with target uncertainties of 3%. Based on the overall published literature indicating a trend of under-estimating of uncertainties, a special focus on a consistent method of uncertainty quantification is laid in this work, detailed in Section 3.1.

2.6 Beyond point kinetics: Spatial description of radiation noise

We will next introduce several versions of neutron noise equations that take spatial effects such as a reflector or decoupled second core, the detector size and higher prompt modes into account. These equations have been derived to explain experimental results that disagreed with point kinetic predictions. These effects are often lumped under the term spatial effects. The presented approaches are all based on deterministic formulations; in showcasing their limitations we motivate our study of Monte Carlo based methods in Chapter 5.

2.6.1 Motivation for spatial and spectral description of reactor noise

Efforts to quantify power reactor noise caused by fuel rod oscillations and bubbles have been initiated in the CROCUS facility [35]. The CROCUS core has, despite its long history, only recently been characterized with regards to its kinetic parameters. Current and future experimental programs [34] will employ noise methods for analysis and require an accurate knowledge of the noise field in the core. Therefore, the interest of investigating spatial effects on noise, specifically in the CROCUS reactor, has become topical.

Decoupled cores [89] naturally arise when the geometry reaches a certain size or fissile regions are well separated, causing local modes to be significant for the kinetics. This raises the question of how large a reactor can be and still obey point kinetics, and how noise measurements are influenced by this. Two point models coupled by transfer terms have been shown to agree well with experimental observations of two separated cores [90].

Noise measurements that have reliable kinetic input data can provide reactivity measurements which in turn offer further options regarding on-line reactivity methods, e.g. for reactor loading surveillance. This is also of interest for (emergency) shutdown states of reactors or spent fuel pools. This is especially true in light of the events in Fukushima, where knowledge of the reactor state after station blackout was not available [91].

In this section the extension of noise theory regarding spatial and spectral effects is described. First, a section on a point kinetic extension model also called the two region model
or the reflected reactor two-point kinetics is presented. The models do not take detector size nor finite space into account. This is accounted for in the explicit spatial method, presented thereafter.

### 2.6.2 Two point model

Derived from [92], the two points or two regions model is a method to determine $\beta_{\text{eff}}$ without knowledge of $F_0$, $D_\nu$, or $\epsilon$. It relies on inverse count rate reactivity measurements and deep sub-critical states ($<-1\%$).

Assuming a point kinetics approach, the spatial dependence is implemented by adding a source and sink of neutrons into a reflector region. The point kinetic equations describing core neutrons $N_c$ and core precursors $c_i$ thus need an additional set to describe neutrons in the reflector $N_r$ with a reflector lifetime $\tau_r$.

\[
\frac{dN_c}{dt} = [k_c(1-\beta_{\text{eff}}) - 1] \left( \frac{N_c}{\tau_c} \right) + f_{rc} \left( \frac{N_r}{\tau_r} \right) + \sum_i \lambda_i c_i + S \tag{2.57}
\]

\[
\frac{dN_r}{dt} = f_{cr} \left( \frac{N_c}{\tau_c} \right) + \frac{N_r}{\tau_r} \tag{2.58}
\]

\[
\frac{dc_i}{dt} = k_c \beta_i \left( \frac{N_c}{\tau_c} \right) - \lambda_i c_i \tag{2.59}
\]

Using a forward-like equation as shown in Section C, the Feynman-$Y$ is found to follow the same basic scheme: An amplitude term $N$ is multiplied by the quasi-hyperbolic function of the reflected reactor decay constants

\[
Y = -2A^{c,r} \left[ \frac{N_{c,r}^{c,r}}{\omega_7} \left( 1 + \frac{1 - e^{\omega_7 T}}{\omega_7 T} \right) + \frac{N_{c,r}^{c,r}}{\omega_8} \left( 1 + \frac{1 - e^{\omega_8 T}}{\omega_8 T} \right) \right] \tag{2.60}
\]

The roots can once again be found by finding the eigenvalues of the system matrix. Explicitly neglecting delayed periods we find

\[
\omega_{7,8} = \frac{g}{2\Lambda_c \Lambda_r (1-f)} \tag{2.61}
\]

with

\[
g = -\left[ (1-\rho)(\Lambda_c + f\Lambda_r) + \Lambda_r (1-f)(\beta_{\text{eff}} - \rho) \right] \tag{2.62}
\]

\[
\pm \sqrt{[(1-\rho)(\Lambda_c + f\Lambda_r) + \Lambda_r (1-f)(\beta_{\text{eff}} - \rho)]^2 - 4\Lambda_c \Lambda_r (1-f)(1-\rho)(\beta_{\text{eff}} - \rho)}
\]

The inhour equation for a reflected reactor is extended by an additional pole proportional to the inverse reflector lifetime $\tau_r$. At criticality, this effect is expected to be not measurable as $\omega_7$ is approximately the prompt decay constant. The further the system diverges from criticality, however, the more pronounced is the effect of the reflector lifetime. Therefore it is concluded, that reflected reactor systems at deep sub-criticality ($<-1\%$) need a two region description for accurate noise modeling [87]. Practically, this implies fitting the measured data with a multi exponent response function to correct for the reflector.
2.6.3 Explicit spatial treatment of neutron fluctuations

A more rigorous treatment of the spatial variable is found using the approach by Sheff and Albrecht [26, 93]. The fluctuation equations are solved by using the Green's functions of the system. The main challenge with the developed equations is then to find the associated Green's functions for a given geometry. The final expression for an auto-correlation for a point detector in an infinite medium was derived to be

$$\phi_{Z_1, Z_2}(\tau) = \frac{C}{2\pi 4L^2} \left( \frac{\alpha}{\pi 4L^2} \right)^{\frac{1}{2}} \Gamma(1/2, \alpha |\tau|),$$

with $L$ being the diffusion length of neutrons in the system, and $C$ a coefficient depending on boundary conditions. This formulation markedly differs from the point auto-correlation in a point medium, i.e. the standard Rossi-$\alpha$ formulation. We compare the analytic expressions for both cases in Figure 2.12. As is visible from the graphs, we can make several statements:

- The correlation functions and therefore power spectra strongly deviate from each other. A measurement in a system with dominant spatial effects would yield highly biased kinetic parameters if analyzed with point kinetic equations.
- The strong difference also implies, that the effect cannot be affecting most measurements that have been performed, as the results agree with point kinetics within small uncertainty bounds.
- Strong spatial effects as presented, caused by a point detector as opposed to an infinite detector, must only be measurable if the detector is very small compared to the diffusion length but also having a high efficiency with respect to the fission rate. Small detectors tend to imply lower efficiency and therefore we expect this effect to not be visible.

2.6.4 Summary of spatial effects on neutron noise measurements

Based on the presented analysis methods for the inclusion of energy and space effects onto noise equations, we summarize the observed implications on noise measurements:

- Noise measurements that were performed in critical or close to critical systems generally yield results in good agreement with point kinetics and point kinetic noise equations, albeit with likely under-estimated uncertainties. In sub-critical systems, however, especially in deeper states ($<-1\%$), spatial effects have been shown to have a non-negligible effect on the result of noise analysis in certain reactors.

- Spatial effects can be summarized to be of the following origin: I) **Reflector or regional effects** due to a non negligible delayed neutron source by a reflector surrounding a core. II) **Energy effects** related to the propagation and detection of thermal vs. fast neutrons. III) **Modal effects** due to the presence of higher prompt shape modes. IV) **Detector size effects** caused by a detector that albeit efficient cannot capture enough neutrons to see point kinetic noise.
• Effects I and III have been experimentally observed in reactors between -1$ and -5$. The inclusion of the presented supplementary terms in the analysis indeed yielded more accurate results for the kinetic parameters. The other effects were shown to strongly impact the measured noise and therefore they cannot be as pronounced as the theory predicts. A survivor bias could play a role here, as published experiments do not contain information about which detectors failed to perform or showed strong disagreement with theory.

The above mentioned methods to quantify spatial effects are nonetheless limited in their application to exact geometries or generally a less constructed model. A point model agnostic approach, such as Monte Carlo based neutron transport, could thus be a method to alleviate these concerns. The study of such a methodology is the content if Chapter 5.

2.7 Gamma noise

Gamma radiation as a by-product of fission and radioactive decay is a major component of a nuclear reactors radiation field. Propagating at the speed of light, gamma radiation transport takes place on much smaller time scales than neutrons — nanoseconds compared to microseconds. Gamma propagation is thus not only well distinguishable from neutrons, but most importantly negligible in the time range of neutron noise measurements. Gamma interactions carry the time information of their birth — time stamping fission — effectively giving the same correlation in time as neutrons. Furthermore, an average of 7 prompt gammas are emitted per fission [94], markedly higher than a thermal fission average for neutrons between two and three. The overall multiplicity is thus higher and adds to the correlation visibility. This is the general hypothesis behind the feasibility of gamma noise.
2.7. Gamma noise

2.7.1 Review of theory

Gamma noise as a complement or replacement of neutron noise was first discussed in theory by Gelinas in 1966 [95]. Later, Kostic [36] derived a spatial dependence of gamma noise compared to neutrons in 1971, concluding that a gamma detector "sees a greater reactor volume" and is thus more sensitive to correlations and can see them from further away. Dam [96] showed that gamma detectors, with respect to noise sources, act as a "smeared out" neutron detector with epithermal sensitivity, confirming the larger "visibility" range. Further boiling water reactor noise considerations [97] conclude that a gamma detector should see the same as a neutron detector, confirming the findings of Kostic.

Gamma rays not only interact on shorter time scales, but also penetrate most media deeper than neutrons. Gamma noise systems could therefore have an advantage insofar that they can be placed further away from a reactor, optimally outside the pressure vessel or coolant. This would also minimize the perturbation of the detector on the system - also because a gamma detector does not require absorbing a neutron to detect the correlations.

2.7.2 Review of experiments

Experimental studies of gamma noise existed very early after the inception of noise analysis in 1968 [98, 99], where CO$_2$ or water based Čerenkov detectors were used to observe the critical gamma CPSD of a low power systems, concluding that gamma noise was in principle observable. Baloh, in 1969 [100], tested small liquid scintillator vials as out of core detectors for power monitoring and in-core perturbation spotting and concluded that correlations are in principle well obtainable. In 1972, Le Vert [101] measured local fluctuations due to flow and voids with two ex-core gamma detector in a TRIGA reactor using liquid scintillator vials and speculated on gamma detectors as being potentially superior to neutron based systems, especially for sodium boiling [102]. Around the same time, Baers measured the auto-correlation function of NaI detectors set close to a sub-critical assembly in Finland [103]. Later these experiments were summarized [104], and concluded that gamma noise is potentially better in time resolution, is less prone to spatial effects, and usable in ex-core locations. A quantification of the limits was not provided, and follow up studies were not conducted. The most recent example of gamma noise measurements was conducted in Japan [105], where two NaI were used to estimate the prompt decay constant - but here as well no follow-up study was published. All of these studies have received little attention since.

A recently discussed application of gamma noise is the assay of fissile material and correlated emission radiation sources. This has been theoretically and experimentally explored by [106] using Cf-252 and Na-22 sources.

The overall consensus of the theoretical considerations is that gamma noise equations have the same shape as their neutron-based counterparts; yet are modified in their asymptotic value. This means that $\alpha$ could be estimated directly from the response function. Whether $\beta$ or other kinetics can be inferred has to date not been quantified.
Chapter 2. Noise measurements in nuclear reactors

Figure 2.13  Left: Analytic examples of PSD neutron, neutron gamma, and gamma PSDs. Right: Gamma PSD for different $\omega_\gamma$. The gamma decay constant, due to the comparatively fast propagation of gamma rays, is expected to be so large so as to not affect the PSD shape induced by the fission chain decay.

2.7.3 Prediction of a power spectral density of gamma noise

The auto-correlation function can be written as a sum of the exponentials with eigenvalues of the combined neutron gamma system being the exponents \[106\]. For neutrons without delayed neutrons the PSD is the Fourier transform of the auto-correlation of the shape

$$P(\omega)_{NN} = 3(A + \sum_{\omega_i} B e^{-\omega_i t}) = A + \frac{B}{\alpha^2 + \omega^2} \quad (2.64)$$

Analogously, considering $\gamma$ or joint $\gamma$-neutron detection, the PSD is a sum of the Fourier transforms of the exponential sum \[36\], \[107\]

$$P(\omega)_{N\gamma} = A + \sum_{\omega_i} \frac{B}{\omega_i^2 + \omega^2} = A + \frac{B}{\alpha^2 + \omega^2} + \frac{C}{\omega_\gamma^2 + \omega^2} \quad (2.65)$$

$$P(\omega)_{\gamma\gamma} = A' + \sum_{\omega_i} \frac{B'}{\omega_i^2 + \omega^2} = A' + \frac{B'}{\alpha^2 + \omega^2} + \frac{D}{\omega_\gamma^2 + \omega^2} \quad (2.66)$$

Note that $\alpha$ and $\omega_\gamma$ are cut-off frequencies. This is illustrated in Figure 2.13, which displays the neutron-neutron, neutron-$\gamma$ and $\gamma$-$\gamma$ PSDs assuming a large $\omega_\gamma$. The other parameters are chosen based off CROCUS; notably we chose a higher amplitude term for the gamma component compared to neutrons, due to the higher amount of prompt emissions per fission. The dominant cut-off frequency is still $\alpha$ for all graphs. Depending on the decay constant of gammas, the shape of the PSD could be different for similar values or close to the same for largely deviating values. In Figure 2.13, the variation of $\omega_\gamma$ in arbitrary units is shown. Since the average lifetime of a photon is much shorter than that of a neutron, $\alpha \ll \omega_\gamma$ is expected, and the shape should be the same aside from a constant shift due to an increased amplitude.
2.8 Chapter’s salient elements

In this chapter the state of the art noise analysis methods were displayed, derived and brought into an experimental context. By introducing the one group point kinetic approximation, observable equations such as the Rossi-$\alpha$, Feynman-$\alpha$, or PSD graphs are found to allow measurements of the prompt neutron decay constant $\alpha$ and/or kinetic parameters such as $\beta_{\text{eff}}$ and $\rho$.

So called spatial effects on neutron noise measurements can be distinguished by exhibiting e.g. an effect on the shape (adding an exponent $\omega_i$) and amplitude of the response function in question. This implies that an accurate measurement of $\alpha$ and $\beta_{\text{eff}}$ should include an analysis of the respective magnitude of said effects and how strongly a result will differ from a point kinetic approximation.

Experiments involving neutron noise analysis must thus take the following into account:

- The point kinetic approximation can yield unreliable results, the magnitude of effects can be difficult to quantify.
- Two region effects: In reactors with reflectors (e.g. water around an active zone) the effective lifetime of a neutron is modified and needs to be taken into account when measuring $\alpha$ in sub-critical states, not however at criticality.
- Modal effects: In sub-critical states, higher harmonics of the flux shape become relevant when assessing noise. This means that a harmonic shape analysis of the reactor needs to be undertaken to distinguish detector positions with either predictable or no harmonic overlap.
- Detector size and position effects: The detector size affects the measurement strongly. A large detector (bigger than several diffusion lengths) will allow for a point kinetic approximation. Smaller detectors will need further treatment e.g. by numerical estimation of Green’s functions of the reactor and detector.
- Gamma noise measurements are expected to exhibit similar correlations as neutrons and allow for measurements of $\alpha$ in the point kinetics approximation. Potentially, gamma noise has a higher flexibility regarding detector positioning, and is less prone to spatial effects.

We are therefore prompted to study the branching noise and it’s field of CROCUS experimentally, using neutron and gamma detectors. The neutron based systems are used to attempt to quantify, if any, spatial effects in the critical or sub-critical noise field of CROCUS – whilst gamma based systems require pioneering work in CROCUS to establish general feasibility.
Chapter 3

Reference noise experiments in CROCUS

In this chapter we set the groundwork for the exploratory experimental study of neutron and gamma noise fields in Chapter 4. Firstly, we introduce the general noise experiment and analysis methodology used in CROCUS in Section 3.1. Secondly, we present the experimental setup, namely the CROCUS reactor and its newly developed noise instrumentation, Section 3.2. Thirdly, in Section 3.3 we detail reference neutron and gamma noise measurements.

3.1 Noise measurement methodology

We first introduce how noise experiments are organized in general before detailing the strategy for estimating uncertainties in noise analysis. The individual steps pertaining to noise measurements are as follows:

- **Detector placement**: The natural first step is to position a "suitable" detector in a "suitable" location. The definition of suitable in this context is multi-faceted, and depends on factors such as particle type observed, detection sensitivity, geometric accessibility, and considerations from noise theory. This is discussed in more detail in Section 3.1.4.

- **Signal collection and amplification**: Depending on the detector type, the particles of interest directly or indirectly generate charges after a detection reaction. These charges require a mechanism of collection and amplification to achieve pulse heights or voltages that do not mandate high precision measurement devices. Any given detector is hence connected to a (pre-)amplification stage that also provides high voltage to the detector when necessary. Low noise\(^1\), impedance matched cabling

\(^1\)"Low noise" in this context can be ambiguous in interpretation, but typically refers to low tolerance manufacturing to ensure little material impurities, precise geometries/crystal structures, and added electromagnetic shielding - all depending on the needs of the application.
is hereby the best practice to avoid added noise sources. For example, we use: Spectroscopy preamplifiers type Canberra 2006 [108]; semi-Gaussian shaping amplifiers type Ortec 572 [109]; in-house current amplifiers (see Section 3.2.2.1); Photomultiplier tubes type Hamamatsu R12421.

- **Discrimination:** Pulse mode signals can be additionally applied to discrimination electronics to cut noise sources, typically done by single channel filters to set a lower level threshold (LLT). For neutron detectors this requires the amplification to be large enough to distinguish a \((n,x)\) pulse height spectrum from noise sources [110]. The choice of LLT for neutron detector calibration has been investigated in detail, e.g., for fission chambers in [111]. For gamma signals this has been investigated sparsely [105], we discuss it therefore separately in Section 3.3.4.

- **Digitization:** The discriminated signal in its analog state is next converted into digital information to be treated by a computer system. This stage is referred to as ADC (Analog-to-Digital-Conversion). The digital information type registered can vary from time tags for pulse events, buffer based systems that register pulses over dwell periods, to voltage signals registered at a certain sampling rate. For example, we use: Mirion DSA-LX [112]; Multi Channel Buffers (MCB) type Ortec ADCAM 926 [113]; Multi Channel Scalers (MCS) type Ortec MCS pci cards [24]; Oscilloscope type Teledyne Lecroy Waveruner 10 [114].

- **Data engineering:** The digitally recorded signal data is next processed on a computer prior to analysis, albeit often done in unison. This can include artifact removal, baseline adjustments, or general formatting of the data to be more convenient and reproducible for research data management. Typical steps for file format conversion include, e.g., .tdms, a typical file format used by LabView interfaces, to ASCII or other binary formats; Data shortening and streamlining with respect to sampling rates and relevant information.

The steps until the actual noise analysis are by design considered to not add bias. This is typically proven by a "linearity check", frequency response and calibration. We can formalize this by assuming all these individual steps to only add white noise with zero mean and constant variance to the response function, as introduced in the state space description of a reactor in Section 2.3.3. The response functions calculated from the experimental data all require a non-linear optimization scheme to estimate the curve parameters as well as their uncertainties. We next introduce the general parameter and uncertainty estimation methodology used in this work.

### 3.1.1 Non linear least squares (NLLS) for noise analysis

All of the introduced noise response functions are non-linear functions, yet with added observation uncertainties – we thus find ourselves requiring to solve an non-linear over-determined inverse problem. Most non-linear optimization problems do not have a closed solution, and require numerical approximation. We naturally strive for the "best" parameter estimate given our observations – we interpret "best" hereby in the statistical

\(^{2}\)The DSA-LX system is a combined amplification, discrimination, high voltage and digitization unit.
sense of being BLUE, a best linear unbiased estimator. This follows the implications of the Gauss-Markov theorem\footnote{The theorem states that a least squares optimization will yield the BLUE of the parameters. The theorem holds in certain cases for non-linear regression, notably for infinite sample sizes \cite{115}. For the purpose of this work, we assume to indeed produce BLUE estimators of the curve parameters when using non-linear-least squares.} used in Ordinary Least Squares (OLS), namely that a least squares based definition of the residual will yield the "best" results by coinciding with a maximum likelihood estimation (MLE) of a normal distribution. "Best" in this sense means giving the smallest uncertainties whilst being unbiased. Smaller uncertainties are naturally desirable to give more statistical weight to an observation. In this work, we use trust region optimization \cite{116}. Whether to use first order, i.e. gradient descent like methods, second order or Newton’s method, hybrid, or stochastic versions is a of current research interest for complex optimization tasks, e.g., in deep learning \cite{117, 118}. It is argued that a consensus is not reached \cite{117} - a current trend appears to be to use trust region methods for low dimensional cases such as noise experiments due to their robustness, despite being computationally more expensive.

### 3.1.2 Model choice

Depending on the response function measured/calculated, we implement the appropriate point kinetic model to define the residual function, see Section 2.4. As we will elucidate in Section 4.1.4.3, we find that multiple exponent (ME) models, e.g., a two exponent decay for correlation functions, can be used to correct for electronic noise and anomalies in Feynman-\(\alpha\) and PSD fitting. Unless indicated otherwise, we hence use ME models in all presented fitted data.

**Starting points**  Due to the existence of local minima in non linear least squares (NLLS), any given optimization scheme is also likely to converge onto a seemingly random set of parameters - the algorithm always needs starting points that may need to be sufficiently close to the estimated value. Heuristics for starting points exist, but do not work reliably when automated. This naturally induces a bias, as the starting point is chosen by the noise analyst, likely based off code predictions such as an IFP scheme. In our experiments in CROCUS, in order for an optimizer to converge, we found that \(\alpha\) needed to be typically provided within 10% of the final estimate. The other starting points could be varied arbitrarily and did not influence the final estimates.

**Weighted least squares**  NLLS problems with unequal observation errors or correlated errors typically require a weighted least squares approach. To again arrive at the maximum likelihood solution, the weights chosen should be the inverse of the observation variance \cite{119}.

**Outlier treatment**  A so-called robust fitting method may, on top of inverse variance weights, introduce another weight based on the distance of the point to the fitted model. Data with extreme outliers with small variance, if not removed by the analyst, can bias
Chapter 3. Reference noise experiments in CROCUS

the final result. The bisquare weighting scheme [120] is hereby a common remedy, as it preserves the least squares approach, yet penalizes outliers.

**Goodness of fit**: A non-linear optimization result needs a further verification by the analyst to ensure that the aforementioned strategies produced an accurate fit model. Here the so-called "goodness of fit" is evaluated using either goodness-of-fit statistics, residual analysis or confidence bounds as comparison metrics. Goodness-of-fit statistics include standard OLS quantities such as \( R^2 \), the Mean Squared Error (MSE), and others. For NLLS, \( R^2 \) is an unreliable measure at best and is not usable [121]. The analyst is often left to graphically interpret residual plots and minimize the confidence bounds and MSE - uncertain about whether the global optimum was reached. Residual analysis includes plotting the residuals to estimate whether they are normally distributed and have equal variance (homoscedasticity) with zero mean to fulfill the assumptions of the Gauss-Markov theorem. Feynman-\( \alpha \) curves often show heteroscedasticity, which leads to variance underestimation [122], a fact we will use to disregard uncertainties yielded by the algorithm.

3.1.3 Uncertainty estimation of NLLS parameters

From a NLLS estimated set of parameters, we aspire to calculate an associated covariance matrix to provide higher order statistics for hypothesis testing, data assimilation, and potentially nuclear data evaluation [13]. The covariance matrix of the experimental result can be estimated by inverting the Hessian matrix \( H \) of the residual function and scaling it by the MSE [123]:

\[
\text{Cov} = \text{MSE} \cdot H^{-1} = \text{MSE} \cdot J^T J.
\]  

(3.1)

The last equation assumes normally distributed parameters and higher order terms to be negligible, leaving first order terms that can be calculated using the Jacobian \( J \) of the residual function - uncertainties estimated this way in our experiments are thus also called first order uncertainties. It is, as discussed, standard practice in integral data benchmarking to provide only first and second order statistics. A more exact estimate of the covariance matrix requires explicit higher order terms that require numerical approximation. This is potentially required when the residuals do not end up following a normal distribution with zero mean.

Alternatively, using a **bootstrap** [124] to calculate the parameter covariance can be shown to include higher order corrections [125]. For noise analysis, this means sampling the signal in segments with replacement and fitting the response function for each sample. The resulting samples of kinetic parameters are then fitted with common distribution functions using MLE. The distribution with the highest log likelihood is then used as the descriptive statistic. This is presented in more detail in Appendix F. Bootstrapping, as a Monte Carlo method, has seen increased use with the rise of available computing power in most uncertainty estimation applications and is likely the best practice when implemented correctly.

For the determination of \( \alpha \), seeing that it depends solely on the shape of the response function measured, a calibration is not required and the provided uncertainty estimate is
3.1. Noise measurement methodology

For $\beta_{\text{eff}}$ (and subsequently $\Lambda$), the absolute amplitude of the response needs to be known. Recalling Section 2.4, the amplitude term $G_{ij}$ for a neutron CPSD is

$$G_{ij} = \frac{2C_i C_j D_\nu}{F(\beta_{\text{eff}} - \rho)^2}. \quad (3.2)$$

This amplitude term depends on the detection efficiency $\epsilon$ in counts per fission. Having access to the count rate $C_i = \epsilon_i F$ of detector $i$ with high accuracy, the main calibration quantity that remains is the fission rate. The Diven factor in CROCUS was shown to contribute less than 1% to the uncertainty budget [24], whilst the fission rate is calibrated at best within 2% [29] — adding to the final uncertainty estimate.

The shape of the response functions are nonetheless prone to biases inherent to their calculation method, i.e. in approximating the infinite integrals of the correlation functions and the Fourier transform. The effect of added white noise on the analysis and the subsequent inherent limitations with regards to uncertainties on kinetic parameters is discussed in the next section.

3.1.4 Limits to noise measurement

Fitting a response function naturally requires that the data is well represented by the imposed model. Assuming an unbiased measurement system, we still find limitations arising from the fact that a detection system is imperfect and adds unwanted random noise (see Section 2.2 when introducing the notion of noise), referred to as additive white Gaussian noise (AWGN). A common metric for noise experiment optimization is the detection efficiency $\epsilon$, which linearly influences the response function or correlation amplitude. We can restate our problem: Measuring kinetic parameters is the task to allow for distinction between a response function of interest within unwanted noise. We characterize AWGN with constant variance $\sigma_W^2$ and zero mean on a signal $S$ by a Signal-to-Noise Ratio (SNR)

$$\text{SNR} = \frac{S^2}{\sigma_W^2}. \quad (3.3)$$

When the SNR is too low, we cannot meaningfully distinguish signal from noise, and the task is failed. Consider the example of an APSD curve calculated using the analytic expression, but with AWGN in the time domain\(^4\) ($\mathcal{N}$ denoting a normal distribution)

$$\text{APSD} = \epsilon F + \frac{\epsilon^2 F D_\nu}{\rho_{\text{eff}}^2} \frac{1}{1 + \omega^2/\alpha_p^2} + \mathcal{N}(\mu_W, \sigma_W). \quad (3.4)$$

We display this expression for different SNR in Figure 3.1, using the kinetic parameters of CROCUS and $\epsilon = 7 \times 10^{-6}$ and a standard AWGN generator. For a SNR of 10 a visual identification of a cut-off frequency becomes impossible, and we more specifically state:

\(^4\)Note that perfect and infinitely measured AWGN is a constant in the frequency domain. For this example we added white noise to the critical ($\rho = 0$) APSD with a mean $\mu_W$ to reflect this.
Chapter 3. Reference noise experiments in CROCUS

Figure 3.1 Calculated APSD using the analytic expression with different levels of AWGN characterized by a SNR in dB. The kinetic parameters are approximate values for CROCUS at criticality (cut-off frequency at \( \sim 25 \text{ Hz} \)) and an efficiency of \( 7 \times 10^{-6} \).

**Noise limitation hypothesis:** *Given a certain AWGN SNR and efficiency, response functions are limited in their accuracy to determine kinetic parameters.*

An experimentalist, in designing a noise experiment, must thus maximize the intertwined efficiency and SNR of the detection system.

To maximize \( \epsilon \) means maximizing the count rate for a given fission rate, i.e. maximizing the detector sensitivity and setting it in a high flux location. This presents itself as a double-edged sword - depending on the electronics and the detector, high count rates can induce dead time (electronic and detector) that in turn results in a loss of information. Count rates need to be balanced depending on the application. In addition, a high count rate is desired to ensure a low observation error. At a typical sampling rate of 1 kHz the counts per bin follow a Poisson distribution, as is clear from Feynman-\( \alpha \) curves at small gate times, see Section 2.4. Each bin needs at least \( 10^4 \) counts to reach relative uncertainty below 1% – meaning a count rate of \( 10^7 \), a value highly beyond dead time limits of high end electronics (\( 10^5 \)-\( 10^6 \)). The inherent observation error for each bin is a source of unwanted noise in the detection process. Analytical expressions for uncertainty bounds based on this observation error were derived for \( \alpha [49] \), these however neglect other AWGN sources.

To maximize the SNR of a detection system is more straightforward. AWGN is added by multiple sources, as discussed in Section 2.2 when introducing the notion of noise. A natural approach to reduce noise levels is twofold: a) **noise source reduction** and b) **signal averaging**. a) is achieved, as already discussed above, by optimizing the experimental setup, e.g., by using high quality equipment and minimizing perturbations.

b) follows directly from sampling theory. A normally distributed data set will exhibit a

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-Noise measurements by virtue of their intent pick up all kinds of events. A mobile phone on top of the current amplifiers would render any measurement useless. A person could cause an oscillation in the high voltage supply by walking too "heavily" close to the equipment.
3.1. Noise measurement methodology

mean value, likely the signal mean if the noise is indeed AWGN. The mean value is estimated by taking many samples, i.e. by observing a noisy signal for a long time and performing segment wise analysis. An important experimental parameter is hence also the measurement time to allow for a sufficient amount of segments. This is standard practice in frequency domain analysis, and is called Bartlett averaging [127]. A more detailed description of the inherent noisiness of the PSD calculation can be found in Appendix G. The bunching technique for Feynman-\( \alpha \) works in a similar fashion [65].

Two metrics have been identified that affect noise measurement success: The efficiency and SNR. We estimated the SNR of several detection systems for noise in CROCUS after 2 hours of acquisition using the strategy outlined in Appendix H. We list them alongside efficiencies in events per fission for frequent measurement locations in Table 3.1. A given SNR and efficiency can thus be used as input for the analytic expression, Equation 3.4, and fitted to estimate a lower bound of the uncertainty for a kinetic parameter. To cover both parameters, \( \epsilon \) and SNR, we simulated PSD curves varying both parameters and fitted the results, trying to reproduce the input values for \( \alpha \) and \( \beta \) and estimate their first order uncertainties. The results are displayed in Figure 3.2. Efficiency and SNR influence the lower uncertainty bounds as expected: With a more efficient detector, we are better able to distinguish correlated from uncorrelated events - with efficiencies above \( 10^{-4} \) the white noise barely influences the retrieved parameter. With decreasing efficiency AWGN smears the response curve, increasing uncertainty bounds. At an SNR of 20, we even observe a mis-estimation of the mean value and uncertainty bounds. This implies that white noise adds enough bias when the SNR is low to a degree where first order approximations of uncertainties are not sufficient. We are encouraged to use bootstrapping even when assuming constant white noise.

![Figure 3.2](image.png)
Table 3.1  Overview of noise instrumentation SNR and efficiency in typical measurement locations in CROCU with inferred lower bounds on $\alpha$ after 2 hours of measurement. All detectors are set at mid core height.

<table>
<thead>
<tr>
<th>Location</th>
<th>$\epsilon=$Events/Fission</th>
<th>$\sim$ SNR (dB)</th>
<th>$\sim$ $\sigma_{\alpha}$ (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CeBr$_3$ Control rod</td>
<td>4.94E-04</td>
<td>30</td>
<td>$&lt;1$</td>
</tr>
<tr>
<td>BGO Against vessel</td>
<td>1.36E-04</td>
<td>26</td>
<td>$&lt;2$</td>
</tr>
<tr>
<td>$^3$He 12NH25/1 Reflector, 4.6 cm</td>
<td>1.36E-04</td>
<td>28</td>
<td>$&lt;1$</td>
</tr>
<tr>
<td>$^{235}$U CFUL01 Reflector, 3.8 cm</td>
<td>7.00E-05</td>
<td>27</td>
<td>$\sim$2</td>
</tr>
<tr>
<td>BF$_3$ MN-1 Control rod</td>
<td>3.92E-06</td>
<td>27</td>
<td>$\sim$5</td>
</tr>
</tbody>
</table>

3.1.5  Summary of uncertainty quantification methodology in noise measurements

All of the above mentioned strategies are now used to optimize a given noise experiment to give the best or even $a$ result. For noise measurements and the kinetic parameter uncertainties we summarize our methodology: A calibrated acquisition system with maximized SNR and best efficiency given the constraints will yield an unbiased data set that allows for response function calculations - auto-correlation, variance to mean, and/or PSD. The curves are then fitted using a non-linear least squares approach, with inverse observed variance as weight and IFP predicted starting points for $\alpha$. Outliers are removed by hand or penalized using bisquare weighting. The covariance matrix of the parameters is either estimated in first order approximation or bootstrapped. Final kinetic parameter estimations include first and second order statistics, common practice in integral databases - and if observed to be relevant, higher orders by including full distributions from bootstrapping. The uncertainties for kinetic parameters in the experiments are consequently bootstrapped or first order approximations.

Other parameters, such as the lower and upper fit bounds, have been considered and are discussed in Appendix G – their overall impact is important, as discussed in [28, 24], but was found to be negligible for inverse variance weighted fits for the experiments in this work.
3.2 Experimental setup for noise experiments in CROCUS

Our stage for experiments is the CROCUS reactor, and we dedicate Section 3.2.1 to presenting the reactor and its characteristics. Thereafter we present the development of new noise instrumentation, namely neutron noise current mode amplifiers, Section 3.2.2.1, and gamma detection capabilities, Section 3.2.2.2. We conclude by detailing experimental setups for noise experiments.

### 3.2.1 The CROCUS zero power research reactor

The CROCUS reactor is a uranium-fueled, light water moderated critical assembly operated by the Laboratory for Reactor Physics and Systems Behaviour (LRS) at the Swiss Federal Institute of Technology in Lausanne (EPFL). A particularly unique feature is the use of two different fuel types in two concentric fuel zones. As a zero power facility used for education and research, its power is limited by regulation to 100W. The reactor geometry has been previously benchmarked for modeling purposes [31].

#### 3.2.1.1 Reactor characteristics

The reactor core is housed in an open aluminum vessel of 1.3m diameter and 1.45m height. A schematic depiction of the reactor vessel and its internals next to the top grid are shown in Figure 3.3. The core is assembled in an approximately cylindrical shape with a diameter of about 58cm and 1m height. De-mineralized light water (H₂O) is used as moderator and reflector. The water is pumped into the core vessel from the bottom in the west and is circulated to an auxiliary tank over a spillway on the top east side of the vessel. The temperature is regulated via intermediary circuits that provide cooling or by heating the
Chapter 3. Reference noise experiments in CROCUS

Figure 3.4  Schematic and dimensions of the radial fuel arrangement of CROCUS. Configuration B-ref-003 with 336 UO₂ and 176 U_nuel fuel rods and two control rods. Two fuel zones are set in approximately cylindrical shape held by two grids. The outer water reflector dimensions are not to scale for illustrative purposes.
3.2. Experimental setup for noise experiments in CROCU

Figure 3.5  Schematic and dimensions of the axial fuel arrangement of CROCU. The water level is controlled by a spillway on the east side of the vessel, where the water can flow off to the auxiliary water tank of the reactor. Dimensions are in mm.

auxiliary tank water, yielding an operational range between 17.5 and 22.5°C - the cooling is normally automatically regulated to keep a temperature of (293.15 ± 0.01) K or 20°C. The core reactivity is controlled by variation of the water level by moving the spillway - it is calibrated with an accuracy of 0.1 mm, i.e. equivalent to 0.4 pcm. Optionally, two control rods containing naturally enriched boron carbide (B\textsubscript{4}C) sintered pellets located in symmetric positions within the outer fuel zone can be inserted. Two different kinds of fuel rods in interlocked zones make up the core. In Figure 3.4 we depict a radial schematic of the fuel arrangement and specifications. The central zone is loaded with 336 U\textsubscript{2}O\textsubscript{2} fuel rods (1.806 wt.%-enriched), set in a square lattice with a pitch of 1.837 cm. The outer zone is loaded with up to 176 U\textsubscript{met} fuel rods (0.947 wt.%-enriched) having a larger diameter and aligned in a square lattice with a pitch of 2.917 cm. All fuel rods have an aluminum cladding and are maintained in a vertical position by the upper and lower grid plates spaced 100 cm apart. Both grid plates incorporate a cadmium layer to limit axial thermal leakage to surrounding structures. The active fuel length starts at the top surface of the lower cadmium layer\textsuperscript{6} and extends 100 cm. This is also the maximum water level that can be set by the spillway. The loading is designed as to limit the maximal excess reactivity at 1000.0 mm water level to less than 200 pcm. A schematic view of the axial geometry of the fuel and grids is shown in Figure 3.5. CROCU has a total of six safety systems, two cruciform cadmium safety blades that are electromagnetically held above the core in operation, and four expansion tanks in the corner of vessel that allow for a rapid water level drop. Below the core a PuBe start-up source is pneumatically inserted or removed

\textsuperscript{6}This surface is used as the reference point for measurements for detector placement, control rod insertions or water level indications.
Table 3.2  Overview of the CROCUS reactor operational states. The reactivity ranges are calculated using a $\beta_{\text{eff}}$ of 750pcm [128].

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Water level</th>
<th>Safety blades</th>
<th>Fission power</th>
<th>Reactivity</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shutdown (ARRET)</td>
<td>Empty</td>
<td>✓</td>
<td>0</td>
<td>&lt;-50</td>
<td>out</td>
</tr>
<tr>
<td>Preparation (ATTENTE)</td>
<td>Filling to 500.0</td>
<td>✓</td>
<td>&lt;1.2</td>
<td>&lt;-10</td>
<td>out</td>
</tr>
<tr>
<td>Start-up (INTER)</td>
<td>500.0</td>
<td>✓</td>
<td>~1.2</td>
<td>&lt;-10</td>
<td>in/out</td>
</tr>
<tr>
<td>Manual (MANUEL)</td>
<td>800.0 to 1000.0</td>
<td>×</td>
<td>up to 100k</td>
<td>[-1.4, + 0.26]</td>
<td>in/out</td>
</tr>
</tbody>
</table>

for operation in a polyethylene shielding on the side of the vessel. In the framework of the COLIBRI fuel oscillation experiment [35] the upper grid has been modified, notably by increasing the cadmium layer thickness to 1 mm. The presented experiments in this thesis are all based on this reactor configuration.

3.2.1.2 Operation and associated constraints

The operational characteristics of CROCUS are unique to its design and standard operation imposes certain boundaries with respect to which water levels can be set for an experiment, if the safety blades are in core, or if the start-up source can be inserted\(^7\). In shutdown state the reactor vessel contains no water and the safety blades are inserted into the core. An operator must follow a set of procedures to start the reactor and eventually have direct control of the spillway to manually set the water level between 800 and 1000 mm. An overview of the reactors operational states is shown in Table 3.2. For example, water levels below 800 mm with the safety blades removed from core are not accessible due to this design.

3.2.1.3 Detection instrumentation

CROCUS is equipped with four operational monitor detectors [129]. Two compensated ionization chambers for general core monitoring in the north and south and two fission chambers (FC) in the eastern and western periphery as calibrated power monitors. CROCUS is additionally instrumented by a range of other neutron detectors, the most prominent options are listed in Table 3.3. Detectors that do not exceed the control rod guide tube inner dimension (17.35 mm) can be set into arbitrary free locations in the U\(_{\text{met}}\) part of

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\(^7\)Regulatory exemptions can be requested for specific non-standard configurations – this naturally increases the complexity of an experiment. Most experiments are conducted within already available authorizations.
3.2. Experimental setup for noise experiments in CROCUS

Figure 3.6 Overview of the CROCUS top grid with fuel arrangement and highlighted detector placement regions. The control rods, as optional tools, can be removed and their guide tubes thus freed for detector placement. Alternatively, any of the remaining free holes in the grid can be used for detector placement. In yellow we highlight the regions accessible with the Plexiglas channels for larger detector setups.
Table 3.3 Non-exhaustive list of commonly used neutron detectors in CROCUS.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Type</th>
<th>Detection material</th>
<th>Electronics mode</th>
<th>Thermal sensitivity $n_{th}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photonis CFUF34</td>
<td>FC</td>
<td>$^{235}\text{U}$ coating</td>
<td>Pulse</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Photonis CFUM21</td>
<td>FC</td>
<td>$^{235}\text{U}$ coating</td>
<td>Pulse</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Transcommerce MN-1</td>
<td>PC</td>
<td>BF$_3$ gas</td>
<td>Pulse</td>
<td>$4 \times 10^{-2}$</td>
</tr>
<tr>
<td>Merlin Gerin CC54</td>
<td>CIC</td>
<td>$^{10}\text{B}$ coating</td>
<td>Current</td>
<td>$8 \times 10^{-2}$</td>
</tr>
<tr>
<td>Photonis CFUL01</td>
<td>FC</td>
<td>$^{235}\text{U}$ coating</td>
<td>Current</td>
<td>1</td>
</tr>
<tr>
<td>Canberra 12NH25/1 (F)</td>
<td>PC</td>
<td>$^{3}\text{He}$ gas</td>
<td>Pulse</td>
<td>12</td>
</tr>
<tr>
<td>Vniitfa CHM73</td>
<td>PC</td>
<td>$^{3}\text{He}$ gas</td>
<td>Pulse</td>
<td>30</td>
</tr>
</tbody>
</table>

the grids. Frequently, the control rods are removed and detectors are placed at mid-core height to achieve the closest proximity to the core center. Larger detection setups can be placed in Plexiglas detector channels [130] that are clamped to the outer edge of the upper grid, giving additional accessible measurement locations. An overview is given graphically in Figure 3.6.

Previous neutron noise experiments employed BF$_3$ proportional counters (PC) that were set in the control rod and peripheral tube positions [33, 24]. They are, however, limited with regards to sensitivity, giving comparatively low correlation amplitudes. A further limitation, applying to all pulse detectors for CROCUS, is their loss of linearity at higher powers. The BF$_3$ for instance yield unreliable results above about 100 mW. This can be attributed to dead time due to pile-up in the detector and the lock-up of the associated spectroscopy charge preamplification electronics. The CICs, albeit operated in current mode, are fixed to the grid and have electronics that are poorly characterized with regards to their frequency response [69], [110].

For higher powers and high efficiency noise experiments we investigated the possibility of using large Canberra CFUL01 FC (1 g deposit of U-235) fission chambers set into the Plexiglas experimental channel in the reflector. As a result, a current mode amplification system was developed, discussed in Section 3.2.2.1.

Gamma detection in CROCUS is either conducted passively with TLDs, or with ambient dose rate meters set in various locations of the facility, including positions close to the vessel. An extension of the general gamma detection capabilities with a perspective towards investigating gamma noise measurements was desired. The result of design considerations [131] resulted in the acquisition of a scintillation based detection array called LEAF, presented in Section 3.2.2.2.
3.2. Experimental setup for noise experiments in CROCUS

Figure 3.7 Schematic of the board logic of the new current amplifier for noise measurements. The high voltage (HV) is passed through and the resulting measured detector current is amplified in the first stage. The second stage provides coarse amplification, filtering, and the output signal. Low voltage (LV) for the operational amplifiers is provided from an external source. The whole circuit is grounded by an additional outer layer to minimize external electromagnetic noise. The detectors output connected to the amplifier is therefore a triaxial cable.

3.2.2 Development of noise detection instrumentation

3.2.2.1 Neutron detection: Current mode amplifiers

In an effort to expand the neutron noise measurement capabilities of CROCUS using high efficiency CFUL01 FC, a custom current mode amplification system was developed [128]. Efficiency is generally an optimization metric in noise applications, a topic explored in detail in Section 3.1.4. Current mode does not allow for straightforward discrimination like in pulse mode, be it for gamma or otherwise induced noise in the system. Fission chambers have an average energy deposit into the gas of more than 100 MeV, as opposed to proportional counters on $^{10}$B or $^3$He basis at about 1-2 MeV. They are thus inherently more suitable for current mode operation\(^8\). CFUL01 fission chambers in current mode were already tested in CROCUS [23] using a CEA developed system called SPECTRON [68], and were deemed suitable for noise measurements. In the following we discuss the design of the current amplifiers.

**Design requirements** As the main purpose of the system is noise measurements, the set of specifications required can be summarized to be:

- Constant DC amplification in the range of nA to mA to allow for standard acquisition tools such as oscilloscopes to read the voltage output.

- Constant, low noise transfer function in the region of interest for noise measurements i.e. 0.1 Hz to 1 kHz to ensure distinguishable correlations from unwanted noise.

\(^8\)This is mostly due to the nature of a reactor’s mixed radiation field of neutrons and gamma rays. Gamma rays, despite their low interaction probability in the detector’s filling gas, can deposit energies comparable to those of $^{10}$B(n,$\alpha$) or $^3$He(n,p).
Overview  The prototype board logic is depicted in Figure 3.7. To date six amplifiers were manufactured, numbered 1 through 6. They thus allow for cross correlation measurements using up to four CFUL01 fission chambers in different power ranges. Each amplifier is hard wired and optimized for a certain reactor power range. Each consists of two main components:

i) A primary stage to pass through the high voltage to the detectors anode and to extract and amplify the current in the detector using a measurement resistance of 10 kOhm and a low noise, low current operational amplifier OPA344NA [132] with a current bias as low as 2 pA to 20 pA. The best results in terms of output noise levels was an effective minimum measurable current in the chamber of 1 nA. In order to minimize induced electromagnetic noise on the first stage amplifier, an independently grounded conductor layer was set around the detector and the amplifier board. The cable between the detector and amplifier is hence a custom triaxial cable. This implies that the detectors connector was modified to fit a BNT [133] triaxial connector.

ii) A secondary stage for filtering purposes and signal amplification to volt ranges for acquisition. This is achieved by using an active double low-pass RC filter which have a respective cut-off frequency of 2 kHz and 3.3 kHz, both giving -40 dB/decade each thereafter. The output is then read via standard BNC9 connectors. The transfer function and linearity of the amplifiers were tested in CROCUS and deemed appropriate given the requirements – details can be consulted in Appendix D.

3.2.2.2 Development of gamma detection instrumentation:

LEAF

On top of new neutron noise capabilities, an experimental program to investigate gamma noise was launched in CROCUS. This is the purpose of the LEAF system: A Large Energy-resolving detection Array for Fission gammas to directly measure gamma ray spectra in CROCUS. The intent of LEAF is twofold, embodied by the two detector types that make up the array. Aside from providing measurement data in previously unexplored parameter regions, namely in-core and high energy (> 2 MeV) spectroscopy of reactors, we chose the detectors to perform specific noise measurement tasks – in-core kinetics and ex-core exploratory experiments. Here, we present the characteristics and examples for measured spectra using LEAF.

Overview  As discussed in Section 2.7, previous gamma noise experiments relied either on comparatively large Cerenkov based detectors or NaI scintillators. Scintillators were the final choice when optimizing for flexible use, price, and nonetheless high efficiency when compared to semiconductor based detectors. In total, an array of four detectors, two small and two large, was acquired from Scionix Holland [134]. The high voltage supply and photomultiplier (PMT) signal treatment was handled by the fully integrated DSA-LX [112] (designated 419 and 420 as per serial number) that allow for count rates of ∼MHz to be treated. In the following, we introduce the respective detectors and present their characteristics. The calibration in energy is detailed in [135].
3.2. Experimental setup for noise experiments in CROCUS

Small sized scintillators: Cerium Bromide (CeBr₃)

The control rod guide tubes of CROCUS are a prominent location for in-core measurements, e.g. the previous neutron noise measurements using BF₃ detectors. Due to the high photon flux in the tubes, a quickly decaying, yet radiation-hard scintillator was required. Cerium(III) Bromide (CeBr₃) – with a decay time of 20 ns and comparatively high light yield of 60k photons/MeV at a density of 5.2 g/cm³ – has been under active research since the early 00s [136] and is a relatively new material [137]. Its radiation hardness was estimated to be adequate for CROCUS [131]. They are designated by the serial numbers 91 and 92. The tubes constrained the cylindrical crystal size to 13 mm in diameter and 15 mm in length as shown in the technical drawing in Figure 3.8. Both detectors house a Hamamatsu Type R12421 PMT.

Large sized scintillators: Bismuth Germanate (BGO)

For ex-core applications, a material with high efficiency was desired, with the intent to maximize the absorption efficiency and thus detect full deposits of photons above 2 MeV. Bismuth Germanate (BGO) – with a decay time of 300 ns and a yield of 9k photons/MeV – has been studied as scintillator since the 70s [138, 139], and been used for reactor power monitoring [140] or, more recently, for multiplicity counting [141] and in medical applications [142]. The size of the detectors was less constrained, and was chosen to give an absorption efficiency of above 95% for 10 MeV photons assuming a density of 7.13 g/cm³. The two finally acquired cylindrical crystals are 127 mm in diameter; the height was constrained by weight (25 kg) and price, and is 250 mm. Both house a Photonis 5" Type XP4578 PMT, as detailed in the technical drawing in Figure 3.9. They are designated by their serial numbers 93 and 94.

Gamma spectra measured in CROCUS using LEAF

We next present in-core and ex-core spectra measured with LEAF. They are primarily intended to provide an illustration and guide as to where lower level thresholds for noise measurements will be determined. For documentation dedicated to their study, refer to [135, 131, 143]. The CeBr₃ detectors were placed at mid core height in the control rod tubes, while the BGOs were set just outside of the main vessel also at mid core height. Both DSA-LX, as industrial products, offer a variety of settings for pulse shaping, pole/zero corrections, etc. For consistent and reproducible signals, we chose the settings for all experiments to be as follows:

- Rise time of 0.2 μs, 0.0 μs flat top.
- -610 V and -1260 V of HV for the CeBr₃ and BGO PMTs, respectively.
- Lower level discrimination at 0.5% of the maximum channel (2⁽¹⁴⁾).
- Coarse gain of 6.4 for all detectors.

In Figure 3.10 we display CeBr₃ and BGO spectra acquired in different reactor configurations. We most strikingly observe several resolved peaks on top of the expected exponential fission spectrum[144]. The CeBr₃ spectra notably exhibit a cut-off at around 3 MeV, whilst the BGO spectrum has a similar cut-off at 10 MeV. Using NIST standard data on photon
Figure 3.8  Technical drawing of the CeBr\textsubscript{3} detector provided by Scionix Holland. The PMT is a Hamamatsu Type R12421.
Figure 3.9 – Technical drawing of the BGO detector provided by Scionix Holland. The PMT is a Photonis 5" Type XP4578 PMT.
Chapter 3. Reference noise experiments in CROCUS

Figure 3.10 Example gamma ray spectra acquired in different reactor states using the LEAF CeBr$_3$ (top) and BGO (bottom) detectors. Prominent peaks are indicated for illustrative purposes. A more detailed analysis of the spectra and their application can be found in [135].
3.2. Experimental setup for noise experiments in CROCUS

mass attenuation lengths \cite{145} and the density mentioned above, we find that photons of the respective energies have a mean free path consistent with the detector geometry. Note that the spectra are not corrected for background or self-radioactivity \cite{146}, as they are negligible with respect to the shown count rates. A more detailed look at the spectra and their outlook is discussed in \cite{135}.

**Linearity of counting** For noise applications the range of linearity for the coupled system of detector and DSA-LX needed characterization. During a standard start-up of the reactor and subsequent power increase to 1 W we monitored the count rates at each stage for 5 min each. As can be seen in Figure 3.11, when compared to the reactor power monitor, we find a consistent increase in count rates up to a reactor power of 100 mW for both detector types. This is mainly reflected in the constant ratio between LEAF detectors and the monitor. Above 100 mW the count rates of LEAF rapidly drop to effectively zero. Note that the Genie2000 Software of the DSA-LX consistently reported dead times below 1%. We infer that not electronic dead time, but paralyzable detector dead time to be the cause of the loss of functionality. The system nonetheless shows - consistently across the detectors - an excellent counting response up to 0.2 MHz, recalling that previous pulse systems, based on gamma spectroscopy equipment, cap at around 10 kHz \cite{24}.

Having presented the measurement setup, analysis and optimization strategies, we next present reference noise experiments using the novel instrumentation in CROCUS, one for neutron and gamma noise each.
Chapter 3. Reference noise experiments in CROCUS

Figure 3.12 Top view of the critical kinetics benchmark core configurations for neutron and gamma noise. The detector locations are indicated by black circles, specifically: 1. CFUL01 fission chambers. 2. CeBr$_3$ scintillators. 3. CFUM21 power monitors. All detectors', except the monitors', sensitive region was centered approximately at mid core height (50 cm).

3.3 Reference neutron and gamma noise measurements in CROCUS

To set an example of the uncertainty estimation methodology and establish a reference measurement to serve as comparison point for later exploratory experiments, we apply our tools to two critical measurements in CROCUS for neutron and gamma noise respectively. Criticality removes the additional parameter of reactivity and thus allows for a direct estimate of $\beta_{\text{eff}}$ and $\Lambda$ after calibration. However, criticality is practically more challenging to maintain$^{10}$. A schematic of the reactor configurations is shown in Figure 3.12. The electronics were set up according to the general scheme introduced in Section 3.1.

3.3.1 Arrangement of noise instrumentation

Templates for neutron noise and gamma noise experimental setups, used for the experiments shown in this work, are depicted in Figures 3.13 and 3.14, respectively.
3.3. Reference neutron and gamma noise measurements in CROCUS

![Diagram](image)

**Figure 3.13** – Schematic of a neutron noise measurement experimental setup in CROCUS. The power monitors (CFUM) are always part of the experiment as they are integrated in the safety systems. The other neutron detectors, e.g. CFUL01, 12NH25/1, or MN-1, are optionally usable, with acquisition possible with up to 4 channels in pulse mode or 4 channels in current.

![Diagram](image)

**Figure 3.14** – Schematic of a gamma neutron noise measurement experimental setup in CROCUS. The amplified gamma signals are taken from the DSA-LX "MON OUT" diagnostics output. Up to 4 channels can be measured in parallel. Since only two DSA-LX were available, up to two gamma and two neutron detectors are measured in parallel.
3.3.2 Neutron noise reference

**Experimental setup** The intents of the experiment is to provide a baseline for kinetic parameters in CROCU S using an established method, and display the uncertainty estimation that will serve as template for other experiments. The experimental conditions were:

- Reactor power of $\sim 900$ mW, or a fission rate of $2.75 \cdot 10^{10}$ $1/s$,
- Reactor fuel and coolant temperature of $(293.15 \pm 0.01)$ K,
- Two CFUL01 fission chambers set in the northern and southern reflector experimental channels, set as close as possible to the fuel,
- Northern and southern CFUL01 connected to current mode amplifiers boxes 1 and 2, respectively,
- Signals acquired on a Teledyne Lecroy 10 Wavesurfer oscilloscope at a sampling rate of 1 kHz,
- Total measurement duration was 5.5 h.

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A critical state is a mathematical description that in reality does not manifest, as is evident from the lack of observed "critical catastrophes" [147, 148]. An operator regularly needs to adjust the spillway by fractions of millimeters to avoid a drift in power, likely because a reactor is always slightly sub- or super-critical.
Due to the oscilloscope’s internal memory limitation, we needed to measure in chunks of 4.9 MS (Mega samples), corresponding to about 1.4 hours, and then manually concatenate the data. The bias induced by this, given an average prompt auto correlation time of 0.01 s, should be negligible. It can, however, induce spectral leakage in PSDs. This is why a Blackman-Harris window was used instead of a uniform window in this analysis [149]. Figure 3.15 shows examples of resulting APSDs and CPSD. We plot alongside fits of the theoretical expression. We find that the PSD shapes are indeed following the expectation, and that the Lorentzian fits visually well onto the data.

**Results** The PSDs were estimated using the Bartlett method, averaging 2435 spectra, giving a spectral resolution of 0.1 Hz. The uncertainties from first order covariance matrix estimates showed an error of 1% on $\alpha$. A bootstrap was performed to estimate the true parameter spread. The resulting distributions compared to Serpent 2 IFP predictions [150] for $\alpha$, $\beta_{\text{eff}}$, and $\Lambda = \beta_{\text{eff}}/\alpha$ are shown in Figure 3.16. Numerical values and MLE parameters chosen on log likelihood basis are found in Table 3.4 on page 70 alongside first order approximations of the parameters.

**Discussion** Neutron noise experiments are the cornerstone of this thesis: They allow for reference kinetics to be measured and analysis tools to be validated using well established
methods. We herein observe the following:

- Using CFUL01s and the developed current amplifiers, we are able to determine kinetic parameters within 1σ of code predictions. Bootstrapped parameter estimates greatly influence how well the results compare to code, e.g. improving a 4% discrepancy of $\beta_{\text{eff}}$ estimated with the CPSD to IFP using JEFF3.3 to 0.6% and yielding a relative uncertainty of 3.5% instead of a likely underestimated 1%. Bootstrapping thus not only influences precision, but also overall accuracy.

- The likelihood estimations suggest that the distributions do not all follow normal distributions. We therefore are encouraged in our method, as this effect would not be visible in first order approximation. The deviations are consistently modeled by a Generalized Extreme Value (GEV) distribution, adding a third reported parameter.

- The results are of the same order as previous experiments using BF$_3$ counters [24], albeit with larger uncertainties. We attribute this to two causes: I) Current mode is more susceptible to pick up electronic noise that influences the signals. II) The uncertainties estimated for this benchmark were bootstrapped, unlike previous data. This leads us to believe that higher order effects are important for current mode systems, or that previous experiments have underestimated uncertainties.

- Neutron noise measurements are able to validate code predictions in general. Due to the comparatively large uncertainties, feedback on which nuclear data library has the more favorable evaluation for CROCUS is not determinable, and nuclear data feedback by providing the integral parameters seems unlikely to benefit. This conclusion contradicts other experiments that achieved 1% uncertainties in first order approximation. A sensitivity analysis could provide insight on whether nonetheless data assimilation could still be used on the provided data [13].

- APSD2 appears to be more sensitive to the bootstrap. As both boxes 1 and 2 are equivalent in their respective design save for their measurement resistances, this indicates a noise source intrinsic to the current amplifier box 2. If indeed a bias, this would affect the CPSD as well – pointing towards a truer estimate of kinetics coming from APSD1.

- The kinetics measured in this experiment now serve as comparison reference for the following gamma noise and noise field experiments.

3.3.3 Gamma noise reference

Experiments using gamma radiation for kinetic parameters in published literature are limited to few cases, as introduced in Section 2.7. Our intent is to thus expand on the previous knowledge: By conducting an experiment using the CeBr$_3$ detectors in in-core locations, we provide evidence for the general usability of gamma detectors for noise applications and compare the performance to neutron noise. Note that the BGO detectors were also validated in this fashion - we discuss the results in the next chapter, as their position just outside the reactor vessel can be considered extraordinary and is thus part of the study on the noise field, see Chapter 4.
3.3. Reference neutron and gamma noise measurements in CROCUS

Figure 3.17  DSA-LX count rates and calculated PSDs of the qualification measurement of the CeBr$_3$ detectors. The black lines are Lorentzian fits added for illustrative purposes.

**Experimental setup**  The experimental conditions were as follows:

- Reactor power of $\sim$20 mW, or a fission rate of about $6 \cdot 10^8$ 1/s.
- Reactor fuel and coolant temperature of $(293.15 \pm 0.01)$ K.
- Two CeBr$_3$ scintillators set in the control rod tubes at mid core height, two BF$_3$ counters in the periphery tubes at mid core height (see Figure 3.12).
- Both detectors connected to the DSA-LX, with the analog output of the amplifier being directed to the PSI/EPFL noise acquisition computer [24] (see Figure 3.14).
- After optimizing the LLT (see Section 3.3.4), the total measurement duration was 2 h, i.e. less than half of the reference neutron measurement duration.

**Results**  Due to the lack of previous literature on gamma noise experiments, we predicted the noise responses based on simplified efficiency considerations [131]. The predictions indicated that noise measurements are only possible with the high efficiency BGO system if set in the reflector. The smaller and less efficient CeBr$_3$ detectors were tested nonetheless for qualification by measuring the APSD and CPSD responses in CROCUS. The results are displayed in Figure 3.17 alongside fits of the theoretical expression. Contrary to the predictions, the CeBr$_3$ exhibit correlation amplitudes about one order of magnitude higher than the CFUL01. This unexpected result encouraged the further study of gamma noise in-core, and the study of ex-core noise using the BGO detectors.
3.3.4 Optimization of gamma noise lower level threshold

The experimentalists attempting gamma noise using NaI [105] noted that the CPSD amplitude would increase with LLT, a potentially counter-intuitive behavior considering our discussion on the efficiency $\epsilon$. An explanation or quantification of the APSD behavior was not provided. We herein quantify the effect for practical purposes and provide a qualitative hypothesis based on the results.

LLTs for neutron detectors are typically determined by acquiring a pulse height spectrum. The characteristic shape of, e.g., \(^3\)He(n,p) is used to discriminate neutron interactions and background. Common neutron detector materials used in this thesis (\(^{235}\)U, \(^{10}\)B, \(^3\)He) are more sensitive to thermal neutrons. With a Q value above 1 MeV we are thus not able to infer the incident neutron energy. If the noise contributions are appropriately cut-off by the LLT, and as correlated neutrons cannot be distinguished from uncorrelated ones, any uncounted neutron results in a direct loss of correlation information and a decrease in correlation amplitude for both APSD and CPSD with a higher LLT.

Gamma spectrometers however also measure the incident particles energy. We hypothesize that gamma rays of higher energy are more likely to have directly originated from fission, thus giving higher correlation information than low energy events. We compare the experimental CeBr\(_3\) APSDs and CPSDs at criticality for various threshold settings in Figure 3.18. The gamma APSD acts similar to a neutron detector, with a simple "more counts is better" indicating one should minimize the threshold. The gamma CPSD amplitude, as opposed to the analytic expectation, increases with LLT. This behavior is probably linked to how likely correlated events are seen in both detectors when varying the gamma energy. A fission gamma spectrum is approximately exponential in shape. Low energy events carry the information as well, albeit more likely to be absorbed or scattered before reaching a detector. Given that fission chains propagate locally on the scale of neutron diffusion lengths, and that lower energy gammas are less likely to reach the detector, we categorize events that lead to "successful" cross correlation, in the sense of observing the same fission chain, ordered by their likelihood:

1. High energy event in detector 1, high energy event in detector 2.
2. Low energy event in detector 1, high energy event in detector 2.
3. High energy event in detector 1, low energy event in detector 2.
4. Low energy event in detector 1, low energy event in detector 2.

Given our observed behavior, we infer that at low energy the likelihood of cross correlating a correlated event is less likely than measuring uncorrelated decay or scattered gammas. By increasing the LLT, we filter these events, effectively changing the correlated to uncorrelated ratio.

The overall variance in the CPSD increases with LLT as well, likely linked to the reduction in event rate. Our observations lead to the following conclusion: The LLT for gamma noise applications should be set as low as possible, to maximize the APSD amplitude and
Figure 3.18 Comparison of gamma PSDs with decreasing LLT (left) compared to the analytical neutron PSD expression with increasing $\epsilon$ (right). A higher LLT is indicated by a deeper red color. The APSD spectra are normalized to their mean value above 0.5 kHz to illustrate the relative amplitude. The gamma CPSD amplitude, as opposed to the analytic expectation and the gamma APSDs, increases with LLT. This behavior is assumed to be linked to how likely correlated events are seen in both detectors when varying the gamma energy. Note how the APSD contains perturbations at 50 Hz at its harmonics due to the power grid which are reduced in the CPSD.
event rate. Practically this means a value just above zero, to cut the inevitable influence of electronics noise in the spectrum. For the experiments presented in this work, we hence use a value of 0.1 V, equivalent to about 0.1 MeV for the CeBr$_3$ and the BGO using the aforementioned amplifier settings.

**Determination of the prompt decay constant using gamma noise** The PSDs were estimated using the Bartlett method, averaging 3515 spectra, giving a spectral resolution of 0.5 Hz. Given the parameters and uncertainties from our neutron noise experiments were likely mis-estimated with first order approximations, we bootstrapped the signals for this experiment as well. The resulting distributions compared to Serpent 2 IFP predictions for $\alpha$ are shown in Figure 3.19. Numerical values and MLE parameters chosen on log likelihood basis can be found in Table 3.4.

The estimates for $\alpha$ were determined precisely, yielding comparatively narrow uncertainties despite bootstrapping. Compared to the IFP predictions we hereby can also provide evidence that gamma noise, in the examined region of the PSD, indeed follows the neutron correlations and is not significantly perturbed by a hypothetical gamma propagation decay constant. We therefore assert that gamma noise measurements provide the true $\alpha$, yielding an accurate measurement method.

### 3.3.5 Comparison of neutron and gamma noise

The uncertainty bounds on $\alpha$ from gamma noise are strikingly smaller, by a factor 2 to 5 — at less than half the measurement duration compared to the neutron measurement\textsuperscript{11}. The reactor power was also over an order of magnitude lower, yielding overall less events per unit time. This means that gamma noise is generally more efficient when regarding an "events per fission" metric. The results even compare well with respect to calculation uncertainties, with both APSD estimates agreeing on the mean value within 0.1%, whilst the

\textsuperscript{11}Note that the CeBr$_3$ scintillators and electronics cost an order of magnitude less than the CFUL01 fission chambers and their electronics.
CPSD estimate is about 3% lower. This accuracy is unprecedented for noise experiments in CROCUSt.

The other kinetic parameters, $\beta_{\text{eff}}$ and $\Lambda$, could not be determined from the gamma noise amplitude term, however. The amplitude terms derived in, e.g. [151], contain additional constants related to the gamma multiplicity in fission, which would require additional modeling. We expect that $\beta_{\text{eff}}$ and $\Lambda$ could be determined with better accuracy than with neutron noise if resolved in future studies.
Table 3.4

Overview of kinetic parameters obtained from the neutron and gamma noise reference experiments compared to Serpent 2 IFP predictions. The neutron noise experiment had a duration of 5.5 h at 900 mW reactor power, while the gamma noise experiment had a duration of 2.5 h at 1000 mW reactor power.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>First order approximation N(μ,σ)</th>
<th>Second order Approximation N(μ,σ)</th>
<th>APSD1</th>
<th>APSD2</th>
<th>CPSD</th>
<th>ENDF/B-8.0</th>
<th>IFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prompt decay constant A (1/s)</td>
<td>1.38</td>
<td>1.38</td>
<td>1.08</td>
<td>1.08</td>
<td>1.12</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>Effective delayed neutron fraction β</td>
<td>748</td>
<td>748</td>
<td>772</td>
<td>772</td>
<td>792</td>
<td>792</td>
<td>792</td>
</tr>
<tr>
<td>Generation time Λ (μs)</td>
<td>48.5</td>
<td>48.5</td>
<td>49.3</td>
<td>49.3</td>
<td>49.0</td>
<td>49.0</td>
<td>49.0</td>
</tr>
</tbody>
</table>

Note that the uncertainties for β and Λ do not include the 2% additional uncertainty due to reaction rate calibration.
3.4 Chapter’s salient elements

Based on our motivation to study the spatial dependence of branching noise, this chapter aimed at providing an overview of the general methodology on noise measurements and analysis, offering a perspective on the incurring limitations.

- We presented how the CROCUS reactor as a zero power facility offers a unique environment to study branching noise, yet lacked the appropriate instrumentation. In order to expand on the noise measurement capabilities, both neutron and gamma noise instrumentation systems were designed and deployed. We note that the developed current mode amplifiers and the CFUL01 detectors were successfully used in perturbation noise studies – such as the bubble velocity experiment VOID [47, 48] and the fuel rod oscillation experiment COLIBRI [152, 35, 153].

- Reference neutron noise measurements using the new instrumentation yielded a new estimate for CROCUS’ critical kinetic parameters. The results are in line with previous noise measurements efforts in CROCUS. Full bootstrapped parameter estimates and uncertainties revealed the need for higher order statistics in the reporting of integral parameters, even at comparatively long measurement durations.

- The new LEAF system for gamma detection applications in CROCUS allowed for an unprecedented study on gamma noise. The results indicate that due to the comparatively higher correlation amplitude, gamma noise offers estimations of $\alpha$ faster, more accurately, and potentially cheaper than neutron noise systems.
Chapter 4

Experimental study of the branching noise field of CROCU S

Noise experiments are often motivated as a non-invasive tool to measure reactor kinetics. As discussed in the previous chapter, it is favorable to maximize the detection efficiency to achieve high precision. Kinetic parameters are however oftentimes measured with in-core instrumentation, creating local perturbations. By moving a detector further away from the core, we perturb the system less, but due to the incurring loss of efficiency smear the result. Given geometrical constraints, the task is thus to optimize for the best detector location within the noise field of the reactor. Yet, as discussed in Section 2.6, a successful measurement also hinges on the validity of point kinetics. Spatial effects could perturb the interpretation of signals from detectors deeper in the reflector, especially in a sub-critical configuration. As an alternative, gamma noise, as compared to neutron noise, is likely to 1) carry the information farther and 2) to be less prone to, if any, spatial effects due to the difference in propagation [36].

The CROCU S core is surrounded by a water reflector of about 35 cm thickness delimited by the vessel. As discussed in Section 3.2.1, we are able to attach an experimental Plexiglas channel to the top grid to position detectors in the reflector. With the experimental environment in CROCU S allowing to study the parameters of space, reactivity and particle type, and inherent needs to quantify these effects within other experimental programs [34], we are prompted to investigate both the neutron and gamma noise field experimentally, at criticality and sub-criticality. In this chapter we attempt to answer the following questions:

- Does CROCU S follow point kinetic predictions, or will measurements at distance and/or sub-criticality be biased using this approximation?
- Given a neutron detection system, how far away from the reactor can one place the detector and still achieve a usable measurement? Can a neutron detector be truly non-invasive?
- With its higher detection efficiency, can gamma noise instrumentation provide the same information as neutrons non-invasively, e.g. outside of the region where a detector can perturb the neutron population?
The proposed questions will be explored from two perspectives:

- The first is to establish a practical measure of the correlations observed in dependence of distance to the core and negative reactivity to maximize, if any, spatial effects. This implies establishing a metric to decide whether correlations are visible and if a long duration measurement could yield accurate point kinetics core parameters. From here, we then aim to define noise measurement limits that can be predicted for arbitrary reactors.

- The second is to measure the prompt decay constant with increasing distance to the core and compare the results to code predictions. Any deviations in the required fit models, uncertainties or mean values can then be used as a starting point to formulate a spatial effect. The null hypothesis for all experiments is that point kinetics is valid for CROCUS throughout, be it for neutrons or gammas and irrespective of the distance and reactivity.

This chapter is structured as follows: In Section 4.1, we present the experiments conducted to examine the neutron noise field and discuss the results. Following suit, in Section 4.2, we display the experimental setup, measurements, and results for the gamma field measurements.

### 4.1 Neutron noise field experiments

#### 4.1.1 Testing of point kinetics behavior of neutron noise

We first define on which base the point kinetics null hypothesis of noise will be tested. From a point kinetics perspective, the space parameter is implicitly expressed by the absolute efficiency $\epsilon_{\text{abs}}(r)$ of the detection process, defined as the measured count rate $C$ over the fission rate $F_0$:

$$\epsilon_{\text{abs}}(r) = \frac{C}{F_0} = \frac{C}{R_{\text{det}}(r)} = \frac{R_{\text{det}}(r)}{F_0}$$ (4.1)

with the intrinsic efficiency $\epsilon_{\text{int}}$ as the ability of a reaction in the detector material to be actually registered as a count, and the geometric efficiency $\epsilon_{\text{geom}}$ as detection reactions per fission. Here we find $R_{\text{det}}(r)$ as a space dependent detection reaction rate defined as

$$R_{\text{det}}(r) = \int dE \, \Sigma_d(r,E) \cdot \phi(r,E) \cdot V_d = \Sigma_d(r) \cdot \phi(r) \cdot V_d.$$ (4.2)

with the macroscopic detection reaction cross section $\Sigma_d$ (e.g. (n,p) in $^3$He counters), the neutron flux $\phi$ and the active detector volume $V_d$.

Based on the point kinetics assumption, moving a detector away from the reactor has two effects: A change in spectrum due to the local difference in moderation ratio, and a strong

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Note that this definition of the intrinsic efficiency is not the standard definition of counts per particle entering the detector volume [154]. Our definition takes the energy of the particle into account and thus is a spectrum independent quantity that can be used for calibration [110].
4.1. Neutron noise field experiments

reduction in absolute flux due to geometric dilution, reducing the efficiency and in turn the correlation amplitude. Our first focus is on the neutron noise field to examine the effects of distance at criticality and sub-criticality on the resulting estimated kinetics. Due to the differing requirements in terms of detection rates we used two different experimental setups, one for critical and one for sub-critical measurements respectively.

4.1.2 Experimental setup

The geometry of the experiments conducted to study the critical and sub-critical neutron noise field is shown in Figure 4.1. The Plexiglas experimental channel for each detector was modified to allow for a displacement in the reflector \cite{130}, pictures of the setup can be found in Appendix I. When setting a distance we moved both detectors symmetrically away from the core.

Figure 4.1  Top view of the critical and sub-critical noise field experiment core configurations for neutron noise. The detector locations are indicated by black circles, specifically: 1a to 1b. Plexiglas experimental channel at different distances to the fuel, holding either CFUL01 FC for the critical experiments, or Helium-3 PC in a polyethylene/cadmium shell. Note that the channel for the He-3 setup is slightly larger in diameter, resulting in different absolute detector distances for the two experiments.
Chapter 4. Experimental study of the branching noise field of CROCS

Table 4.1 Measurement duration, calibrated fission rate, mean signal, and inferred absolute efficiency in counts per fission for each of the critical neutron noise field experiments using CFUL01 fission chambers. Using the respective amplifier’s characteristics, the mean signal in volt was used to calculate the mean current in the FC, yielding an average count rate using manufacturer given sensitivities [155].

<table>
<thead>
<tr>
<th>Distance (cm)</th>
<th>4</th>
<th>7.5</th>
<th>10</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acquisition time (h)</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Fission rate ( \times 10^{10} \text{ s}^{-1} )</td>
<td>2.41</td>
<td>2.44</td>
<td>2.59</td>
<td>2.57</td>
</tr>
<tr>
<td>Mean Signal Box 1 (V)</td>
<td>3.01 ± 4%</td>
<td>2.38 ± 5%</td>
<td>1.81 ± 8%</td>
<td>0.36 ± 12%</td>
</tr>
<tr>
<td>Mean Signal Box 2 (V)</td>
<td>3.79 ± 4%</td>
<td>2.96 ± 5%</td>
<td>2.14 ± 8%</td>
<td>0.41 ± 12%</td>
</tr>
<tr>
<td>Efficiency ( \epsilon_1 ) ( \times 10^{-5} )</td>
<td>7.13 ± 7%</td>
<td>5.57 ± 7%</td>
<td>4.01 ± 8%</td>
<td>0.80 ± 10%</td>
</tr>
<tr>
<td>Efficiency ( \epsilon_2 ) ( \times 10^{-5} )</td>
<td>7.13 ± 7%</td>
<td>5.52 ± 7%</td>
<td>3.95 ± 7%</td>
<td>0.76 ± 12%</td>
</tr>
</tbody>
</table>

4.1.2.1 Critical neutron noise field

The critical neutron noise study was carried out using the CFUL01 fission chambers already presented in Section 3.1.5. Initial tests sought to determine the approximate efficiencies to estimate whether a noise measurement is likely to give a distinguishable response. The CFUL01 showed an efficiency drop below \( 10^{-5} \) at 19 cm from the fuel – we thus increased the measurement time with distance to aid resolving noise metrics. The reactor power was chosen to be similar to the reference measurement, around 800 mW. An increase in power, as shown in Section 2.4, does not increase the correlation amplitude. Higher powers could potentially yield a higher SNR, but at the cost of requiring us to use different amplifiers and the incurring challenge of operating CROCS at higher power. Table 4.1 shows a complete list of distances to the fuel, absolute detector efficiencies and measurement durations for the critical neutron noise field experiment. The signals were sampled at 1 kHz.

4.1.2.2 Sub-critical neutron noise field

CROCS has to date not been studied in its sub-critical states below -20 e[24]. We recall that CROCS' unique design allows having the water level\(^2\) being set manually between 800 and 1000 mm. Changing these parameters, we studied five different water levels \( H \) close to criticality (see Figure 4.3) using five different distances of the detectors to the fuel \( d \) at each water level; a total of 25 experiments were conducted. Going as low as approximately -1.4$ in reactivity, and 20 cm in distance, we hypothesize that any spatial effects are maximized in these states. For the sub-critical experiments we employ more efficient Helium-3 detectors in the same experimental channel to compare the relative expected increase in efficiency.

\(^2\)Note that we mostly indicate the water level instead of a reactivity, as the water level is an experimental quantity, whilst the reactivity requires an estimation method. The reactivities given are predicted with Serpent 2.
Two thin Canberra 12NH25/1 Helium-3 tubes were set into a cylindrical polyethylene block covered by a cadmium layer, cutting the thermal flux and moderating the flux that reaches the detector (see Figure 4.2). The detector setup had two motivations: Firstly, the setup cancels thermal component of the flux. This, hypothetically, would lead to only the fast mode of the noise to be measured [156]. Secondly, the He-3 tubes used were found to be too efficient to be operated at close distances or close to criticality due to dead time. The Cd/PE layer reduced the count rate, and correspondingly the efficiency, by about an order of magnitude. Only very close to critical did we find this setup to be influenced by its dead-time. As the detectors were connected to pulse counting electronics, the analysis methods included variance and covariance to mean (VTM/CTM) estimates next to PSD. The setup for the closest distance to the core is displayed in Figure 4.2. The respective measurement times in dependence of the other parameters are listed in Table 4.2. As a reference for $\alpha$ we will compare the results to explicit predictions using Serpent 2 IFP using JEFF 3.3 (C/E-1). If a systematic deviation was to be found, both a spatial effect or a bias in the IFP calculation could be attributed as the cause. We therefore expand our experiment by measuring the sub-critical $\alpha$ also with the CeBr$_3$ scintillators used in the same experimental setup as for the gamma noise reference, see Section 3.3. This allows us to compare the He-3 tube results to in-core gamma noise measurements at the same reactor configurations ("E/E-1"). As discussed in Section 2.7, we expect a spatial effect to be less pronounced in gamma noise – a similar systematic trend to that of the IFP predictions should thus to be visible relative to the in-core gamma reference.
Figure 4.3 Side view of the sub-critical reactor configurations at 950 mm (left) and 800 mm (right).

Table 4.2 Measurement time in hours for each position and water level $H$ with estimated reactivity using Serpent 2 JEFF 3.1.1 in dollar.

<table>
<thead>
<tr>
<th>Water level (mm)</th>
<th>$\rho (\text{$})$</th>
<th>Distance to fuel (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>-1.4</td>
<td>4.65 6.85 11.2 14.25 20.15</td>
</tr>
<tr>
<td>850</td>
<td>-0.9</td>
<td>2.0 2.0 2.0 2.0 2.0</td>
</tr>
<tr>
<td>900</td>
<td>-0.5</td>
<td>2.9 2.9 2.9 2.9 2.0</td>
</tr>
<tr>
<td>925</td>
<td>-0.3</td>
<td>2.9 2.9 2.9 2.9 2.0</td>
</tr>
<tr>
<td>950</td>
<td>-0.1</td>
<td>2.0 2.9 3.0 3.0 2.0</td>
</tr>
</tbody>
</table>

4.1.3 Comparison to code: Serpent 2 model of the experiments

We chose Monte Carlo calculations using Serpent 2 as the method to calculate k-static estimates of $k_{\text{eff}}$ and IFP estimations of a sub-critical $\alpha$. The critical reference water level for the configuration with the detectors set at 4.65 cm was experimentally determined to be (967.2±0.1) mm. The estimated $k_{\text{eff}}$ corresponds to the absolute reactivity $\rho_{\text{sim}}$ of the simulated system, and not the experimental value $\rho_{\text{exp}}$. An over-prediction of $k_{\text{eff}}$ by about 230 pcm is commonly observed in the Monte Carlo simulation of CROCUS benchmarks with respect to the measured critical state [17]. We will thus compare relative decay constants or reactivities, e.g. as is common practice for rod worth estimations. The IFP-$\alpha$ is calculated using

$$\alpha = \frac{(\rho_{\text{corr}} - \beta_{\text{eff}})}{\Lambda}$$

(4.3)

and

$$\rho_{\text{corr}} = \frac{1}{k_{\text{eff}}(967.2 \text{ mm})} - \frac{1}{k_{\text{eff}}}$$

(4.4)

$k_{\text{eff}}$ being the k-static estimate yielded by the Monte Carlo code.
Figure 4.4 — Examples of measured CPSD and CTM curves for different water levels and distances in CROCUS. The CPSDs are normalized to their respective low frequency plateau average (2-5 Hz).

4.1.4 Analysis of experiments

4.1.4.1 Calculation of VTM/CTMs and APSD/CPSDs

The analysis of the sub-critical signals was performed using the PSD and Feynman-α methods. Sample curves of CPSD and CTM curves for the different water levels are displayed in Figure 4.4.

The PSDs were calculated using the method outlined in Appendix G.3. For all experiments the segment length was chosen to be $2^{13}$ or $2^{14}$ using a Blackman-Harris window with no overlap to achieve approximately 0.1 Hz spectral resolution. The VTM/CTM were calculated using a summed-bunching-technique to reduce the relative noise by the intrinsic Poisson-like source. The whole signal was hereby split into segments of equal length corresponding to a given gate duration. These segments were then added together and their (co-)variance and mean calculated.

4.1.4.2 Discussion of anomalies

The experiments performed were subject to noise and anomalies that required non-automatized treatment. For example, several experiments showed up to 1s long spikes of up to four order of magnitude in count rates. These are usually attributed to high voltage instabilities, but could also be caused by grid instabilities, WiFi, mobile network, etc. These spikes were filtered out of the respective signals, as they significantly influenced the results. In order to ensure further time synchronization of the two detectors for cross-correlation, any time segment that was cut-out from one signal was also removed in the other.

We also examined the dead time effects at higher relative count rates. An example of signals influenced by dead time is displayed in Figure 4.5.
Figure 4.5 Comparison of APSD/CPSD and Feynman-$\alpha$ VTM/CTM curves at different water levels at 4 cm distance for anomaly illustration. The CPSDs are normalized to their respective low frequency plateau.
In the APSD 2 signal, an anomaly causing high frequency noise (> 100 Hz) and distorting the PSD shape can be seen at 950 mm. The APSD 1 signal is also affected, but appears to only exhibit an increased white noise plateau. Also as discussed in [157], the cross correlation filters these individual effects out and yields the expected shape for the PSD. The cause of this effect can be found in the VTM curves: At 950 mm we observe a starting point below 1 for the curves at closer distances. This is usually attributed to dead time and can be corrected [61] by allowing a subtracted constant as additional fit parameter. The CTM curves, unlike the PSDs, appear to retain a portion of the noise in the shape of a linearly increasing ratio above the gate time around \(3 \times 10^{-2}s\). We conclude that the dead time also affects the CTM, yet cannot be as trivially corrected for.

The Feynman-\(\alpha\) curves overall exhibit a higher sensitivity to unwanted noise and are more inconsistent. For example, we observe a large difference in the VTM ratio between the two detectors at 925 mm with no clear causal link. Overall, we will focus the analysis of experiments on the cross correlation (CPSD and CTM), as they will provide the least amount of bias induced by the individual electronics.

### 4.1.4.3 Model fitting

As the experiments use water levels as low as 800 mm, which Serpent 2 predicts to be as low as -1.4\$ in reactivity, we entertain the possibility of non-negligible higher alpha modes, clustering, or unknown spatial effects specific to CROCUS. The VTM/CTM and APSD/CPSD curves were fitted both with single exponent and two exponent models, labeled henceforth SE (for single exponent) and ME (for multi exponent) respectively, to allow for the additional degree of freedom to account for these effects [8]. The starting point of \(\alpha\) was set via Serpent 2 predictions to be able to converge below \(10^4\) iterations. The second exponent of the ME fit, denoted \(\alpha_2\), was found to not be sensitive to its or other starting points. We explicitly compare SE and ME models and their effects on the fit in Section 4.1.5.2 when discussing the sub-critical experiment results.

### 4.1.5 Results and discussion

#### 4.1.5.1 Critical neutron noise field

The PSDs of the CFUL01 signals were calculated using a window size of \(2^{13}\) to achieve a spectral resolution of approximately 0.1 Hz for each measurement. The resulting APSDs and CPSDs at each distance are shown in Figure 4.6. We determined the prompt decay constant and the effective delayed neutron fraction for each distance, as listed in Table 4.3. The uncertainties given are first order approximations, see Section 3.1.3. Visually we are able to clearly discern a flattening of the PSD shape with distance, as expected from the point kinetics hypothesis. We further observe that at 19 cm the shape is visually indistinguishable from a constant – indicating why the algorithm was able to fit a Lorentzian with reasonable accuracy.

The results agree within \(1\sigma\) to \(2\sigma\) for \(\alpha\) with the reference measurement of Chapter 3. The variation appears typical for noise experiments, and points to a valid point kinetics
Figure 4.6  PSDs of CFUL01 fission chambers displaced in the CROCUS reflector away from the fuel. At 19 cm the PSD shape visibly flattens out, giving an indication of neutron noise limits due to the loss of efficiency.

Table 4.3  Critical kinetics parameters determined with CFUL01 fission chambers at different distances to the fuel. The listed uncertainties are first order approximations, neglecting the 2% uncertainty due to fission rate and $D_o$ determination, and are likely underestimated.

<table>
<thead>
<tr>
<th>Distance (cm)</th>
<th>4</th>
<th>7.5</th>
<th>10</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{APSD1}$ ($1/s$)</td>
<td>155.1±2.1</td>
<td>159.5±2.3</td>
<td>151.2±4.5</td>
<td>150±94</td>
</tr>
<tr>
<td>$\alpha_{APSD2}$ ($1/s$)</td>
<td>162.7±2.2</td>
<td>154.6±3.1</td>
<td>153.7±7.2</td>
<td>150±147</td>
</tr>
<tr>
<td>$\alpha_{CPSD}$ ($1/s$)</td>
<td>151.3±2.0</td>
<td>152.1±1.7</td>
<td>153.6±5.2</td>
<td>157±253</td>
</tr>
<tr>
<td>$\beta_{APSD1}$ (pcm)</td>
<td>751±16</td>
<td>761±17</td>
<td>772±30</td>
<td>820±153</td>
</tr>
<tr>
<td>$\beta_{APSD2}$ (pcm)</td>
<td>789±15</td>
<td>788±28</td>
<td>787±18</td>
<td>925±270</td>
</tr>
<tr>
<td>$\beta_{CPSD}$ (pcm)</td>
<td>750±8</td>
<td>799±12</td>
<td>794±14</td>
<td>2005±1400</td>
</tr>
</tbody>
</table>
response for distances up to 10 cm from the fuel. The agreement for $\beta_{\text{eff}}$ fluctuates more, with it being increasingly over-estimated at higher distance. This is however consistent with the overall spread of measurements of $\beta_{\text{eff}}$ using other detectors [23], [24]. The kinetics parameters determined at 19 cm all display relative uncertainties above 50%, with the mean values converging close to the starting point – in this range the estimates can be deemed unreliable and we find neutron noise measurements to be ineffective at 19 cm. Note that CFU01 fission chambers are comparatively high efficiency detectors and in the list of available neutron detectors are only second to Helium-3 PC.

### 4.1.5.2 Sub-critical neutron noise field

An example of a typical data set after CTM and CPSD estimation of the He-3 signals is displayed in Figure 4.7, with the results for $\alpha$ and $\alpha_2$ depicted alongside. In analyzing the data of the sub-critical experiments, we noticed strong disagreement between the Feynman-$\alpha$ and PSD estimates when using single exponent fits. We thus first compare SE and ME fits with their respective residual plots. We summarize our findings:

- A SE fit for CTM clearly does not minimize onto the curve as expected or a residual distribution that a comparative likelihood estimation estimates as normal. This

![Figure 4.7 - Example of curve fit results on measurement data with residuals for SE and ME fits for CTM and CPSD. For this example we display the data for He-3 placed at the closest distance (4 cm) at 900 mm water level.](image-url)
results in a under-prediction of $\alpha$ and apparent under-estimated uncertainties. This is on top of the inherent variance estimation bias that we are likely to encounter when comparing VTM/CTM due to heteroscedasticity seen in the residuals.

- A ME fit for CTM improves this considerably: We find a normal distribution in the residuals, i.e. the fit is visibly minimized onto the curve. In this specific example, the ME fit is not able to capture $\alpha_2$ with less than 100% uncertainty.

- The starting point of $\alpha_2$ did not change its final estimation. Also, we were not able to fit fast decay modes ($> 1000 \text{ l/s}$), indicating no measurable effects of higher modes. We infer from this that the effect we fit is indeed related to electronic noise.

- The CPSD is less sensitive to the fit model, and yields $\pm 1\sigma$ consistent results for either SE or ME fits. The uncertainties are comparable to the CTM ME case.

- $\alpha_2$ was also found using the CPSD, albeit mostly with $> 100\%$ uncertainties.

The general behavior of $\alpha$ with water level and fit model is shown in Figure 4.8, also contrasted with the results of IFP predictions for $\alpha$ given by the Serpent 2 model using JEFF 3.3.

We confirm that CTM with SE models largely mis-estimate $\alpha$ and yield underestimated uncertainties. The ME models for both CPSD and CTM yield overall consistent results. Note that the minor apparent heteroscedasticity, even in the PSD residuals, encourages us to bootstrap uncertainties - due to the computational cost we will however use only first order approximations and favor CPSD over CTM estimates.

**Comparison to code predictions and gamma noise** One of our comparison metrics is the in-core gamma noise reference, we therefore first display the C/E-1 results for the gamma noise CPSD results of $\alpha$ for all water levels compared to Serpent 2 IFP in Figure 4.9. We find 1 to 2$\sigma$ agreement with no systematic disagreement.
4.1. Neutron noise field experiments

In Figure 4.10 we display the C/E-1 ratios of the He-3 estimates to the Serpent 2 predictions, alongside their relative standard deviations inferred from first order approximations of the covariance matrices, and the C/E-1 z score. Most of the 25 experiments fall into a ±5% range of agreement with code predictions - the larger deviations being at large distances and low water levels. This is reflected in the relative uncertainty, with the "bottom left" corner of the parameter space displaying the highest uncertainties. Both CTM and CPSD estimates achieve z scores below 1 (i.e. the C/E-1 discrepancy is within one standard deviation) for several experiments, but an overall discrepancy with an average.

Furthermore, we find that large deviations from code predictions are, except for the 800 mm, 19 cm case, only inherent to a single method and thus weaken the hypothesis of strong spatial effects. The high efficiency of the $^3$He detectors compared to previous experiments allows to set a more refined practical limit for neutron noise, notably reactivity dependent. With increasing reactivity the range of successful noise measurements increases.

We next attempt to discover a weak spatial effect, i.e. to discuss whether discrepancies from code arise systematically. As can be seen in a C/E-1 display of all 25 experiments in Figure 4.11, a general trend of the CPSD underestimating the IFP predicted $\alpha$ can be attested to. With increasing distance at a given reactivity, we are able to observe a slight linear decreasing trend, as indicated by the black lines. When plotting the same data in dependence of distance, neglecting CTM due to the aforementioned biases, the effect is more visible: An up to -5% bias at higher distance arises when moving the detector away from the fuel. The uncertainties at this distance are nonetheless important, and this observation in itself could also point to a bias in the calculation of $\alpha$ with increasing sub-criticality.
Chapter 4. Experimental study of the branching noise field of CROCU5

Figure 4.10  **Top:** C/E-1 for $\alpha$ yielded by Serpent 2 IFP using JEFF 3.3 relative to CPSD or CTM analysis for the He-3 detectors. **Middle:** First order approximations of the relative standard deviations of the experiments E. **Bottom:** z score for C/E-1, i.e. the distance of a discrepancy between calculation and experiment in multiples of the C/E-1 standard deviation.
4.1. Neutron noise field experiments

Figure 4.11 C/E-1 results comparing the prompt decay constant from all 25 sub-critical experiments to Serpent 2 IFP predictions using JEFF3.3, sorted by reactivity (top) or distance (bottom). A systematic trend amounting up to -5%, indicating a spatial effect or calculation bias.

We further compared the prompt decay in dependence of distance to sub-critical gamma noise measurements in-core using the CeBr$_3$ detectors, see Figure 4.12. A similar, in this case +5% systematic trend can be seen, albeit again with important uncertainties. Using the z scores, we cannot reject the point kinetic hypothesis within 1σ to 2σ for most measurements. Since we are using first order approximations of the uncertainties (i.e. assuming normality), we furthermore expect the real uncertainty bounds to be higher or skewed, and more significantly include the point kinetics hypothesis and render a z score less meaningful.

4.1.6 Summary of neutron noise field study

Critical noise field  Within 2σ of the measurement uncertainty, we conclude that point kinetics offer a reliable estimate for the critical kinetics of the system up to 10 cm distance from the fuel. The effective delayed neutron fraction β$_{eff}$ was successfully determined at distances up to 10 cm, agreeing within 1σ with Serpent 2 IFP code predictions using JEFF3.3 with the uncertainties estimated in first order approximation.

At 19 cm the resulting low efficiency implied the need for longer measurement times. Although measuring for 6 h, β$_{eff}$ could only be determined with a high uncertainty and strongly overestimated mean value. The efficiency is too low at this distance, so that
Figure 4.12  $E_{\text{He-3}}/E_{\text{CeBr}_3}^{-1}$ results comparing the prompt decay constant from all 25 sub-critical experiments to in-core gamma noise sorted by distance of the He-3 experiment to the core. Given that we observe a similar systematic effect of up to 5% with distance as with neutron noise compared to IFP, we may infer a weak spatial effect.

fit convergence becomes unlikely. We therefore find an experimental limit of absolute efficiency of $4 \times 10^{-5}$, corresponding to 19 cm distance, for large fission chamber for noise measurements in CROCUS.

Sub-critical noise field  CROCUS' sub-critical kinetics were successfully measured at configurations corresponding to about -1.4\$, agreeing well with Serpent IFP predictions using JEFF3.3, using both neutron and in-core gamma noise. We conclude that noise measurements in CROCUS can be effectively fitted by ME models, but mostly to account for strong biases probably stemming from electronic noise. However, despite a small systematic trend of up to 5% confirmed in two different ways by comparing to IFP and gamma noise, we cannot reject the point kinetics hypothesis in CROCUS in first order uncertainty approximation, and find that the second exponent cannot be interpreted systematically.
4.2 Gamma noise field experiments

We next apply ourselves to the gamma noise field. The gamma field is theoretically more vast than the neutron noise field, i.e. yields a distinguishable correlation at further distances, and less prone to potential spatial effects. Gamma noise could thus be used to measure kinetic parameters of a reactor where neutron noise is not efficient enough, e.g. due to restricted access to the core or vessel.

Initial calculations using a simplified photon flux efficiency estimated that ex-vessel measurements are unlikely to yield a usable result \[131\]. To test these hypotheses experimentally, we used the BGO detectors of the LEAF system. With their size and scintillator typing (high Z) they are designed to count almost every photon that enters the crystal volume.

4.2.1 Experimental setup

The BGO detectors, as indicated in Section 3.1.5, showed an absolute efficiency of the order $10^{-4}$ events per fission despite being set against the vessel, approximately 60 cm further away than the CFUL01 at maximum distance. This was in itself an unexpected result, and allowed the measurement of the prompt decay constant outside of the reactor vessel. We thenceforth tested the correlation amplitude at even larger distances, as shown in the final experimental location map for the BGOs in Figure 4.13. Pictures of some setups are shown in Appendix I. The final position, designated number 11 at around 6.77 m distance to the core center, was the farthest location achievable within the CROCUS facility with a direct line of sight to the core center. In order to achieve this direct line of sight, the reactors irradiation channel in the west side of the concrete cavity was opened. The channel is approximately centered at mid core height. We note that the channel acts as a collimator, possibly reducing random scattering\(^3\). The mapping of the correlation amplitude was mostly conducted at 900 mm water level, i.e. a sub-critical state of 0.5\$, to minimize the time periods between experiments due to in-cavity dose rates and thus to maximize the parameter space covered. The resulting tests were carried out for 30 min each, enough to determine an efficiency but not an accurate prompt decay. We nonetheless can give qualitative estimates of when a correlation amplitude was non-trivial, listed with the measurement times and reactor states in Figure 4.14. The signals were sampled at 2 kHz.

4.2.2 Results and discussion

4.2.2.1 PSD amplitude of gamma noise with distance

In Figure 4.15 we compare the APSD and CPSD amplitudes with increasing distance to the core. Note that the spectra are smoothed with a moving average filter for illustrative purposes. Evidently, we observe the same behavior already remarked in Section 3.3.4

\(^3\)Given that photon transport occurs on a much shorter time scale than neutron transport, we expect even scattered photons to carry the relevant neutron time information.
Chapter 4. Experimental study of the branching noise field of CROCUUS

Figure 4.13  Top view of the BGO detector locations for the gamma noise field characterization. The numbers refer to a specific measurement, e.g. experiment 6 was conducted with one BGO in the cavity irradiation channel and one in the NW corner of the cavity.

Figure 4.14  Overview of the measurement duration for the gamma noise field characterization. Due to time restrictions the measurements were conducted for 30 min to qualify the responses. Red colors indicate a flat or noisy response curve that did allow for a converged fit. Green indicates a benchmark measurement for validation.
4.2 Gamma noise field experiments

Figure 4.15 Comparison of the measured BGO APSD (left) and CPSD (right) amplitudes with distance to the core. The reactor was in a sub-critical state with source at 900 mm, corresponding to approximately -0.5. The curves were smoothed with a moving average filter over a span of 100 points for illustrative purposes. The observed effect is similar to that of increasing the LLT, see Section 3.3.4. The APSD amplitude decreases with distance, hypothesized to be due to the geometric dilution and the incurring loss of efficiency. The CPSD amplitude increases at first with distance, but reaches a maximum at \( \sim 3.20 \text{ m} \) and decreases thereafter.

When discussing the effect of LLT on gamma noise responses. With increasing distance the APSD decreases with the loss of efficiency due to geometric dilution\(^4\). The CPSD exhibits a trade-off response: At first, until about 3.2 m distance to the core, the amplitude increases linked to our hypothesis that low energy events are less likely to reach the detector farther away, see Section 3.3.4. After this approximate optimum at 3.20 m distance, the CPSD amplitude decreases. Here the loss of efficiency weighs stronger than the relative gains in correlation, supporting our initial hypothesis.

We also observed the co-variance to mean (CTM) and variance to mean (VTM) responses with distance, shown in Figure 4.16 – they were used as the preliminary indicator for correlation. The CTM does not show the trade-off behavior. This could be explained by the fact that electronics noise exhibits itself as a linear contribution to the overall shape, as shown in Section 4.1.4.2, changing the amplitude and not allowing for a conclusion similar to the PSD observations.

4.2.2.2 Ex-vessel prompt decay constant reference

Similar to our in-core gamma noise reference using the CeBr\(_3\) detectors, we present a reference determination of \(\alpha\) for the BGO measurements at position 1, next to the core vessel at about 0.9 m from the core center. The measurement duration was also 2 hours, at

\(^4\)The more common description would be the inverse square law of rays. We will nonetheless refer to geometric dilution, as the inverse square law is specific for point sources.
Chapter 4. Experimental study of the branching noise field of CROCUSS

Figure 4.16 Comparison of VTM and CTM curves obtained from the $\gamma$ noise field experiments.

Figure 4.17 Probability distribution of the bootstrapped kinetics parameters estimated for the BGO gamma noise benchmark measurement at 0.9 m distance to the core.

criticality with a reactor power of roughly 20mW. The resulting bootstrapped distributions are displayed in Figure 4.17. Despite the large distance to the core compared to neutrons, we find that the uncertainties are of the same order as the gamma noise reference for APSD2. The broad distribution of $\alpha$ estimated via APSD1 indicates a problem in the electronics of the detector, which was tested to indeed induce unexplained electronic noise from the PMT. The CPSD is thus biased as well, albeit that the mean value for $\alpha$ is determined consistently, as listed in Table 4.4.

4.2.2.3 Far field correlation validation

Given that correlations could be detected at the farthest position with a direct line of sight to the core, we conducted a far field benchmark measurement of 2 hours duration to attempt to determine the kinetics parameters at 6.77 m distance. In Figure 4.18 we directly contrast the CTM curves for the close position outside the vessel and the far field location. The numerical values are listed in Table 4.4. Given the computational cost, only the PSD uncertainties were bootstrapped. The PSD and (co-)variance to mean results for ex-vessel overall agree well within 1 to 3$\sigma$ with the in-core benchmark. Note that the VTM for BGO 1 was found to be strongly influenced by a linear noise contribution that the cross
4.2 Gamma noise field experiments

Figure 4.18 Comparison of γ CTM curves after 2 hours of measurement of a critical configuration of CROCUS at 20 mW, at the reactor vessel (left) and at the far end of the reactor hall (right).

Table 4.4 Critical kinetics parameters determined with LEAF scintillators at different distances to the reactor. The listed uncertainties are first order approximations for the VTM/CTM results, whilst the PSD results are bootstrapped. GEV refers to Generalized Extreme Value, WBL to the Weibull, TLOC to the location-scale, N to the normal distribution.

<table>
<thead>
<tr>
<th>Position Distance (m)</th>
<th>α in-core (CeBr$_3$)</th>
<th>α out of vessel (BGO)</th>
<th>α far field (BGO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α$_{APSD1}$</td>
<td>GEV(159.2,2.1,-0.23)</td>
<td>WBL(154.4,1.9)</td>
<td>N(130,65)</td>
</tr>
<tr>
<td>α$_{APSD2}$</td>
<td>GEV(159.5,2.1)</td>
<td>TLOC(151.3,1.5,1.0)</td>
<td>N(204,140)</td>
</tr>
<tr>
<td>α$_{CPSD}$</td>
<td>GEV(154.8,2.5,-0.45)</td>
<td>WBL(153.4,2.7)</td>
<td>GEV(194,90,0.4)</td>
</tr>
<tr>
<td>α$_{VTM1}$</td>
<td>N(156.3,1.8)</td>
<td>N(47.6,3.0)</td>
<td>N(188,70)</td>
</tr>
<tr>
<td>α$_{VTM2}$</td>
<td>N(157.4,1.6)</td>
<td>N(157.9,2.4)</td>
<td>N(264,120)</td>
</tr>
<tr>
<td>α$_{CTM}$</td>
<td>N(150.8,1.2)</td>
<td>N(156.2,0.2)</td>
<td>N(167,53)</td>
</tr>
</tbody>
</table>

correlation notably filtered out. This is consistent with our observation of a wide spread in the APSD1 values of the reference measurement. The far field measurements gave overall results agreeing within 1σ with the benchmark, albeit mainly due to the large uncertainties above 50%. We are able to conclude, that gamma noise measurements are indeed capable of capturing the core behavior from much farther than neutron noise. In the closest tested position at 0.9 m ex-vessel the BGOs outperform the neutron noise reference measurements in accuracy.

Summary of gamma noise field The BGO detectors were able to observe correlations at unprecedented distances to the core, with the closest tested location at 0.9 m already outperforming the benchmark of the neutron based measurements in terms of precision. Due to the close agreement with the gamma benchmark with regards to α, we can state that gamma noise at around 1 meter distance to the core can observe a point kinetics reac-
tor in the most non-invasive manner. As spatial effects are expected to be less pronounced with gamma noise measurements, we are able to give experimental evidence to substantiate these theoretical claims. Furthermore, the limitations of gamma noise within CROCUS were tested. We found that correlations in certain sub-critical states are not visible after a given distance - indicating a trade-off due to the lower event rate at sub-criticality. At the farthest position the BGOs could be installed, still in line of sight of the reactor at almost 7 meters distance, we were able to distinguish correlations in the signal and extract $\alpha$, albeit with large uncertainties. These results allow us to set practical limits to gamma noise within CROCUS, yet we could not achieve a distance high enough to not distinguish the correlations. This is particularly encouraging as a result with regards to other applications of noise measurements, indicating a strong possibility of ex-core, ex-vessel and generally far-field measurements to be useful.

4.3 Normalization of reaction rate maps: Geometric efficiency as driver of noise success

For general experimental design purposes, we determined the reaction rates of common detection materials (namely $^3$He, $^{235}$U, and $^{10}$B) in arbitrary locations of the CROCUS reactor. The focus was hereby to allow for an estimation of expected efficiencies in dependence of detector material amount and location. Horizontal and vertical plane mesh grid detectors were hereby used in Serpent 2 to map the entire core of CROCUS. Using Equation 4.1, we calculated absolute efficiencies given previously calibrated intrinsic efficiencies [110]. The resulting maps are displayed in Figure 4.19 page 95.

The color map was chosen to reflect the approximately exponential nature of decrease of geometric efficiency with distance. Black regions reflect an efficiency below $10^{-6}$, the empirical limit below which a noise measurement is likely to not distinguish correlations from noise or require unreasonable measurement times [70]. The neutron noise limits for a Helium-3 based system are the farthest, as expected from their higher sensitivity. We overall observe a drop in efficiency by about one order of magnitude every 10 cm from the fuel - consistent with our experimental observations. Nonetheless, our Cd/PE Helium-3 is limited to measurements within about 50 cm to the core center, or about 20 cm away from the fuel. A Helium-3 without shielding could potentially allow for measurements 10 cm further, given that the efficiency is expected to rise by an order of magnitude. This distance of 60 cm to the core just falls short of the vessel boundaries of CROCUS (65 cm). At this distance the perturbation on the neutron population can be considered negligible, but the accuracy of a noise experiment is likely to be poor. Our tested BGO, with an efficiency of $10^{-4}$ at 90 cm to the core center, clearly outperforms Helium-3 in both non-invasiveness and accuracy.

94
4.3. Normalization of reaction rate maps: Geometric efficiency as driver of noise success

Figure 4.19  Normalized total efficiency (Counts per fission) for different detection materials in the CROCUS reactor. The intrinsic efficiency was estimated using Serpent 2 reaction rates and compared to experimental efficiency.
4.4 Chapter’s salient elements

The experimental study of the noise field in CROCUS was motivated by several reasons: Firstly, to prove or disprove a point kinetics hypothesis by attempting to maximize any spatial effect by investigating sub-critical states and higher distances to the core. Secondly, to study the differences between gamma and neutron noise in terms of overall performance, and thirdly, to provide data that can be used for code validation.

- The neutron noise field of CROCUS, studied at critical and sub-critical states, displayed a point kinetics behavior throughout. High efficiency fission chambers could not distinguish correlation from noise when located above 19 cm away from the fuel.

- The sub-critical noise field experiments showed a point kinetics behavior at close distances, and a weak systematic effect at higher distances. Due to the high first order uncertainties, we conclude that CROCUS can be reasonably approximated with point kinetics, even in reactor configurations down to -1.4$ and distances of up to 20 cm to the fuel.

- The gamma noise field of CROCUS was studied using two large BGO scintillators that showed a higher than expected noise response outside of the reactor vessel, at 60 cm from the fuel. The behavior of the CPSD correlation amplitude with distance was shown to behave similarly to LLT experiments, notably by displaying an initial increase with distance before decreasing after around 3.2 meters distance to the core. This effect provides evidence for the hypothesis regarding LLT settings of gamma noise, stated in Chapter 3.

- Using gamma noise, the prompt decay constant was measured with reference precision outside of the reactor vessel, and comparing well to IFP code predictions. We substantiate our hypothesis that gamma noise is indeed an effective tool, if not more than neutron noise, for kinetic parameters measurements.

- We were furthermore able to show that the gamma noise field extends as far as direct line of sight to the reactor is preserved, giving a successful correlation measurement at almost 7 meters from the core center.
Chapter 5

Monte Carlo simulation of branching noise

In this chapter we present the motivation, implementation, and validation of Monte Carlo based schemes to simulate noise experiments. The purpose is twofold. We firstly desire to produce predictions for noise responses that require less assumptions than e.g. point kinetics, i.e. a model agnostic approach. Secondly, given the amount of noise experiments performed at different sub-critical states, for different detector locations, and for different particle types, we hope to provide enough data to validate the code implementing this approach and subsequently use of it for arbitrary geometries.

In Section 5.1 we introduce the history and recent developments of Monte Carlo calculations in neutronics for noise, and discuss why Monte Carlo methods are indeed a suitable candidate for noise simulations. Then, in Section 5.2, we present the explicit methodology for noise simulations in Monte Carlo by verification of two codes, namely TRIPOLI-4 and Serpent 2. We attempt to then validate both codes by comparing the results to the sub-critical experiments, Section 5.3. In Section 5.4, we discuss the simulation of a decoupled reactor system and how noise methods could potentially allow for efficient analysis of its kinetics. We finally conclude in Section 5.5.

5.1 Monte Carlo methods for noise simulation

The results of noise measurements can directly be used for validating Monte Carlo (MC) methods designed to predict kinetic parameters, such as standard k-static estimates of $k_{\text{eff}}$ or IFP – as is standard for common neutronics MC codes such as TRIPOLI-4 (T4) [43], MCNP6.2 [158] or Serpent 2 (S2)[150].

These codes are designed to yield quantities that relate to the Boltzmann formulation of neutronics, i.e. fluxes, reaction rates, etc. and do not offer a simple option to simulate noise. MC particle transport is nonetheless a close representation of the actual underlying physics. The fission chains are simulated directly and as such the simulation captures explicitly the time fluctuations of neutron interactions. This means that noise measurements
could be explicitly simulated by registering the time stamps of interactions of interest, such
as fission or detection reactions, e.g. \((n,p)\) for He-3 tubes. The registering of time stamps
is agnostic to models such as point kinetics, and as such should yield accurate results. A
user could hypothetically be able to predict kinetic parameters or \(k_{\text{eff}}\) without needing
IFP, k-static or other methods of prediction - just by using noise analysis.

Simulating neutron noise has been of interest since the inception of Monte Carlo methods
[159], with its applications being either in multiplicity counting for safeguards and nuclear
weapons or in noise measurements, specifically sub-critical reactivity assessment. First
more widely available codes were KENO-NR [160] and MCNP-DSP [161], using analog
transport and power spectral density analysis methods. The validation was mainly per-
fomed for coincidence measurements [162] and Cf-252 driven experiments [160]. A newer
MCNP based code that incorporates physical fission models for neutrons and photons as
well as truer time treatment of emissions is MCNP-PoliMi [163] - validated again for mul-
tiplicity experiments of Pu-spheres and \(\text{UO}_2\) samples [164].

With the theoretical considerations of an Accelerator Driven System (ADS) reaching pop-
ularity [165], [166] the interest in studying sub-critical systems was followed by further
research into the applicability of noise methods in Monte Carlo for sub-critical systems. A
first look into non-analog variance reduction methods for noise calculations in Monte Carlo
was undertaken by Yamamoto [167], concluding that \(\alpha\) is unaffected by weighting meth-
ods, but that the amplitude terms (used for \(\rho\) and \(\beta_{\text{eff}}\)) are if only a single detector is used.

The PhD thesis of Szieberth [168] has a preliminary study on how to calculate noise curves
from MCNP compared to experiments [169], but are not-optimized and only preliminary
with unexplained discrepancies. Sub-critical simulations of the Indian reactor BARC [170],
with subsequent measurements [171] attempt to further verify the method. They compare
diffusion based kinetic parameters and multiplications to the noise measurements, similar
to experiments in CROCUS [24] or MINERVE [58].

In the KUCA sub-critical facility, a study to validate kinetic parameters from determinis-
tic and Monte Carlo codes on sub-critical experiments was undertaken [172]. The study
presents experimental count rate decay curves that are compared to MCNP6 time stamp
data and deterministic codes (ERANOS, VARIANT and PARTISN). The experimental
setup was a proton beam pulse of 100 \(\mu\)s width on an assembly at \(k_{\text{eff}} \sim 0.97\). The de-
tectors then measured the decay in neutron population in the time domain using counting
electronics. MCNP6 was patched in order to correctly reflect the time of arrival in a
detector, with the time stamps being collected via F8 or PTRAC cards i.e. comparing
Rossi distributions in addition to results for \(\alpha\) and \(k_{\text{eff}}\). The study was able to reproduce
the pulse decay within typical uncertainties in MCNP6. These calculations were however
focused on \(\alpha\) analysis of the actual decay slope, and not on steady state source driven
experiments.

A new study [173] complements the previous pulse decay noise analysis with steady state
and pulsed source driven experiments that were also directly simulated using MCNP6 time
5.1. Monte Carlo methods for noise simulation

stamps to produce simulated Rossi and Feynman curves. To our knowledge, this is the first study to actually compare noise experiments to noise simulations - a set of 9 experiments using a pulsed proton source were simulated with good agreement for $\alpha$ reported between MCNP time stamps and experimental results using Rossi, Feynman, and PSD methods. $k_{eff}$ was hereby compared to experiments by introducing a correction factor to account for source biases, since the KUCA facility at $k_{eff}=0.97$ could not be calibrated at criticality.

As presented in Chapter 3, the CROCU5 reactor noise field was analyzed in a wide range of sub-critical and critical states that offer the potential to serve as validation data. In this chapter we aim to expand on this previous knowledge. The motivations can be summarized to be as follows:

- The study of sub-critical experiments and their direct Monte Carlo simulations were studied in KUCA in pulsed proton source configuration - the data from CROCU5 is "classical" sub-critical source driven noise measurements and thus provides another angle for validation.

- The MCNP6 simulations of KUCA required a patched version to use F8 and PTRAC tallies to save time stamps at higher than millisecond precision. We will show an alternative that is potentially more flexible and more general in its validity to produce time stamps, in this case in T4 and S2.

- Only MCNP-PoliMi uses a more sophisticated model for fission. The simplified fission treatment in MCNP6 to simulate the KUCA experiments is potentially to be improved by allowing for such a fission model. FREYA, a method of the LLNL fission library, is a way to allow for a "standard" Monte Carlo code such as T4 to access such a model and assess its impact on noise simulations [174].

- The experiments in CROCU5 were also undertaken at criticality. Simulations of critical configurations for noise purposes have not been tested to our knowledge.

Based on these motivations, we formulated overall our research queries:

- Use T4 and S2 to model a set of noise experiments conducted in CROCU5 – the former as reference being an already established code with FREYA coupling, the latter to allow for code-to-code comparison;

- Establish a method on how to produce noise data to be used for validation - based on the hypothesis that analog Monte Carlo transport coupled to a fission library such as LLNL FREYA is required to achieve the best results;

- Validate the method with experimental data and conclude on the applicability of the method;

- Allow for the modeling of exotic configurations, such as a CROCU5 core that is split by moderator to probe for beyond point kinetics behavior. The experiments in CROCU5 and KUCA are sufficiently explained by point kinetics. The noise method could allow access to higher modes of $\alpha$ or a simpler way to assess the criticality of a decoupled system using noise methods.
5.1.1 Monte Carlo for neutronics

Monte Carlo methods in neutronics are used to microscopically simulate a given amount of individual neutron histories by imposing only interaction and transport physics of a neutron. Using specific so-called scoring methods one is able to infer ‘macroscopic’ transport quantities such as flux or reactivity from the bulk behavior. Nuclear data are used to sample for the different parameters such as the cross section, the average amount of neutrons emitted per fission $\bar{N}$, the energy of fission neutrons, etc. In this context we next distinguish the analog from the non-analog treatment of interactions.

5.1.2 Analog Monte Carlo transport

Analog transport is likely the most intuitive implementation of Monte Carlo transport. Single neutrons are initialized with angle and energy, referred to as source definition, and then transported through a given geometry, using transport laws following from the definition of cross section. Interactions are simulated closely to actual physics, i.e. absorption leads to the end of a neutron history. If a fission is sampled, the neutrons energy and angle is re-sampled from nuclear data or a given fission model and any additionally sampled neutrons are added to a to-be-transported list. The energy is typically sampled from a Watt spectrum and the angle by assuming isotropy.

5.1.3 Implicit capture Monte Carlo transport

Analog transport is, however, computationally expensive. Implicit capture refers to the method most commonly employed in Monte Carlo transport codes used for computational optimization and can be seen as an omnipresent variance reduction technique. Every tracked neutron receives an associated weight (usually a neutron is born with weight 1). Instead of removing a neutron in a physically removing interaction, such as absorption or fission, the weight is reduced by the non-absorption or non-fission probabilities. Neutrons are now only truly removed below an arbitrary weight cut-off, energy cut-off or upon leaking from the geometry. This forced survival perturbs the local physical fission chain behavior, but allows for the variance of scores of interest (flux, current, reaction rate, etc) to be effectively reduced. Global average quantities of interest such as reactivity and fluxes are preserved and computationally more efficiently achieved in desired bounds. In the context of this work, we will refer to implicit capture as Non-Analog calculations as opposed to pure Analog.

5.1.4 Fission neutron sampling

In addition to implicit capture perturbing the actual physics, the manner in which fission neutrons are sampled is likely also to affect the trueness of the simulation.

In most established MC transport codes, the number of neutrons emitted per fission $\nu$ is determined by taking the floor function of the nuclear data average $\bar{\nu}$, and then sampling the removed decimal fraction to decide whether to add another neutron or not. In $^{235}\text{U}$ fission from JEFF 3.3, for example, this means that always two or three prompt neutrons
5.1. Monte Carlo methods for noise simulation

Figure 5.1  Example of fission neutron yield for an analog calculation using TRIPOLI4.

are emitted per fission. Delayed neutrons are sampled among these prompt neutrons from
given delayed to prompt ratios. The neutrons’ angles and energies are hereby sampled
independently, i.e. not taking correlations into account.

More sophisticated fission libraries to sample a more physical amount of fission neutrons,
as well as their correlated angles and energies, exist for analog calculation applications.
The resulting fission yields, albeit approximately giving the same average amount of
neutrons per fission, exhibit a broader distribution from 0 to up to 10 neutrons per fission. As
additional hypothesis, one may assume the fission model to affect the fission chain behav-
ior. In this work we will use the LLNL fission library with FREYA fission event treatment
[175] to investigate the differences. A coupling of this fission library with T4 has already
been implemented and tested for a spontaneous fission source model [174], and we will use
T4 as our main reference. In the context of this work, FREYA was also coupled to S2
to allow for code-to-code comparison. An example distribution of fission neutrons tracked
in the T4 model of CROCUS using standard sampling compared to FREYA is displayed
in Figure 5.1. Its effects on the simulation and whether FREYA or analog transport are
necessary for noise analysis in Monte Carlo are discussed in Section 5.2.10.

5.1.5 Capturing time correlation in Monte Carlo

Monte Carlo codes can directly provide the time of arrival or time of fission usually by
keeping track of velocities and distances traveled by the neutron to estimate time intervals
between interactions. We interpret these time stamps of events directly as detection events.
The created time stamp arrays are thus equivalent to detection time interval distribution
measurements in reactors. We therefore have a method to directly create simulated neutron
noise. A summary of the final response curve estimation methodology can be found
in Figure 5.8.
Chapter 5. Monte Carlo simulation of branching noise

Given the introduced strengths of MC and common biases that could perturb the simulation, we next present the numerical experiments conducted in T4 and S2 to verify our methodology and discuss parameters of interest.

5.2 Verification: Design of a numerical experiment

Our codes of choice for noise simulations are T4 and S2. By default, both use implicit capture and simplified fission neutron sampling. Analog transport, i.e. keeping neutron weights always at unity, is a standard feature in both codes. For the specific needs of noise simulation, we also require an explicit or more complex modeling of the fission process. For T4, we herein present a methodology making use of the T4 neutron track feature that was adapted for noise purposes. A coupling of the LLNL fission library with T4 has already been implemented and tested for a spontaneous fission source model [174]: In this work we present a first comparison and validation of this method for neutron noise problems. In a similar fashion, FREYA was coupled to S2 in the frame of this work to allow for code-to-code comparison.

5.2.1 Whole core fission tracking

As main verification of the functionality of the noise method in Monte Carlo to produce data comparable to experiments, we focus on the tracking of every fission event in the CROCUUS models. This is equivalent to having a detection system with perfect efficiency, i.e. 1 count per fission. Unless indicated otherwise, we use JEFF3.1.1 as nuclear data library for all calculations. As the reactivity estimated for different water levels is code and library dependent, we will indicate reactor configurations by water level.

The parameters that can and must be tuned are identified as follows:

- Code specific input parameters, demonstrated with a code example in Section 5.2.2.
- Noise analysis method parameters, discussed in Sections 5.2.4 through 5.2.6.
- Reactor configuration and simulation options such as delayed neutrons, Section 5.2.7.
- CPU time requirements, elucidated in Section 5.2.8.
- Source location, energy, angle, and intensity, in Section 5.2.9.
- Simulation type: Implicit capture, Analog, Analog+FREYA, presented in Section 5.2.10.

In the following subsections we will try to optimize these parameters one by one, attempting to find heuristics or optimums to achieve observables that are comparable to experiments. The steps towards comparing simulations to experiments are presented in Sections 5.2.11 to 5.3.1.
5.2. Verification: Design of a numerical experiment

5.2.2 T4 input specifications for noise simulation

The input file for TRIPOLI follows the standard structure as outlined in the handbook [176]. The geometry is given as a ROOT file [17] with an accompanying material composition file. As no tallies were used, the scoring and meshing cards were omitted. The tracking of events was done in post-processing using the ROOT-tracks feature, which allows the saving of all neutron tracks in .root.tracks file format. The corresponding input file skeleton can be found in Appendix E. Using this input scheme an output file is produced that contains all neutron tracks that were optionally filtered for interactions of interest. In the following sections an example data set of the CROCUS model at 800 mm water level will be used to illustrate the process to treat the tracks and create the typical noise analysis curves. The data set contains 25.7 million fission time stamps, or 400 MB of ASCII data after a calculation time of \( \sim 1 \) day on a single CPU.

5.2.3 S2 coupling to FREYAY

The FREYAY coupling to Serpent 2 (Version 2.1.31) [15] required changes in the collision and fission event subroutines, as schematically shown in the flowchart in Figure 5.2. More specifically, the fission event subroutine of Serpent was modified to call a FREYAY fission event instead of sampling a neutron amount from the given \( \bar{\nu} \). A FREYAY event contains a list of outgoing neutrons with their respective correlated angles and energies, which were then used to overwrite standard Serpent subroutines for energy and angle sampling (sampleENDFlaw.c). Simultaneously, the time stamp of the fission event is saved to a text file to track \((n,f)\) for verification. \((n,x)\) time stamps are saved from the collision subroutine when a reaction of interest has been successfully sampled. For future applications with respect to gamma noise, we also sketched where photon interactions are processed. The pulse height detector subroutine should allow for gamma time stamp tracking to explicitly model prompt gamma noise.

In the following subsections we present the general methodology on how to produce typical noise observables, such as the Rossi-\( \alpha \) or Feynman curves, from Monte Carlo simulations. The aforementioned simulation parameters are now extended by the parameters of the respective noise analysis method. Thereafter we attempt to directly compare simulations to experiments.

5.2.4 Noise from simulation: Rossi-\( \alpha \)

The Rossi-\( \alpha \) method is the simplest to implement. Once the time stamps of a reaction of interest, in this case fission, are extracted from the tracks file (T4) or the time stamp text file (S2), a histogram of said time stamps is directly the Rossi distribution. This is illustrated in Figure 5.3. This requires that all source particles are emitted at time zero, which is the default setting for both T4 and Serpent. For the parameter study that follows we will use data sets that use a time cut off at 0.1 seconds to reduce the overall data amount due to longer chains caused by delayed neutrons. We will first present the general outcomes of time stamp binning and compare to the expectations based on point kinetics and physics.
Binning of Rossi-histograms  The binning method to create Rossi-distributions (or any binned distribution) biases the final result. As most approaches are heuristics and empirical in nature, we examine the effect of varying the bin size explicitly to attempt to deduce an adequate value.

Prompt decay constant  The immediate adequacy metric shall be how the prompt decay constant can be produced via NLLS fitting of the created histogram. We recall that the Rossi-$\alpha$ distribution can be fitted with a single exponential decay model:

$$R(t) = a \cdot e^{\alpha \cdot t}$$  \hspace{1cm} (5.1)

with $a > 0$, $\alpha < 0$. In Figure 5.4, we compare how the bin amount changes the distribution and the fitted exponent. An immediate observation is the obvious change in occurrences per bin. Assuming a Poisson-like statistical model we could decide to require $10^3$ occurrences per bin to ensure statistical errors below 1%. Given the data set size, however, these occurrence rates are only guaranteed for $10^2$ bins.

The found exponential decay constant is also displayed in Figure 5.4. For $10^2$ bins the coarse resolution increases the uncertainty of the fit. Overall we find the mean value of $\alpha$ to be found consistently irrespective of bin amount, with a small systematic bias due to bin choice of the order of 0.2%.
Verificiation: Design of a numerical experiment

(n,f) time stamps
0.0000156...
0.0014522...
0.0424692...
0.0000051...
...

**Figure 5.3** Basic principle to create a Rossi-α distribution from Monte Carlo transport calculations. A reaction (n,x) can be treated as point of interest. In this figure we chose fission event time stamps measured from a source birth at time zero. A histogram gives an exponentially distributed survival probability of the prompt chain, i.e. a decay governed by the prompt decay constant. Delayed neutrons spuriously add 'background counts' and require more expensive calculations to exhibit their respective decays.

**Delayed decay constant** As the experiments and simulations could ultimately also be used to explore the regions outside of the prompt decay, i.e. higher prompt modes, delayed decay and spatial effects, we compare the single exponent models to two exponent models:

\[ R(t) = a \cdot e^{\alpha_1 t} + b \cdot e^{\alpha_2 t} \quad (5.2) \]

In Figure 5.5 we compare the extreme cases, $10^2$ and $10^6$ bins, fitted with a two exponent model. Setting the starting point of $\alpha_2$ in the order of 1 1/s to capture delayed effects, we find that despite the "Poisson"-heuristic motivated above, the distribution with more bins finds the delayed decay constant with smaller confidence bounds. This could however just

**Figure 5.4** Effect of changing the amount of bins ($10^2$ to $10^6$ bins) used for creating a Rossi-histogram of fission time stamps. The data was generated using T4 and the CROCUS model at 800 mm water level.
be a bias due to the high amount of points used for fitting. The prompt decay constant itself is adjusted by <1% using a two exponent fit. The delayed decay constant was estimated with S2 using JEFF 3.1.1 to be 0.26 l/s at 800 mm. This discrepancy could hint at the hypothesis that the "slow" process described by the second exponent could be the reactor’s reflector rather than a delayed neutron effect.

**Higher prompt mode decay constants** Trying to fit the 1E6 bins distribution with a rapid exponential results in a decay constant in the order of 10^6 l/s, as displayed in Figure 5.6. Higher alpha modes of CROCUS have yet to be estimated to provide an order of magnitude. In the plot we also drew the 'delayed' two exponent fit for comparison. Note that a three exponent model, albeit attractive in theory for this application, rarely converges and was tested in this case to not yield converged results even if the starting points are chosen with tight bounds. Whether the observed 'prompt' decay is physical or not cannot be assessed. It is irrespective of its physical nature not observable in experiments due to its high frequency of >200 kHz and not further discussed.

### 5.2.5 Noise from simulation: Feynman-α

Feynman variance to mean curves are not as straightforward to calculate. In the original implementation publication [174], the neutron (spontaneous fission) source time was distributed between \( t_0 = [0,1000] \) s. For a given gate length, the detectors time stamps were tallied for a random time segment. Repeating this time stamp tallying until the data is exhausted resulted in an array of count rates per gate length - and in turn a variance and mean of said array.

However, in this work we propose a different method that is mathematically equivalent. Assuming that the source is stationary (no clusters or cluster movements), then we propose starting each source event at \( t_0=0 \). Each neutron interaction that is tallied now requires an
5.2. Verification: Design of a numerical experiment

Figure 5.6  Fit results for two exponent models on the \(10^6\) bins distribution using the binning test data set. The higher prompt decay seen in the figure is unexplained.

additional "source event identifier"\(^1\) to enable the association of time stamps to a source event. The Feynman curve is then calculated as follows:

- The time stamps are grouped into arrays corresponding to an individual source event.

- Each source event array of time stamps can now be distributed in time by adding a uniformly sampled \( t = U[0,T] \), with \( T \) of the order 1000 s or longer to allow for larger gate (\( >1 \) s) statistical convergence.

- Any time stamp bigger than \( T \) is divided by \( T \) to artificially append the beginning with delayed neutrons. Alternatively the initial "build-up" of delayed sources can be cut out.

- All newly distributed time stamps are now joined into a single array. Arbitrary gates are opened over the data, i.e. the time stamps are put into histogram with a given gate length used as bin width.

- For each requested gate length the histogram bins’ mean and variance is determined, yielding the desired variance to mean curve per gate.

Note that increasing \( T \) with a given amount of neutron histories is equivalent to decreasing the source strength or reactor power. The methods are equivalent and can be used interchangeably if the time stamps carry the mentioned history identifier, allowing for arbitrary redistribution in post-processing. Compared to the typical load of the saved time stamp array in double precision, we find an additional integer to be minor in added memory requirements.

\(^1\)This identifier can be just an integer denoting which source event was sampled, e.g. a number running from 0 to the amount of neutrons used as input source.
Maximum time $T$ of redistributed time stamps  The effect of $T$ on Feynman-$\alpha$ curves from simulated data is displayed in Figure 5.7. If $T$ is below 10 seconds, we find that the shape is strongly influenced, likely due to the lack of statistics for the longer gate lengths. Above 100 s no significant difference can be seen, confirming the expectation that variance to mean curves are independent of reactor power, see Section 2.4.

![Effect of maximum time $T$ on Feynman-$\alpha$](image)

**Figure 5.7  Comparison of Feynman-$\alpha$ curves obtained from noise simulations with differing maximum time $T$.**

5.2.6 Noise from simulation: PSD

The PSD or Cohn-$\alpha$ curve can be, given our methods for Rossi and Feynman, calculated from the data in two ways:
I) By taking the Fourier transform of the Rossi distribution.
II) By taking an appropriate histogram while calculating the Feynman curve. For a gate width of 500 $\mu$s, i.e. a sampling frequency of 2 kHz, we can calculate directly the PSD using, for example, the Welch method.

Both methods have different user input parameters depending on the method. Fourier transforming the auto-correlation is mathematically unique but noisy due to the white noise behavior of the periodogram, as discussed in Appendix G. We will for the overall verification and validation use only Rossi-$\alpha$ to reduce the added complexity and parameters needed for PSD or Feynman-$\alpha$. A summary of all methods in their concept and the user inputs is displayed in Table 5.1 with a graphical representation of the method shown in Figure 5.8.
5.2. Verification: Design of a numerical experiment

Figure 5.8  Illustration of how to treat time stamp data from MC simulations to create the observable noise data allowing for curve fitting with the Rossi, Feynman or PSD expressions.

Table 5.1  Summary of methods and their free parameters for determining noise curves from Monte Carlo transport.

<table>
<thead>
<tr>
<th>Method name</th>
<th>Principle</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rossi-α</td>
<td>Histogram of time stamps</td>
<td>Bin width</td>
</tr>
<tr>
<td>Feynman-α</td>
<td>Moments of redistributed histories</td>
<td>Max. time T</td>
</tr>
<tr>
<td>Cohn-α</td>
<td>Fourier transform or Welch PSD</td>
<td>FFT/PSD parameters</td>
</tr>
</tbody>
</table>
5.2.7 To $\beta$ or not to $\beta$

At first glance it might appear that delayed neutrons are necessary in order to allow for noise methods to yield a physical result. Practically, by disabling delayed neutrons we decrease the reactivity of the system by $1\%$, thus avoiding potential computational pitfalls close to criticality due to very long fission chains. In Figure 5.9 we display the results of either using or not using delayed neutrons on the Rossi distribution. The obvious difference is the missing "background" or delayed tail. Nonetheless, we find that this difference does not affect the prompt decay constant inferred from the distributions. Delayed neutrons could potentially be completely ignored when assessing the multiplication of the system. Nonetheless, unless indicated otherwise, delayed neutrons are included in all further calculations.

5.2.8 Computational requirements

In Figure 5.10 we present the Rossi distributions in dependence of single CPU time used to quantify a "sufficient" time for noise analysis. We can observe that even after $10^{-4}$ of a CPU day (≈8 seconds, 2.5k time stamps), we begin to see the expected value for the prompt decay constant emerge. As with all Monte Carlo methods, a brute increase of CPU time yields better results. Above $10^{-2}$ CPU days (≈14 minutes, 250k time stamps) the delayed decay becomes visible. As we are interested in assessing the accuracy compared to experiments, we will use 1 CPU day equivalent amounts of time stamps for fission tracking to ascertain the prompt fitting. Fission tracking, generally speaking, indeed proves to be an effective method to produce time stamps. We were not able to parallelize T4 or S2 when using FREYA, which is a natural next step to improve performance.
5.2. Verification: Design of a numerical experiment

The calculations were done with T4 using analog transport and FREYA fission.

5.2.9 The Source problem

An important matter for Monte Carlo calculations is the choice of initial source locations and neutron energies. In k-static calculations so called inactive cycles are used to find the critical equilibrium fission source distribution, based on an initial guess e.g. a Watt spectrum uniformly distributed in fissile volumes. For sub-critical states the k-static approach leads to a biased flux shape\footnote{Also, it leads to a biased estimation of \( k_{eff} \) for systems far from criticality. "Far" is hereby very dependent on the geometry and materials of the problem \cite{38}.}, as any sub-critical interrogation depends also on the source location and intensity. We hypothesize that a uniform source distribution will underestimate the true fission chain length compared to a unbiased source, as sampling sources in boundary regions lead to shorter chains on average.

For all the presented calculations, we thus required an explicit source modeling of all measured sub-critical states of CROCUS. CROCUS uses a PuBe source that is pneumatically inserted below the core for start-up and withdrawn for (super-)critical states; it is not modeled for usual k-static calculations. In order to calculate the sub-critical fission source distribution, a cylindrical surface source in the core bottom was modeled using a spectrum found in \cite{177} and \cite{110}, displayed in Figure 5.11. The resulting difference in source distribution for a reactor configuration at 800 mm is shown in Figure 5.12, visually confirming a relevant difference between the two distributions.

In Figure 5.13 we display the differences in Rossi distributions and fitted \( \alpha \) when using a k-static critical source or a sub-critical source. At 950 mm the relative difference is 0.4\%, while at 800 mm it is 2.1\%. This result is consistent with the fact that the source’s influence should diminish closer to criticality. Also, it substantiates the hypothesis regarding

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_10}
\caption{Rossi distributions and fit results for single exponent models for increasing single CPU time use. The calculations were done with T4 using analog transport and FREYA fission.}
\end{figure}
Figure 5.11  Left: T4 model of CROCUS at 800 mm water with sketched location of the PuBe start-up source. Right: Neutron energy spectrum of the source used for the sub-critical source distribution estimation [110].

Figure 5.12  Left: Fission source distribution after 200 cycles at 10000 particles per batch for a T4 k-static calculation. Right: Fission source distribution when using the explicitly modeled PuBe start-up source below the core as input, $10^5$ particles.
5.2. Verification: Design of a numerical experiment

Figure 5.13 Rossi and PSD fission distributions and fits in black for single exponent models for different water levels. The calculations used analog transport and FREY A fission.

Fission chain lengths being dependent of the source location. Hence, for any sub-critical configuration the interrogation source location needs to be explicitly modeled to remove this bias.

5.2.10 Is analog or FREY A necessary?

As introduced in Section 5.1, a hypothesis with regards to trueness of noise simulations is that analog transport and sophisticated fission treatment such as FREY A is necessary to capture the true noise behavior of the core and to produce the truest observables when compared to experiments. In order to study the difference between "implicit capture", "analog", and "analog with FREY A", we conducted simulations of CROCUS in T4 in the configurations at 800mm and 950 mm water level with all three modes of simulation. We compare the resulting Rossi and Feynman distributions in Figure 5.14.

Figure 5.14 Rossi distributions and fit results for single exponent models for different simulation modes: Analog, analog with FREYA and standard implicit capture.
We observe that the results for $\alpha$ are similar – the simulation mode affects the fission chains only to a small degree. Visually, one is able to ascertain that the Rossi and Feynman distributions approximately follow the same pattern. The results for $\alpha$, when comparing analog to analog with FREYA, do not immediately suggest a superiority of either method. The overall judgment on which method yields the truest result is based on the validation when comparing to IFP and experiments, see Section 5.3.1.

5.2.11 What does the noise method measure?

In this section we further investigate the effects of different reactor configurations on the results of noise method analysis when tracking all fissions in core. For this purpose we compare different water levels of CROCUS - the same water levels as used in the sub-critical experiments, to approach the final goal: comparison of simulations to experiments. In Figure 5.15 we display the Rossi distributions for the five experimental water level configurations using both T4 and S2 with analog transport and FREYA fission.

A naive hypothesis is that the simulated Rossi-\(\alpha\) values are those seen in experiment. A more sophisticated hypothesis is that the fission chain length corresponds to the absolute reactivity $\rho_{\text{sim}}$ of the simulated system, and not the experimental value $\rho_{\text{exp}}$. Equivalently, IFP-method calculated kinetic parameters fall under this bias. Indeed, a consistent over-prediction of $k_{\text{eff}}$ by $\sim 230$ pcm is observed in benchmarks using T4 [17], MCNP6.2 or S2 with respect to the true critical state - an indication for nuclear data to be the source of bias. The experimental values should differ by this amount from the simulations. A comparison with experiments is only possible by comparing relative reactivities (e.g. as is common practice for rod worth estimations).

In Table 5.2 and Figure 5.16 we attempt to provide evidence for the sophisticated hypothesis - we compare the results for \(\alpha\) determined via IFP or the noise method. The IFP-\(\alpha\) was calculated using

$$\alpha_{\text{IFP}} = \frac{(1 - 1/k_{\text{eff}}) - \beta_{\text{IFP}}}{\Lambda_{\text{IFP}}}. \quad (5.3)$$

with $k_{\text{eff}}$ being the estimate provided by a k-static calculation.

The results agree until 900 mm within 1$\sigma$ for T4, yet fail to agree closer to criticality. The results for S2 are generally in disagreement except for 850 mm. A possible reason could be the fact that the simulations at 925 mm and 950 mm have $k_{\text{eff}} > 1$ - and the fission chain behavior is perturbed by giving longer than expected chains due to the super-criticality. The full comparison will require the comparison to experimental data, which is the content of the next section.
5.2. Verification: Design of a numerical experiment

Figure 5.15  Rossi fission distributions and fits in black for single exponent models for different water levels. The calculations used analog transport and FREYA fission.
Figure 5.16  Prompt decay constant for the different experimental configurations in CROCUS, as predicted by S2 using IFP, T4 using IFP, and T4 using noise methods.

Table 5.2  Comparison of the prompt decay constant $\alpha$ for different water levels of CROCUS for IFP predicted values and fit results for the noise method using both T4 and S2. The uncertainties listed arise either from statistical estimates for the simulations or 95\% confidence bounds on fits for the noise method.

<table>
<thead>
<tr>
<th>Water level (mm)</th>
<th>$\alpha_{IFP}$</th>
<th>$\alpha_{Noise,Rossi}$</th>
<th>$\alpha_{IFP}$</th>
<th>$\alpha_{Noise,Rossi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>-339.2 ± 0.6</td>
<td>-354.5 ± 0.4</td>
<td>-340.8 ± 2.6</td>
<td>-340.2 ± 0.4</td>
</tr>
<tr>
<td>850</td>
<td>-254.6 ± 0.5</td>
<td>-255.2 ± 0.3</td>
<td>-253.9 ± 1.5</td>
<td>-251.1 ± 0.6</td>
</tr>
<tr>
<td>900</td>
<td>-182.9 ± 0.5</td>
<td>-200.5 ± 0.5</td>
<td>-186.2 ± 2.3</td>
<td>-190.3 ± 0.6</td>
</tr>
<tr>
<td>925</td>
<td>-150.9 ± 0.5</td>
<td>-166.7 ± 0.3</td>
<td>-152.1 ± 2</td>
<td>-167.2 ± 0.3</td>
</tr>
<tr>
<td>950</td>
<td>-122.4 ± 0.5</td>
<td>-136.7 ± 0.3</td>
<td>-128.0 ± 2.2</td>
<td>-142.7 ± 0.4</td>
</tr>
</tbody>
</table>
5.3. Validation: Comparison to IFP and experiments

Having discussed the prerequisites to simulate noise using Monte Carlo codes, we next attempt to validate our methodology with the sub-critical experiments introduced in Section 4.1.2.2. We first compare (n,f) tracking results, as they are computationally efficient (∼CPU days) to calculate. Tracking every fission agnostic of its location effectively assumes point kinetics, however, and we present the explicit detector models and results for $^3$He(n,p) reaction tracking thereafter.

5.3.1 $\Delta \alpha$ for different configurations

A direct comparison of prompt decay constants is strictly speaking not meaningful if absolute $k_{eff}$ of simulated and real systems do not agree. This disagreement usually stems from nuclear data biases – as discussed in the simulation benchmarks for CROCUS [30], [17]. We hence define the relative decay constant for validation purposes as

$$\Delta \alpha = \alpha_{ref} - \alpha_{pert}. \quad (5.4)$$

In Table 5.3 we compare $\alpha$ and $\Delta \alpha$ obtained both with T4 and S2 using IFP and noise analysis to the CPSD estimates of the He-3 experiments at the closest distance to the core, i.e. 4.65 cm to the fuel. The experiments are also directly compared to $\alpha$ obtained from S2 IFP when using the experimental critical water level of 961.8 mm as reference – as discussed in Section 4.1.3. In addition, we display $\Delta \alpha$ for the experiments and noise simulations. Note that the reference for $\Delta \alpha$ is the respective value at 800mm$^3$. The overall comparison of calculation methods compared to the experimental values of $\Delta \alpha$ is shown in a C/E-1 form in Figure 5.17.

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$^3$Note that choosing the reference at 950 mm gives the same results, i.e. this choice is arbitrary and does not affect the conclusion.
### Table 5.3

Comparison of the prompt decay constant $\alpha$ for different water levels of CROCUS for experiments, IFP predicted values for S2 and T4, and results for T4 noise using Rossi distributions on full core (n,f) tracking. The experimental values are statistical estimates for the simulations or 95% confidence bounds for the noise methods. We also compare relative $\Delta \alpha$ values with 800 mm as reference for the experiments and T4 Noise.

<table>
<thead>
<tr>
<th>Water level (mm)</th>
<th>$\alpha$ (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>98.6 ± 0.6</td>
</tr>
<tr>
<td>850</td>
<td>114.0 ± 0.1</td>
</tr>
<tr>
<td>900</td>
<td>137.8 ± 0.2</td>
</tr>
<tr>
<td>925</td>
<td>149.9 ± 2.3</td>
</tr>
<tr>
<td>950</td>
<td>173.0 ± 2.8</td>
</tr>
<tr>
<td>800</td>
<td>198.7 ± 1.1</td>
</tr>
<tr>
<td>850</td>
<td>229.5 ± 1.6</td>
</tr>
<tr>
<td>900</td>
<td>256.9 ± 1.1</td>
</tr>
<tr>
<td>925</td>
<td>281.9 ± 0.9</td>
</tr>
<tr>
<td>950</td>
<td>304.5 ± 0.7</td>
</tr>
<tr>
<td>800</td>
<td>329.2 ± 0.6</td>
</tr>
<tr>
<td>850</td>
<td>352.1 ± 0.7</td>
</tr>
<tr>
<td>900</td>
<td>375.9 ± 0.5</td>
</tr>
<tr>
<td>925</td>
<td>398.5 ± 0.3</td>
</tr>
<tr>
<td>950</td>
<td>420.0 ± 0.4</td>
</tr>
<tr>
<td>800</td>
<td>440.8 ± 0.2</td>
</tr>
<tr>
<td>850</td>
<td>461.7 ± 0.1</td>
</tr>
<tr>
<td>900</td>
<td>482.6 ± 0.1</td>
</tr>
<tr>
<td>925</td>
<td>503.5 ± 0.1</td>
</tr>
<tr>
<td>950</td>
<td>524.4 ± 0.0</td>
</tr>
</tbody>
</table>

The uncertainties are propagated from the aforementioned values.
5.3. Validation: Comparison to IFP and experiments

Table 5.4  Overview of number of time stamps and results for $\alpha$ from $^3$He(n,p) tracking simulations for close distance (4.65 cm) detector location water level variation. $\alpha$ is determined by using the Rossi method. All calculations used analog transport and FREYA for fission.

<table>
<thead>
<tr>
<th>Water level (mm)</th>
<th># of time stamps</th>
<th>$\alpha_{\text{Noise,Rossi}}$ (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>950</td>
<td>11k</td>
<td>$-144.6 \pm 7.7$</td>
</tr>
<tr>
<td>925</td>
<td>13k</td>
<td>$-178.8 \pm 6.3$</td>
</tr>
<tr>
<td>900</td>
<td>12k</td>
<td>$-179.9 \pm 5.4$</td>
</tr>
<tr>
<td>850</td>
<td>12k</td>
<td>$-259.9 \pm 6.4$</td>
</tr>
<tr>
<td>800</td>
<td>12k</td>
<td>$-339.7 \pm 10.3$</td>
</tr>
</tbody>
</table>

The observations can be summarized as follows:

- S2 and T4 IFP agree within $2\sigma$, indicating a generally agreeing inner workings when evaluating IFP. IFP also increasingly under-predicts $\alpha$ with deeper sub-criticality, but appears to be corrected for when using the actual experimental critical state as reference.

- The comparison of $\Delta\alpha$ with IFP reveals that the respective IFP methods give similar results in relative terms, with a larger than $1\sigma$ deviation found at 950mm. The IFP predictions nonetheless overestimate the relative change in $\alpha$, agreeing with the above mentioned general under-prediction with sub-criticality.

- Using FREYA elevates the results for $\alpha$ from being different by hundreds of pcm to only a few pcm for the Rossi distribution fit results, when comparing to the IFP predictions – we use this as evidence to state that an explicit fission model is necessary for noise simulation.

- When comparing $\Delta\alpha$, the codes show a different respective behavior. While the S2 Rossi-$\alpha$ predictions follow its uncorrected IFP results, the T4 results are remarkably closer to the experiments and appear to not suffer from the bias. We infer that T4 Rossi-$\alpha$ gives the better prediction for $\alpha$ than T4 IFP, indicating a difference in implementation of FREYA between T4 and S2.

5.3.2 Explicit modeling of He-3 experiments

Whole core tracking not only assumes a perfectly efficient detector, but also a detector that covers the entire geometry, effectively assuming point kinetics. We thus next examine the explicit model in T4 of the He-3 sub-critical experiments conducted in CROCUSto test if consistent results emerge and to study the expected increase in CPU time required. Here, the tracks files are filtered for (n,p) interactions in the Helium-3 detector volumes to track detection events.

The simulations covered 9 of the 25 experiments: For the closest distance we varied the water level to all five configurations, and for the 925mm configuration we modeled each
Figure 5.18  Top: Comparison of Rossi distributions for all five water levels resulting from (n,p) tracking in the Helium-3 detectors, all at 4.65 cm distance. Bottom: Comparison of Rossi distributions for all five detector distances resulting from (n,p) tracking in the Helium-3 detectors at 925 mm. The data was produced using T4 in analog mode with FREYA fission.
5.3. Validation: Comparison to IFP and experiments

of the five detector distances. The Rossi distributions for each variation are displayed in Figure 5.18. The values for \( \alpha \) can further be compared to the experimental, IFP, and fission tracking values listed in Table 5.3.

**Results** In Table 5.4 we display the amount of time stamps gathered for each water level for the closest distance experiment simulations. The resulting fit uncertainty on \( \alpha \) is consistent with our previous considerations with respect to CPU time when tracking all fissions. The relative amount of time stamps per CPU hour for \((n,f)\) to \((n,p)\) in He-3 should correspond to the experimental efficiency. Indeed, \((n,f)\) tracking gives about \(10^6\) time stamps per CPU hour, while \((n,p)\) in a single detector gives \(4.4 \times 10^2\) time stamps per hour. The ratio is \(2.1 \times 10^{-4}\) - compared to the experimental efficiency of \(1.1 \times 10^{-4}\), we can infer an intrinsic efficiency of around 50%, a typical value for He-3 tubes. This assures us in the trueness of the simulations.

Overall we observe an agreement of T4 \((n,p)\) Rossi-\(\alpha\) estimates with the fission tracking within \(2\sigma\), largely due to the large uncertainty associated with the explicit model. As discussed above, we find that the experiments agree best with T4 Rossi predictions. This implies the following: Due to the large computation cost associated with the explicit model (which is increased by a factor \(1/\epsilon\) compared to fission tracking), we can only conclude that the detectors are likely to observe a point kinetics core. This means that the explicit models only aid in estimating the computation cost to prove this with comparable uncertainties to the experiments. The simulation approach should nonetheless be able to reveal more about the physics of the system, e.g. if it acts as a point reactor without requiring expensive calculations.

5.3.3 Location of first and last fission of an observed fission chain

Given that MC codes simulate the fission chains close to the physical nature, we are able to track general behaviors to examine the physics of the system. When detecting \((n,p)\) reactions in a Helium-3 tube, we observe a neutron that was probably liberated by a nearby fission. The notion of nearby is hereby likely related to the average distance a neutron travels in a given reactor. CROCUS is water moderated and we expect a mean free path of around 3cm, and the detector would thus observe only fissions occurring locally in a sphere of several cm radius.

The tracking of \((n,f)\) also allows the user to save location coordinates for each fission. This allows us to plot the location of the last fission (see Figure 5.19, 925 mm configuration) of a chain that took place before causing a detection \((n,p)\) in the northern or southern periphery of the core. Figure 5.19 is colored with a kernel density estimate of the point distribution to illustrate the relative concentration. Indeed, we find that a detector "sees" only its immediate fissile surroundings, in this case the CROCUS metal fuel zone. This observation indicates that detectors are indeed prone to spatial effects e.g. if the fission chains are locally perturbed to not traverse the whole core and mostly cause fission chains of shorter length than point kinetics would predict.
Figure 5.19 – Top view of a scatter plot of coordinates for last fission locations of a chain that results in a (n,p) detection.

In the specific case of CROCUS, we next investigate the location of the first fission that resulted in a detection, displayed in Figure 5.20. We find that the first fission happens throughout the core – this means that fission chains indeed traverse the whole core in CROCUS. This could be seen as evidence for the point kinetic hypothesis: Any neutron injected into the system is likely to cause fission chains that traverse most of the core and give temporal information corresponding to the whole core. We observed that most of the prompt chain died out within 0.1 seconds. Using a prompt generation time of roughly 50 µs (typical for CROCUS), we infer that around 2000 generations of fission may occur, apparently enough to traverse most of the core. Arbitrary reactor geometries could be probed in this way to assess where a detected fission chain originated from, and if the whole core can contribute to the time correlations observed.

5.4 Extrapolation: Noise simulations for physics interrogation

The noise simulation method was, in the previous sections, used to model the well characterized experiments in CROCUS to provide a reference and validation case. Our analysis concludes that T4 coupled to FREYA appears to best agree with the experiments, and as such is our tool of choice for beyond validation applications.
5.4. Extrapolation: Noise simulations for physics interrogation

![Graph showing explicit He-3(n,p) tracking and location of first fission](image)

**Figure 5.20** - Top view of a scatter plot of coordinates for first fission locations of a chain that results in a (n,p) detection.

5.4.1 Decoupled kinetics: Split core CROCUS

As a final test of the capabilities of the noise method, we attempt to cause a "departure" from point kinetic behavior. Decoupled kinetics and multiplication factor calculations are of interest for modern reactor designs with intermediate breeding zones or for spent fuel criticality assessment [178, 179]. Such a decoupling likely manifests as Rossi distribution exhibiting two or more decays in the prompt region. Using the CROCUS model we assumed that a core splitting, i.e., a spatial separation of the core into two halves, could allow for decoupling of the kinetics. In Figure 5.21 we display the T4 model of the CROCUS model for this experiment, split by a distance equivalent to 4 pitches of the $U_{met}$ lattice ($\sim11.7$ cm) with added fuel to test the compensation of reactivity loss.

For this problem we examine again the Rossi distributions for $\langle n_f \rangle$ being agnostic about location - as shown in Figure 5.22 in dependence of the amount of distance quantified in units of $U_{met}$ pitches. The source was chosen uniformly in the fissile volumes – we thus expect a bias when comparing to a localized source. The resulting distributions and fits follow the expected behavior: For a larger separation the sub-criticality increases. For a separation of 4 $U_{met}$ pitches with and without added fuel, however, we find that a single exponential does not fit the prompt decay adequately. In Figure 5.23 we display a detailed view of the latter case. Indeed, a single exponential only captures the average prompt decay, while a two exponential model more accurately yields two decay components: the larger at $\alpha_2 \sim -8165/\text{s}$ likely the decay constant in the respective core halves - and the smaller at $\alpha_1 \sim -1788/\text{s}$ being the decay constant of both core halves when coupled.
Figure 5.21 – Top view of the T4 model of CROCUS in split core configuration at 4 $U_{\text{met}}$ pitches distance with added fuel.

Figure 5.22 – Rossi distributions with single exponent fits for the CROCUS split core configuration. The distance between the core halves is given in units of the pitch of the $U_{\text{met}}$ grid. The calculations used analog transport and FREYA fission.
We summarize our findings:

- CROCUS can probably be decoupled by splitting the core in halves - albeit in a very sub-critical state.

- Noise methods can capture more complex behavior outside the realm of point kinetics - likely more reliably than k-static calculations of decoupled systems.

- If $\beta_{eff}$ and $\Lambda$ of the system are known, we would now be able to infer half core and full core $k_{eff}$-values.

- We provided evidence that probing the fission locations that result in fission can indicate whether a reactor will act according to point kinetic predictions.

The split core simulations could be complemented by examining (n,p) via the noise method – and thus determine if this decoupled behavior is also seen in a detector at all, and if so, how it is spatially dependent. Another free parameter is the source location and intensity with similar implications for interrogation experiments.
5.5 Chapter’s salient elements

We presented the full methodology, numerical experiments, and comparison to experimental data of noise methods applied to Monte Carlo calculations. The reference is hereby point kinetics and the CROCUS sub-critical noise experiments. Our findings are as follows:

- A standardized methodology on how to calculate noise curves from Monte Carlo calculations had hitherto not existed. We presented a possibility in detailed fashion, highlighting parameters of interest and displaying their impact - such as the binning of Rossi-\(\alpha\) or CPU time.

- Using analog, FREY A or implicit fission impacts the noise curves and their fit results significantly - in comparison to IFP predictions FREY A fission gives the best results.

- The noise method allows for the prediction of \(\alpha\), and we find that Rossi-\(\alpha\) agrees well with IFP predictions for configurations that are sub-critical.

- Tests using a split core model of CROCUS showed that decoupled problems can potentially be efficiently solved using the noise method – if validated with an experiment, a new method for fuel criticality assessment and loosely coupled systems could be established.
Chapter 6

Conclusion and Outlook

6.1 General summary and conclusion

Noise analysis applied to nuclear reactor physics is a powerful tool to investigate kinetic parameters, and more generally the underlying physical processes determining core behavior. In the prospect of aiding the evaluation process for nuclear data, we sought to provide a sound parameter and uncertainty estimation scheme. In complement to that general purpose, we expanded the detection capabilities of the CROCUS research reactor by improving the neutron noise instrumentation, and developing a dedicated gamma noise detection system.

The improved detection capabilities allowed the precise determination of the kinetic parameters of CROCUS using both neutron and gamma noise. The results displayed how the common first order approximation of the covariance matrix mis-estimates the parameters and underestimates the uncertainties. We thus conclude that noise measurements require an improved scheme, such as bootstrapping. The reporting of full (non-normal) distributions of kinetic parameters is required in order to accurately represent the experimental result. This is particularly important for providing feedback on nuclear data evaluation, as Monte Carlo based data assimilation can directly benefit from explicit distributions. New reference neutron noise measurements were conducted using the developed current amplifiers connected to high efficiency fission chambers. With a comparatively long measurement time of almost six hours, the kinetic parameters of CROCUS were determined within 1σ of Monte Carlo code predictions, largely due to the improved uncertainty estimation technique. In-core gamma noise measurements conducted for two hours revealed their advantage compared to neutron noise, with an unprecedented accuracy for the prompt decay constant measured in CROCUS, yielding uncertainties comparable to the bias arising from nuclear data library choice. Given the detectors available to us, we conclude that gamma noise is likely superior in terms of uncertainty, cost-efficiency, and overall accuracy compared to neutron noise for the considered case of prompt decay constant measurement.

Having obtained a sound parameter estimation scheme and the appropriate instrumentation, the biases stemming from point kinetic assumptions and the practical limits of noise experiments were studied experimentally. We investigated the critical and sub-critical noise
field of CROCUS by varying the parameters of distance to the core, negative reactivity, and particle type. The neutron noise field study allowed to conclude that neutron noise measurements are likely to be limited to the immediate proximity of the core (up to around 0.2 meters in the case of CROCUS), and that CROCUS can be reliably modeled as a point kinetic reactor. A systematic trend pointing to a weak spatial effect biasing distant neutron noise measurements by up to 5% could nonetheless be shown. The observable gamma noise field using high efficiency BGO detectors was shown to extend well beyond the observed neutron noise limits. We were able to determine the prompt decay constant in an ex-vessel location, at about 0.9 meters to the core center, with comparable accuracy to that of in-core reference experiments. In addition, we discovered that gamma noise can be observed in CROCUS as long as a direct line of sight to the reactor is preserved, enabling a successful noise measurement at an unprecedented 7 meters to the core center. The mean value was estimated with 4% relative difference to predictions, albeit with uncertainties of around 50%, pointing to an uncertainty over-estimation.

Finally, a simulation scheme to estimate noise responses was developed to allow for a validated code to provide predictions for arbitrary geometries, but also aid in the interpretation of experiments. Our approach was to model the experiments using analog Monte Carlo transport coupled to an explicit fission model. We used both an established code with an existing FREYA fission library coupling, TRIPOLI-4, and developed a coupling of FREYA to Serpent 2 for code-to-code comparison. We presented the full methodology, numerical experimentation, and validation of noise simulations to CROCUS experiments. We further investigated the general behavior of fission chains in CROCUS to deduce a qualitative explanation as to why point kinetics is a valid approximation. The validated methodology is then used on a split core model of CROCUS to investigate its capabilities to capture beyond point kinetic behavior. The results indicate an effective tool to model decoupled systems.

6.2 Outlook and recommendations

The results obtained in the context of this thesis offer several paths forward.

- In order to contribute to data assimilation and nuclear data evaluation, a defined target has been set on the accuracy of estimates of the effective delayed neutron fraction $\beta_{\text{eff}}$. To determine $\beta_{\text{eff}}$, however, one requires the fission rate or a calibration using nuclear data as input. The incurring loop of using nuclear data to provide an estimate of a parameter to improve nuclear data could be broken by developing a method to estimate a calibration-less quantity related to $\beta_{\text{eff}}$, such as the PSD plateau value, that could be used in data assimilation.

- Gamma noise has been shown to be potentially more accurate and cost-effective as compared to neutron noise. The information of $\beta_{\text{eff}}$ is also coded in the gamma correlation amplitude, yet a method to extract the relevant parameters and associated uncertainty quantification does not yet exist. By studying the theoretical expressions, the here presented experimental data could be analyzed for an estimate of $\beta_{\text{eff}}$ as well.
6.2. Outlook and recommendations

Figure 6.1  Power spectral density obtained from BGO gamma detectors set at approximately 7 meters from the core center. We compare three experiments: Two single lateral fuel rod oscillations in the COLIBRI experiment [35] at 0.1 Hz and 5 mm amplitude, producing a local perturbation compared to a static measurement. Even at this large distance, the oscillation is visible in the gamma noise. Due to large unwanted noise sources found in this test setup, the measurement requires a repetition for a more sound conclusion.

![Gamma oscillation: BGO 93 APSD at 6.9 m to core center](image)

- The distance at which gamma correlations is distinguishable in a detector opens the possibility to study noise measurements in previously inaccessible geometries. Reactors with restricted access for neutron detectors could be characterized by ex-vessel gamma detectors, given a similar line of sight to that of CROCUS.

- A rough estimate of whether perturbation or power reactor noise is distinguishable is whether or not branching noise can be seen. Given the observed range of gamma noise, we hypothesize that gamma detectors can be used for perturbation noise analysis, both in research reactors or in nuclear power plants. To test this, we conducted a single rod oscillation experiment in CROCUS, and observed the result in the BGO detectors at almost 7 meters distance, see Figure 6.1. These first tests indeed indicate a powerful tool to observe perturbations.

- Having developed a methodology to explicitly simulate noise experiments, the possibility to expand on this study can be multi-faceted: The split core numerical experiment could be examined with explicit source and detector locations to study the spatial effect on the measured response. We studied the spatial reach of fission chains in CROCUS and found it to encompass the whole core. A test case using larger geometries could provide insight as to when a reactor decouples internally. Given enough computational time investment, the explicit models of our experiments could be evaluated with high accuracy to provide further insight to the level at which point kinetics are valid.
Appendix A

Glossary of terms

The used terms are found more formally in stochastic theory books such as [180] and [181].

- **Stochastic process**: Is a process in which the involved quantities have a random time dependence.

- **Moment**: Is a measure defined onto a set of points, e.g. signal over time data pairs. In statistical terms, the $n$-th moment is defined as

$$E(x^n)$$  

(A.1)

The first moment is also referred to as the mean value. The variance (see below) can also be expressed by the moments:

$$\text{Var}(x) = E(X^2) - E(X)^2$$  

(A.2)

- **Stationary process**: Is a stochastic process that has time invariant moments.

- **(Stochastic) Markov process**: A memory-less stochastic process, i.e. it can be described entirely by the current state without any information about the history of events. This is also called the Markov property.

- **Diffusion process**: Is a Markov process that has continuous trajectories. The probability densities of a diffusion process can be shown to be following the stationary Chapman-Kolmogorov equation.

- **Kolmogorov forward equation**: Also called the Fokker Planck equation for a diffusion process. It is a stochastic differential equation to describe the time dependency of a probability density $P(y,t)$ in function of an input $y$ and time $t$. The term forward refers to the fact that the equation is set up to describe the distribution at time $t$ in a future state $t'$:

$$\frac{\partial}{\partial t} P(y,t) = -\frac{\partial}{\partial x} [\mu(y,t) P(y,t)] + \frac{\partial^2}{\partial x^2} [D(y,t) P(y,t)]$$  

(A.3)

with $\mu$ describing drift and $D$ describing diffusion contributions.
Kolmogorov backward equation: This equation also describes the temporal behavior of a probability distribution, but viewed from a future time $t$ to predict past states at time $t'$.

Covariance: Given a probability density function $f(x,y)$ for two continuous random variables $x,y$ we define the covariance as

$$\text{Cov}_{xy} = \int dx f(x,y)(x - \mu_x)(y - \mu_y)$$ \hspace{1cm} (A.4)

Variance: A measure for the spread of a random variable around it’s mean and equivalent to the covariance in the case $x = y$.

Cross correlation function: Given two time dependent signals, it is the convolution $\ast$ of both with respect to a time lag $\tau$

$$R_{xy}(\tau) = (x(t) \ast y(t))(\tau) = \int_{-\infty}^{\infty} x(t + \tau)y(t)dt$$ \hspace{1cm} (A.5)

Auto correlation function: Is correlation of signal with itself and equivalent to the cross correlation for $x(t) = y(t)$.

Correlation coefficient: Is a normalized covariance and indicates the magnitude of correlation $[0,1]$ with respect to a linear relation:

$$C_{xy} = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$$ \hspace{1cm} (A.6)

Cross power spectral density (CPSD): Following the Wiener-Khinchin theorem, the cross spectral (power) density can be found by Fourier transforming the cross correlation function:

$$G_{xy}(\omega) = \mathcal{F}(R_{xy}) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt x(t + \tau)y(t)e^{-i\omega\tau}$$ \hspace{1cm} (A.7)

The cross power spectrum gives the magnitude or contribution power of a frequency to the crossed signal’s variance.

Auto power spectral density (APSD): Is analogously the Fourier transform of the auto correlation function.

Coherence function: Is the frequency domain analogy to the correlation coefficient, and gives a magnitude of correlation $[0,1]$ with respect to individual frequencies:

$$\gamma_{xy}(\omega) = \frac{|G_{xy}(\omega)|^2}{G_{xx}(\omega)G_{yy}(\omega)}$$ \hspace{1cm} (A.8)
Appendix B

Probability generating function method

The probability generating function (PGF) is the power series representation of a probability mass function with \( p(x) \) being the probability \( P(X = r) \). For \( X \) being a random discrete variable, we define the PGF \( G(z) \) for complex numbers with the sufficient condition \(|z| \leq 1\)

\[
G(z) = E(z^X) = \sum_{x=0}^{\infty} p(x)z^x \tag{B.1}
\]

with \( p \) being the probability mass function of \( X \). Equivalently we can write for a multivariate case \( X = (X_1, X_2, ..., X_d) \):

\[
G(z_1, z_2, ..., z_d) = \sum_{x=0}^{\infty} p(x_1, ..., x_d)z_1^{x_1}z_2^{x_2}...z_d^{x_d}. \tag{B.2}
\]

For illustration of the application we consider the intuitive forward equation of a simple birth process [182], [183]:

\[
\frac{dP_N(t)}{dt} = \lambda(N - 1)P_{N-1}(t) - \lambda NP_N(t) \tag{B.3}
\]

with \( P_N(t) \) being the probability distribution of \( N \) samples that are subject to the birth process. The change in population around \( P_N \) (described by the time derivative on the left hand side) is then equal to a birth process from a population of the size \( N - 1 \) with birth probability \( \lambda \) minus the probability to already have a \( N \) sized population and no birth happening in the infinitesimal time step. We now introduce a PGF to examine the moments of the distribution:

\[
G(x,t) = \sum_{x=0}^{\infty} P_N(t)x^N \tag{B.4}
\]

Note that this example and most others can surely be solved directly. The PGF is a method of solving stochastic differential equations without the use of Ito or Stratonovich integrals (maybe expand theory, SDE and Ito calculus). The derivative of the generating
Appendix B. Probability generating function method

function with respect to time is

$$\frac{\partial G}{\partial t} = \sum_{x=0}^{\infty} \frac{\partial}{\partial t} P_N(t)x^N$$  \hspace{1cm} (B.5)

and with respect to \(x\)

$$\frac{\partial G}{\partial x} = \sum_{x=0}^{\infty} N P_N(t)x^{N-1}$$ \hspace{1cm} (B.6)

From the definition of the moment, we can see immediately that

$$\frac{\partial G}{\partial x}\big|_{x=1} = \mathbb{E}(N)$$ \hspace{1cm} (B.7)

This is one of the strong features of the generating function method: The moments can be found via derivatives evaluated at 1.

We now multiply Equation B.3 by \(x^N\) and sum from 0 to \(\infty\):

$$\frac{\partial G}{\partial t} = \sum_{x=0}^{\infty} \lambda(N - 1)P_{N-1}(t)x^N - \sum_{x=0}^{\infty} \lambda NP_N(t)x^N$$  \hspace{1cm} (B.8)

Re-indexing the first sum gives

$$\sum_{x=0}^{\infty} \lambda(N - 1)P_{N-1}(t)x^N = \sum_{x=-1}^{\infty} \lambda NP_N(t)(x)^{N+1} = \sum_{x=0}^{\infty} x\lambda NP_N(t)(x)^N$$  \hspace{1cm} (B.9)

The last equality holds when assuming a zero probability for negative populations \(P_{n<0} = 0\). We now find

$$\frac{\partial G}{\partial t} = \lambda(x - 1) \sum_{x=0}^{\infty} P_N(t)x^N$$  \hspace{1cm} (B.10)

or, using the above found derivation,

$$\frac{\partial G}{\partial t} = \lambda x(x - 1) \frac{\partial G}{\partial x}.$$  \hspace{1cm} (B.11)

Taking the derivative with respect to \(x\) again yields:

$$\frac{\partial}{\partial x} \frac{\partial G}{\partial t} = \frac{\partial}{\partial x} \left[ \lambda x(x - 1) \frac{\partial G}{\partial x} \right]$$  \hspace{1cm} (B.12)

we can now find the expected value by evaluating \(x = 1\)

$$\frac{d}{dt} \mathbb{E}(N) = \left. \frac{\partial}{\partial x} \frac{\partial G}{\partial t} \right|_{x=1} = \lambda \mathbb{E}(N)$$  \hspace{1cm} (B.13)

The PGF has several properties which can be exploited in statistical analysis such as

- \(G(0) = P(X = 0)\)
- \(G(1) = \sum_r P(X = r) = 1\)
- \(G'(1) = \mathbb{E}(X)\)
- \(G''(1) = \mathbb{E}(X(X-1))\)
- \(\text{Var}(X) = G''(1) + G'(1) - (G'(1))^2\)
Appendix C

Kolmogorov forward approach to derive point kinetics

Based on the stationary Chapman-Kolmogorov equation [184], we define a branching process approach to describe the evolution of a probability distribution function \( P \) [8]. Consider the joint probability distribution of parameters such as neutron population \( N \), the precursor concentration \( C \) and the counts in a detector \( Z \) at time \( t \):

\[
P(N,C,Z,t|N_0,C_0,Z_0,t_0) = \sum P(N',C',Z',t'|N_0,C_0,Z_0,t_0)P(N',C',Z',t' \rightarrow N,C,Z,t)
\]  

(C.1)

with the left hand side signifying the probability of finding \( N \) neutrons, \( C \) precursors, and \( Z \) counts in the detector at time \( t \), if we began at initial values for all parameters with subscript 0. This equation states that the final probability is the sum over all intermediate states multiplied by the respective transition rate. As the distributions considered are only dependent on time, we are considering a point reactor model. We now define the forward equation, based on known transition rates in a nuclear reactor, i.e. \( \lambda_f \) for fission, \( \lambda_d \) for detection, \( \lambda_c \) for capture, and precursor decay \( \lambda \)

\[
\frac{dP(N,C,Z,t)}{dt} = \lambda_c(N+1)P(N+1,C,Z,t)
\]

\[
+ \lambda_d(N+1)P(N+1,Z-1,t)
\]

\[
+ \lambda_f \sum_{n=1}^{N+1} \sum_{m=0}^{C} (N+1-n)p_f(n,m)P(N+1-n,C-m,Z,t)
\]

\[
+ S \delta(N-1,C,Z,t) + \lambda(C+1)P(N-1,C+1,Z,t)
\]

\[
- ((\lambda_f + \lambda_c + \lambda_d)N + \lambda C + S)P(N,C,Z,t)
\]

(C.2)

The terms on the right hand side are respectively the joint probabilities of capture of a neutron, detection of a neutron, fission, source neutron addition, delayed emission and no event at all with constant population. \( p_f \) denotes the probability of emitting \( n \) prompt neutrons and \( m \) precursors per fission. This equation is impractical to solve directly and is therefore expanded by polynomials to form probability generating functions (PGF). We define

\[
G(x,y,v,t) = \sum_{N} \sum_{C} \sum_{Z} x^N y^C v^Z P(N,C,Z,t)
\]

(C.3)
Appendix C. Kolmogorov forward approach to derive point kinetics

as the PGF of $P$ and

$$g_f(x,y) = \sum_n \sum_m x^n y^m p_f(n,m)$$  \hspace{1cm} (C.4)

the PGF of $p_f$. $x$, $y$, and $v$ are complex numbers which are assigned to the respective physical quantity. An overview of PGFs and their properties as well as an instructive example can be found in Appendix B. Expanding Equation C.2 by the infinite polynomial series accordingly yields

$$\frac{\partial G}{\partial t} = [\lambda_f (g_f(x,y) - x) - \lambda_d(x - v)] \frac{\partial G}{\partial x} + [\lambda(x - y)] \frac{\partial G}{\partial y} + S [x - 1] G$$ \hspace{1cm} (C.5)

With the condition:

$$G(x,y,v,t = t_0) = 1$$ \hspace{1cm} (C.6)

This equation is generally not solved for $G$ rather than differentiated with respect to the variable representing one state parameter (i.e $x$ for $N$ etc.) yielding the first moment equations

$$E(N(t)) \equiv N(t) = \frac{\partial G(x,y,v,t)}{\partial x} |_{x=y=v=1},$$ \hspace{1cm} (C.7)

$$E(C(t)) \equiv C(t) = \frac{\partial G(x,y,v,t)}{\partial y} |_{x=y=v=1},$$ \hspace{1cm} (C.8)

and

$$E(Z(t)) \equiv Z(t) = \frac{\partial G(x,y,v,t)}{\partial v} |_{x=y=v=1}$$ \hspace{1cm} (C.9)

Equivalently, expressions for the generating function derivatives of the fission emission probabilities are found

$$\frac{\partial g_f(x,y)}{\partial x} = \sum_n \sum_m n p_f(n,m) \equiv \langle \nu_p \rangle \equiv \langle \nu \rangle (1 - \beta)$$ \hspace{1cm} (C.10)

$$\frac{\partial g_f(x,y)}{\partial y} = \sum_n \sum_m m p_f(n,m) \equiv \langle \nu_d \rangle \equiv \langle \nu \rangle \beta$$ \hspace{1cm} (C.11)

The first moment i.e. mean equations are then

$$\frac{dN(t)}{dt} = \frac{\rho - \beta}{\Lambda} N(t) + \lambda C + S$$ \hspace{1cm} (C.12)

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} N(t) - \lambda C(t)$$ \hspace{1cm} (C.13)

$$\frac{dZ(t)}{dt} = \epsilon \lambda_f N(t)$$ \hspace{1cm} (C.14)

With reactivity, prompt generation time and efficiency defined as

$$\rho = \frac{\langle \nu \rangle \lambda_f - (\lambda_f + \lambda_c + \lambda_d)}{\langle \nu \rangle \lambda_f}$$ \hspace{1cm} (C.15)

$$\Lambda = \frac{1}{\langle \nu \rangle \lambda_f}$$ \hspace{1cm} (C.16)
C.1. Derivation of Feynman-α from the forward approach

\[ \epsilon = \frac{\lambda_d}{\lambda_f} \]  
\( \text{(C.17)} \)

Evidently, the point kinetics, see Equations C.12 and C.13, have been derived from probabilistic grounds, which were also found earlier from the transport equation in Section 2.1.2. Equation C.14 takes the detection process into account, and as expected, yields a linear proportionality between the neutron density and the counts. This is also the first observable equation we have derived. As stated in Appendix B, the generating function method can also be used to calculate higher moments of the distribution. In Appendix C.1 we sketch the second moment equations of detection and how the Feynman-α formulation follows.

C.1 Derivation of Feynman-α from the forward approach

We seek to find a formulation for the variance of the detector counts \( Z \). We use Equation C.5. The second moments are found by differentiating another time with respect to the introduced variable:

\[ \mathbb{E}(Z(Z-1)) = \frac{\partial^2 G(x,y,v,t)}{\partial v^2} |_{x=y=v=1} \]  
\( \text{(C.18)} \)

For illustrative purposes, the modified variance \( \mu_{XX} = \text{Var}(Z) - \text{Mean}(Z) \) is introduced, finally yielding:

\[ \frac{d\mu_{ZZ}(t)}{dt} = 2\epsilon\lambda_f\mu_{NZ}(t) \]  
\( \text{(C.19)} \)

This equation is coupled with the modified covariance \( \mu_{NZ} = N\overline{Z} - \overline{N}\overline{Z} \)

\[ \frac{d\mu_{NZ}(t)}{dt} = -\alpha_{p}\mu_{NZ}(t) + \lambda\mu_{CZ}(t) + \epsilon\lambda_f\mu_{NN} \]  
\( \text{(C.20)} \)

which in turn is coupled with the delayed neutron modified covariance equation

\[ \frac{d\mu_{CZ}(t)}{dt} = -\lambda\mu_{CZ}(t) + \frac{\beta}{\lambda}\mu_{NZ}(t) + \epsilon\lambda_f\mu_{NC} \]  
\( \text{(C.21)} \)

In order to solve for \( \mu_{ZZ} \) the equations are Laplace transformed to allow for simpler decoupling. After some algebraic massage, we find the previously motivated Feynman-α equation, here displayed in its full form accounting for \( i \) delayed neutron groups [50]

\[ \frac{\mu_{ZZ}}{Z} = \sum_i Y_i \left\{ 1 - \frac{1 - e^{-\alpha t}}{\alpha t} \right\} \]  
\( \text{(C.22)} \)

with the Feynman \( Y \) for the amplitude of the excess fluctuation compared to a Poisson distribution

\[ Y_i = \frac{2\epsilon D\nu A_i H(-\alpha_i)}{-\alpha_i} \]  
\( \text{(C.23)} \)

\( H(\alpha) \) denotes the zero power transfer function found in Equation 2.44 and \( A_i \) are the residues of the transfer function. Considering only one group of delayed neutrons, we can simplify the equations further to

\[ \frac{\mu_{ZZ}}{Z} = \frac{\text{Var}Z}{Z} - 1 = Y_1 \left\{ \left( 1 - \frac{1 - e^{-\alpha t}}{\alpha t} \right) + \frac{Y_2}{Y_1} \left( 1 - \frac{1 - e^{-\alpha_d t}}{\alpha_d t} \right) \right\} \]  
\( \text{(C.24)} \)
with

\[ Y_1 = \frac{\epsilon D_v}{(\rho - \beta)^2} \]  
\[ Y_2 = \frac{\epsilon D_v}{(\rho - \beta)^2} \left[ \left( \frac{\rho - \beta}{\rho} \right)^2 \left( 1 + \frac{2\nu_p \nu_d}{\nu_p(\nu_p - 1)} \right) - 1 \right] \]  
\[ \alpha_d = \frac{\lambda \rho}{\beta - \rho} \]
Appendix D

Current Amplifier Details

Here we present details pertaining to the characteristics and calibration of the current amplifiers. A figure displaying the final design of the board geometry is shown in Figure D.1. The detailed electronics schematics can be found in Figure D.2. An example of a measured transfer function is shown in figure D.3, indeed showing the desired flat low frequency response and a cut-off at more than 2 kHz. A linearity test was performed in CROCUS, the results are shown in Figure D.4.
Figure D.2  Detailed electronic schematic of the current mode amplifier 2. The board logic is the same among the other amplifiers manufactured to date.
Figure D.3  Measured transfer function of the current amplifier box 1 using a pulse generator.

Figure D.4  Linearity tests of the current mode amplifiers boxes 1 and 2 in CROCUS.
Appendix E

Monte Carlo code input decks

E.1 TRIPOLI-4 input deck

An input file for TRIPOLI-4 used for noise simulations resembles the following (arbitrary user inputs are denoted with "X", comments after "//"): 

ROOT_GEOMETRY // Geometry input file generated via a PyROOT script geometry.root END_ROOT_GEOMETRY

FILE // Material composition input file generated via a PyROOT script composinit.jdd

LIST_SOURCES
// Source input, options range from monochromatic point sources to // analytical formulae for space and energy. END_LIST_SOURCES

TRACKS // Option to activate track saving used for noise simulation
INTERACTION 1 33 // Optional: Example of filtering: Only tracks with // interaction number 33 (fission) will be saved to the root.tracks-file
VOLUME 1 N // Optional: Example of filtering: Only tracks that cross // volume number N will be saved to the root.tracks-file. END_TRACKS

SIMULATION

FIXED_SOURCES_CRITICALITY // Analog is only available in fixed source mode BATCH X SIZE X RANDOM MT19937_RANDOM SEED X FORCED_UNIT_BASE_INTERPOL
Appendix E. Monte Carlo code input decks

PARTICLE 1 NEUTRON

ENERGY_INF NEUTRON 1.0E-11
ENERGY_SUP NEUTRON 20.0

PROPT_FISSIONONLY // Optional: Delayed neutrons are not sampled.
ANALOG NEUTRON // Analog fission model keyword to force equal weights
// after interactions.
FISSIONMODEL LLNL_FISSION // Keyword to enforce FREYA fission events as
// source for fission yield, energies and angles.

SUPER_HISTORY 10000 // Optional: Generation cut-off for neutrons.
// Used for close to critical or super-critical calculations
// to ensure finite calculation times.
END_SIMULATION

E.2 Serpent 2 input deck

Generally, we highlight the following lines that are required after geometry and material
definitions to use analog transport:

set nps X // External Source mode: X number of neutrons
src Nsource n sf X // Input source file X
set delnu 1 // Optional: Delayed neutrons are sampled. Default is 0.
set impl 0 0 0 // Implicit capture settings, 0 for no implicit capture.
set tcut 10 // Optional: Time cut-off.
set gcut 10000 // Optional: Neutron generation cut-off.
set nbuf 1e6 // Neutron memory buffer, set higher than default.
Appendix F

Bootstrap and MLE for noise parameter estimation

In order to achieve a truer estimate of the fitted noise response parameters and their uncertainties, we employed a bootstrap method and analyzed the results using MLE. The bootstrap was conducted by segmenting the input time series, sampling from it with replacement to create shuffled signal. The MLE was then computed from the shuffled signal, yielding distribution parameters for the noise response parameters.

Figure F.1  Schematic describing the bootstrap method for PSD analysis.
Appendix F. Bootstrap and MLE for noise parameter estimation

placement to create a new time series, and then using the re-sampled series to calculate a sample PSD. The sample PSD was then fitted to extract the kinetic parameters. The time series re-sampling was typically done 1000 times to achieve smooth distributions. The resulting distributions of kinetic parameters were then fitted with 20 common distributions using MLE, and their negative log likelihood was compared to choose the most appropriate. The process is visualized in Figure F.1. The distributions we considered are those directly available to MATLAB 2019b [185]:

- Bernoulli Distribution
- Beta Distribution
- Binomial Distribution
- Birnbaum-Saunders Distribution
- Burr Type XII Distribution
- Uniform Distribution (Discrete)
- Exponential Distribution
- Extreme Value Distribution
- Gamma Distribution
- Generalized Extreme Value Distribution
- Generalized Pareto Distribution
- Geometric Distribution
- Half-Normal Distribution
- Inverse Gaussian Distribution
- Logistic Distribution
- Loglogistic Distribution
- Lognormal Distribution
- Nakagami Distribution
- Negative Binomial Distribution
- Normal Distribution
- Poisson Distribution
- Rayleigh Distribution
- Rician Distribution
- Stable Distribution
- t Location-Scale Distribution
- Uniform Distribution (Continuous)
- Weibull Distribution
Appendix G

Power Spectral Density method: Understanding inherent bias

The uncertainty quantification in common noise experiments typically include only the least squares parameter confidence bounds as well as fission rate variance. Final estimates of kinetic parameters therefore often exhibit low uncertainty values (<1%), which were often found to be too small to explain parameter discrepancies to code [23, 24, 110]. Parameters for the PSD estimation are, presumably, set via trial and error to optimize for code prediction agreement. This inevitably introduces a bias based on the prediction tool. As we will discuss in the following Sections, a strategy on how to choose the PSD parameters can indeed be found.

G.1 Frequency domain analysis: Overview

Frequency domain analysis is based on the Fourier transform \( \mathcal{F} \), applied to a continuous function \( x \) over time:

\[
\mathcal{F}(x(t)) = F(f) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \tag{G.1}
\]

All digital Fourier transform methods rely on discrete Fourier transforms (DFT) in order to calculate the frequency of a sequence of complex valued samples, e.g. an analog to digital converted electrical signal in volts over time \( t \):

\[
\mathcal{F}(x(t)) = \sum_{0}^{N-1} x_n e^{-ifn2\pi/N} \tag{G.2}
\]

A popular algorithm to implement DFT is the fast Fourier transform (FFT). The free parameter is hereby the sample size \( N \), which is typically the total length of the signal.
Periodogram calculation The power spectral density (PSD) is found via the Plancherel theorem. The periodogram estimate of the PSD is the FFT multiplied by its complex conjugate:

\[ G_P(f) = F(f) \cdot F^*(f) \]  (G.3)

This periodogram needs to be normalized, such that the integral over it gives the signal power \( P \):

\[ P = \sum_f \hat{G}_P(f) \]  (G.4)

Taking only positive frequencies into account (i.e. the so-called one-sided PSD), the normalized periodogram reads

\[ \hat{G}_P(f) = \frac{2 \cdot F(f) \cdot F^*(f)}{f_{min}} \]  (G.5)

\( f_{min} \) is the spectral resolution (or minimum resolvable frequency), inferred from the time resolution of the sample signal. \( \hat{G}_P(f) \) is the so-called periodogram PSD estimate and is what is usually meant when referring to "pure" FFT calculations.

For random processes, strictly speaking, the power spectrum cannot be directly calculated using the above integrals/sums, as stochastic processes are generally not (Lebesgue) integrable. The PSD is rather estimated via the Wiener-Khintchine theorem, using expectations values for a stationary random process.

For a periodic signal the Fourier transform is unique, i.e. the value of \( G_P \) (subscript \( P \) for periodogram) and its uncertainty are given only by the sample values \( x_n \) and uncertainty \( \sigma_{x_n} \). However, one finds that with increasing sample length \( N \), the variance of the power spectrum is just the power spectrum squared [186]:

\[ \lim_{N \to \infty} \sigma_{G_P}^2 = G_P^2 \]  (G.6)

The variance is therefore not arbitrarily small with increased sample length. Statistically this means the periodogram method is not a consistent estimate of the PSD, albeit the mean value is asymptotically the true PSD value. Also, as consecutive PSD values are uncorrelated, the periodogram will exhibit white noise erratic behavior [187].

Bartlett and Welch method In order to arrive at a consistent estimator for the PSD, the Bartlett method can be employed. It consists in splitting the sample into \( M \) segments of length \( W \), calculating the periodogram of each section, and then averaging the periodograms to receive the total PSD. Averaging these segments is possible under the assumption of ergodicity. Assuming independence of the neighboring segments, reasonably warranted when neglecting the small contribution of delayed neutrons, the variance of a Bartlett-estimated PSD (subscript \( B \) for Bartlett) for a Gaussian signal is now:

\[ \sigma_{G_B}^2 = \frac{1}{M} G_B^2 \]  (G.7)
G.1. Frequency domain analysis: Overview

Figure G.1

With a decreasing variance with $M$, we have found a consistent estimate of the PSD. Figure G.1 shows the calculated cross-periodogram compared to the Bartlett method CPSD for the reference measurement in CROCUS (described in Section 3) of 5.5h. Clearly, the periodogram has a high variance and just by visual inspection does not meet the standard for a high accuracy fit. The averaged Bartlett CPSD, found with an arbitrary segment duration of 25s, visually exhibits the Lorentzian shape and low variance for curve fitting purposes. The number of DFT points was chosen to be the window or sample length. These arbitrarily chosen parameters are indeed the subject of the following section.

We note that standard algorithms treat a lower number of DFT points (compared to the segment length) by averaging, while a higher number is dealt with by interpolation. Both effects do not add, rather remove, PSD information and therefore just add bias.

The Welch [188] method is equivalent to the Bartlett method aside from allowing for overlap of successive segments to retain more information. The variance of a Welch PSD estimate at 50% overlap is approximately [127]

$$
\sigma_{GW}^2 = \frac{9}{16} \sigma_{GB}^2
$$

Hence we expect a smaller total variance due to more information being processed in the PSD estimation. We note that the calculated variance of the PSD estimate is often used to weight the subsequent kinetic model fit. This will be discussed in Section 4.2.5.
Appendix G. Power Spectral Density method: Understanding inherent bias

G.2 Summary on PSD estimation method

Based on the summary of the Section before, we propose a terminology and method of PSD estimation:

- The PSD estimation in noise experiments is mostly done using the following methods: *Periodogram, Bartlett or Welch* estimation.

- The ability to quantify uncertainties depend on the method, with the periodogram generally being the least favored choice due to the inconsistency and high variance.

- In the choice of a method, the sampling frequency and sample length (+ window size/type) are necessary data for reproducibility. In typical noise experiments, the values, albeit mostly available, tend to be arbitrarily chosen via predictor biased optimization.

G.3 PSD spectral resolution and shape

A minimum of 10 averaged spectra is suggested by [127] to reduce the standard deviation by a factor three compared to the periodogram, at the expense of spectral resolution. In practice, 20-50 averaged spectra is common literature optimal, with the spectral resolution demands being application dependent. We will now examine how the number of spectra influences the PSD estimation in neutron noise.

We first consider the PSD bandwidths effect on the PSDs shape. The bandwidth is given by the minimum and maximum frequency, $f_{\text{min}}$ and $f_{\text{max}}$, of the estimated PSD. $f_{\text{max}}$ is half the sampling frequency $F_s$, also called Nyquist frequency [189], and should be chosen high enough to capture the desired frequencies. In the case of zero power reactor noise analysis, $f_{\text{max}}$ should be chosen to be about an order of magnitude higher than the cut-off frequency of the reactor. Rates below risk cutting into relevant signal, especially at $\rho < -\beta_{\text{eff}}$ sub-critical states with higher cut-off frequencies. $f_{\text{min}}$, the spectral resolution, is the inverse of the window duration - dependent on the sample or window length $W$ and the bin duration or dwell time $\Delta t$:

$$
f_{\text{min}} = \frac{F_s}{W} = \frac{1}{\Delta t \times W} \quad (G.9)
$$

Note that the window (and thus signal) are required to be of length $2^n$ with $n \in \mathbb{N}_0^+$ in order to not be truncated or zero padded by common FFT algorithms. This equation can be used to determine minimum measurement lengths, $t_{\text{min}}$, for $M$ averaged spectra with a given sampling rate and window length:

$$
t_{\text{min}} = M f_{\text{min}} \quad (G.10)
$$

We will use the reference neutron noise measurement presented in Section 3.3 to determine the effect of number of averaged spectra on the estimated PSD, quantifying the shape by examining the low frequency region between 0 and 4 Hz, denoted $a$, and $\tilde{c}$, the high
G.3. PSD spectral resolution and shape

Using a Bartlett PSD estimate on the reference data, we compare the effective spectral resolution to the required window length and thus amount of averaged spectra. Table G.1 displays said data with Figure G.2 showing the APSD north and CPSDs in dependence of number of averaged spectra for comparison.

The observation of Figure G.2 reveals that averaging is firstly a method to visually distinguish the PSD shape from zero and from white noise - in this case at 32 averaged spectra. This is consistent with the literature estimate of 20-50 averaged spectra. Note that electronic noise at 150Hz became only visible after about 8 spectra, clearly discernible at 30. This enforces how signal quality assurance is reliant on spectral averaging.

A visual discernibility of the PSD from white noise is nonetheless an expert opinion and thus biased. In order to arrive at a consistent and unbiased metric for PSD estimation success, we suggest to observe the distributions of the PSD parameters $a + \tilde{c}$ and $\tilde{c}$ in dependence of spectral resolution. In Figures G.3 and G.4 we display the APSD south and CPSD histograms of the frequency ranges 1-5Hz to approximate $a + \tilde{c}$ and 300 to 350Hz for $\tilde{c}$.

With more spectra averaged, following the central limit theorem, we expect the values for $a + \tilde{c}$ and $\tilde{c}$ to follow a normal distribution, irrespective of their initial distribution. Indeed, we can observe for the APSD the shift from an initially log-normal distribution to a normal distribution. For the CPSD the shape for few averaged spectra resembles an exponential distribution, with a normal distribution arising above $\sim$100 averaged spectra.

For kinetic parameter determination, a non-linear least squares fit is usually performed. Without prior knowledge of the distribution of estimated PSD values, we therefore assert that a reasonable PSD estimation requires enough averaged spectra to indeed have locally
Appendix G. Power Spectral Density method: Understanding inherent bias

Figure G.3  Southern APSD $a + \tilde{c}$ and $\tilde{c}$ parameter distributions for the number of averaged spectra.

Figure G.4  Southern CPSD $a + \tilde{c}$ and $\tilde{c}$ parameter distributions for the number of averaged spectra.
normally distributed PSD values. This also ensures a least squares fit to converge on the mean values of the parameter distributions.

To prove this, we fitted the data in Figures G.3 and G.4 with log normal distributions. In Figure G.5 we display the mean value and standard deviation of the associated normal distribution of $a$ with number of averaged spectra. For the APSDs this requires the subtraction of $a + \tilde{c}$ by $\tilde{c}$. The standard deviation is divided by 10 for illustrative purposes.

<table>
<thead>
<tr>
<th>Window length</th>
<th>No. of averaged spectra</th>
<th>Spectral Resolution (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^{10}$</td>
<td>19476</td>
<td>1.0</td>
</tr>
<tr>
<td>$2^{11}$</td>
<td>9738</td>
<td>0.5</td>
</tr>
<tr>
<td>$2^{12}$</td>
<td>4869</td>
<td>0.2</td>
</tr>
<tr>
<td>$2^{13}$</td>
<td>2435</td>
<td>0.1</td>
</tr>
<tr>
<td>$2^{14}$</td>
<td>1217</td>
<td>6.1E-02</td>
</tr>
<tr>
<td>$2^{15}$</td>
<td>609</td>
<td>3.1E-02</td>
</tr>
<tr>
<td>$2^{16}$</td>
<td>304</td>
<td>1.5E-02</td>
</tr>
<tr>
<td>$2^{17}$</td>
<td>152</td>
<td>7.6E-03</td>
</tr>
<tr>
<td>$2^{18}$</td>
<td>76</td>
<td>3.8E-03</td>
</tr>
<tr>
<td>$2^{19}$</td>
<td>38</td>
<td>1.9E-03</td>
</tr>
<tr>
<td>$2^{20}$</td>
<td>19</td>
<td>9.5E-04</td>
</tr>
<tr>
<td>$2^{21}$</td>
<td>10</td>
<td>4.8E-04</td>
</tr>
<tr>
<td>$2^{22}$</td>
<td>5</td>
<td>2.4E-04</td>
</tr>
<tr>
<td>$2^{23}$</td>
<td>2</td>
<td>1.2E-04</td>
</tr>
<tr>
<td>$2^{24}$</td>
<td>1</td>
<td>6.0E-05</td>
</tr>
<tr>
<td>Full signal</td>
<td>1</td>
<td>5.0E-05</td>
</tr>
</tbody>
</table>

Overall, as expected, we find a decreasing variance in the fitted distributions with more averaged spectra.

The CPSD mean exhibits a mostly constant estimate of $a$. The mean values of the APSDs, however, exhibit a saddle region behavior, giving a constant value between 5 and $\sim 5000$ averaged spectra. Regarding Table G.1, the spectral resolution above 5000 averaged spectra decreases to 0.5 Hz. This resolution can be considered too coarse to capture the PSD behavior for the specific case of a critical CROCUS core, as indicated by the dip in mean value for all three $a$ estimates in Figure G.5.

Note that overlapping by 50%, i.e. using the Welching method with a rectangular window, yielded negligible differences ($< 10^{-8}$/Hz $\sim 1\%$) to the presented results. A smaller estimated variance due to a higher amount of averaged spectra is thus the only effect.
From these considerations and calculations, we can conclude that, given the constraints of a given measurement time and sampling rate, a minimum of 20 averaged spectra with a resulting spectral resolution of 0.1 Hz or better yields a consistent APSD estimate. We shall hereby define this minimum a robust zero power PSD estimate. For zero power noise measurements at a sampling rate of 1000 Hz this implies a minimum measurement time of 200 s. A too coarse spectral resolution $> 0.5$ Hz was also found to impact the parameter distributions. The CPSD is only impacted by the coarse spectral resolution bias, and appears otherwise to yield consistent mean values.

In the following sections we examine the effect of measurement time and short measurements, detection noise, and overlapping of segments.

### G.4 Measurement time, robust PSD estimation and overlapping

For longer measurements, we next inquire on how long a noise measurement needs to last in order to estimate, if possible, the true PSD. As previously discussed, a minimum of 20 averaged spectra with a resolution of 0.1 Hz is necessary for a robust PSD estimation. More averaged spectra whilst preserving the spectral resolution reduces the variance of the PSD parameters. The consolidation of the mean value, however, is of main interest with regards to kinetic parameter determination.

In Figure G.6 we compare how adding robust measurement segments to APSD/CPSD $a + \tilde{c}$ and $\tilde{c}$ estimates resulting in 20 averaged spectra, in randomized and non-randomized order.
Figure G.6 Display of the $a + \tilde{c}$ and $\tilde{c}$ PSD parameters for robust segment adding to the signal of which the PSD is estimated. The segments are randomized in order in comparison to estimate bootstrapped uncertainties.

from the reference measurement. The randomization is to account for bias induced by power drifts, albeit small, and to bootstrap the uncertainty within.

As is visible from Figure G.6, the variance of the PSD parameters decreases over the first 20 added segments, corresponding to 4000s or 1.1 hours. Hereafter, the mean value only changes marginally while the variance decreases significantly only in the non-randomized case.

Indeed a randomized segment choice of a signal allows to estimate an upper bound of uncertainty, but will overestimate longer measurement variance improvements. Ultimately, a minimum of $\sim$1 hour is necessary to achieve a stable mean value, without a significant improvement in variance visible thereafter.

Overlapping shorter measurement durations is a method used to extract more information out of a Bartlett PSD. In Figure G.7 we compare the CPSD parameters $a$ and $\tilde{c}$ for a measurement length of $2^{17}$ at 1000Hz, i.e. 2.2 min. Indeed, as opposed to the negligible effect on long measurements, a distinction between the distributions of $a$ and $\tilde{c}$ becomes possible after 12 spectra. The distinction metric is hereby the overlap of $1\sigma$ and the mean value of the respective other parameter. Full distribution distinction, i.e. the non-overlap
Appendix G. Power Spectral Density method: Understanding inherent bias

Figure G.7 Southern CP SD $a$ and $\bar{c}$ parameter distributions for the number of averaged spectra.

of both standard deviations, is also achieved after 48 spectra when using a Welch estimate as compared to 97 for the Bartlett estimate, indicating a more effective use of data when considering short acquisitions. This was shown to be true for perturbation noise studies [48].

With regards to the PSD parameters, based on observations made on experimental data of CROCUS, we hereby conclude:

- Increasing the measurement length reduces the variance of the parameter distributions, with a minimum measurement of 1 hour necessary to achieve a stable mean estimate.

- Welch overlapping allows for a shorter measurement to provide more information i.e. a robust estimate of the PSD with fewer averaged spectra. The minimum amount reduces from 20 to $\sim$12, which is incidentally the variance reduction factor of $9/16$ for Welch estimation. This gain diminishes for longer measurements ($>30$ min).
Appendix H

SNR for CROCUS noise instrumentation

SNR for a given measurement system depends on its initial noise sources and on how long we have observed it. We tested the typical measurement setups for neutron and gamma noise, such as the ones shown in Section 3.2. The previous considerations on AWGN allowed us to derive an empirical expression to estimate the SNR of a given PSD, as schematically displayed in Figure H.1. Note that the window length was chosen to result in a spectral resolution of 0.1 Hz, a choice motivated in Appendix G.3. We compare the SNR of the CFUL01 APSDs and CPSDs based on this approximation in Figure H.2. A general observation is thus that the cross-correlation increases the SNR by about 5 to 10.

In order to illustrate why a bootstrap might be necessary to capture the true uncertainty in a noise measurement, we show the first order parameter and uncertainty estimation over measurement time in Figure H.3 for $\alpha$ and $\beta_{\text{eff}}$. Albeit the mean values are found with smaller than 1% uncertainties, we observe fluctuations in the estimator that are not captured by the uncertainty.
Appendix H. SNR for CROCUUS noise instrumentation

Synthetic data

Analytic PSD with AWGN

PSD

100 Hz

Frequency

Histogram of >100 Hz region

MLE of histogram, yielding $N(\mu, \sigma)$

width of histogram | AWGN SNR
---|---
6.3 | 10
1.5 | 20
0.1 | 30
... | ...

Exponential fit of SNR(width)

SNR estimation model

Experimental data

Measured APSD

PSD

100 Hz

Frequency

Histogram of >100 Hz region

MLE of histogram, yielding $N(\mu, \sigma)$

Experimental SNR

Figure H.1 Overview of the experimental SNR estimation method.
Figure H.2 SNR over time for the CFUL01 and the current amplifiers.

Figure H.3 Kinetic parameters and their first order uncertainties with measurement duration.
Appendix I

Noise field measurements: Photos

I.1 Neutron noise: Experimental channel

The Plexiglas channel for detector placement in the water reflector surrounding CROCUS is depicted in several chosen locations in Figure I.1.
Appendix I. Noise field measurements: Photos

(a) Position 1, 4 cm from the fuel.

(b) Position 3, 10 cm from the fuel.

(c) Position 4, 19 cm from the fuel.

Figure I.1 Pictures of chosen CFUL01 detection setups for the neutron noise field experiments.
I.2 Gamma noise: BGO placement

The BGO detectors were placed in various locations in and outside the CROCUS cavity, we depict a chosen set of locations in Figures I.2 and I.3.
Appendix I. Noise field measurements: Photos

Figure I.2 Pictures of chosen BGO detection setups for the gamma noise field experiments.

(a) Position 1, 0.9 m from the core center.

(b) Position 4, 1.4 m from the core center.

(c) Position 6, 1.9 m from the core center.
I.2. Gamma noise: BGO placement

(a) Position 7, 3.2 m from the core center.

(b) Position 8, 3.3 m from the core center.

(c) Position 11, 6.9 m from the core center.

Figure I.3  Pictures of chosen BGO detection setups for the gamma noise field experiments.
Appendix I. Noise field measurements: Photos
Bibliography


Bibliography


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OSKARI VILLE PAKARI  
Nuclear Physicist

PERSONAL INFORMATION

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EDUCATION

- **Ph.D. Physics**  
  2016 - 2020 | EPFL, Switzerland  
  Experimental and numerical study of stochastic branching noise in nuclear reactors

- **M.Sc. Nuclear Engineering**  
  2014 - 2016 | ETH Zurich & EPFL, Switzerland  
  Semester project at PSI, Master's thesis at EPFL: Development of current mode noise measurements

- **B.Sc. Physics**  
  2011 - 2014 | KIT, Germany  
  Minor in Computer Science  
  Thesis at the Tritium Laboratory Karlsruhe: Passive detection of Tritium in Helium mixtures

EXPERIENCE

- **Ph.D. candidate**  
  2016 - Present | EPFL, Switzerland  
  **Teaching**  
  Lecturer and TA for "Radiation Detection", "Neutronics", and "Reactor experiments"  
  **Supervision**  
  4 Semester project students  
  1 Master's thesis  
  **Presentations**  
  International conferences: ANIMMA 2017 & 2019, PHYSOR 2018  
  Summer schools: EJC 2017, FJOH 2018  
  **Recipient of EU ENEN+ research exchange grant with IRSN & CEA Saclay/Paris-Sud**  
  Licensed reactor protection expert  
  **Licensed reactor operator for CROCUS**

EXTRACURRICULAR ACTIVITIES

- **Member, Model United Nations EPFL**  
  Oct 2017 - Present  
  Crisis delegate (Advanced) at the LIMUN 2019 Conference

- **Co-President, EnergyBPT EPFL**  
  Oct 2016 - Oct 2017  
  Four events per semester. Lead the creation of the first editions of: EPFL Energy Night (50-100 participants, 10 labs, 4 companies)  
  Energy Student Conference (40 participants, 3 companies + Workshop)

- **Member, Model United Nations EPFL**  
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  Energy Student Conference (40 participants, 3 companies + Workshop)

LANGUAGE SKILLS

- English C2, native
- German C2, native
- Finnish C1, mother tongue
- French B2, conversant
- Spanish A2
- Mandarin A1

PROGRAMMING SKILLS

- MATLAB, Python (NumPy, SciPy, Pandas), C++

INTERESTS

- Tae-kwon-do, skiing, surfing
- Music, playing guitar
- Philosophy and ethics

AWARDS

- "Best presentation" - FJOH 2018 Summer school on nuclear reactors
- 2x "Award of the city of Karlsruhe for extraordinary achievement in sports" for 2x second place at the German Universities Tae-kwon-do Championships