A Multi-Criteria Decision-Support Workflow for Early-Stage Neighborhood Design based on Predicted Solar Performance

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ABSTRACT: Despite recent developments, the need for adequate guidance and support in the early decision-making process of urban planners, designers, and architects has recurrently been recognized. Traditional performance assessment methods, which are often based on partial and independent dynamic simulations evaluating individual metrics, are better suited for detailed design and are particularly complex and time-consuming at the urban scale. They typically follow a linear design-and-test approach, limiting the user-guidance features. Taking a different approach, this paper proposes a multi-criteria decision-support workflow that evaluates the daylight and passive and active solar potential of early-stage neighborhood designs. The performance evaluation is done through a predictive mathematical function for the passive solar and daylight potential, requiring little information from the user. The implemented workflow is introduced, and the development of the underlying performance assessment engine is summarized, along with the results from a proof-of-concept study to probe the validity boundaries of the predictive functions. Results show the proposed workflow to be promising as an interactive and real-time performance-based design support.
Keywords: urban design, early-stage, decision-support, predictive modeling, solar performance

INTRODUCTION

The early-design phase of neighborhood projects corresponds to a temporal and spatial scale of great importance regarding the design decisions that are being made. These define, among others, the building typology (shape), massing, and layout, which affect the heat losses and solar gains and strongly influence the future energy performance of the design. To help architects, urban designers and planners make informed decisions and reach the ambitious energy goals set by various norms and labels, the exploration of early-design alternatives from a performance-based perspective must be facilitated and supported.

While the amount of studies in the field of meso-scale performance assessment is increasing, very few have taken the form of design decision-support tools. Notable examples are the Rhino-based environment Umi, evaluating energy use, walkability and daylight [1], the CitySim software, performing the simulation and optimization of energy fluxes of urban districts [2] and ArchiWIZARD [3], a simplified 3D thermal and daylight simulation software. A shared feature among such tools is the linear workflow they induce for evaluating and comparing design alternatives [4], through a manual iterative process starting with a user-defined 3D model, often with detailed information usually known only at advanced design stages, and ending with a performance value for each design iteration. This intrinsic 'generate-and-test' process yields limited guidance and supports analysis rather than design [5]. Emerging paradigms are instead promoting a performance-based design workflow, involving simultaneously generating and evaluating multiple design variants [4]. The Animated Building Performance Simulation framework by [6] follows such an approach, allowing the solar performance evaluation of multiple building-scale design variants.

To enable this type of approach, current limitations linked to the underlying performance assessment engine on which most tools are relying must be addressed. This engine usually solves equations that simulate the behavior of a building [7], a process that requires detailed inputs and quickly becomes complex and time-consuming at the meso scale. The increasing computational power as well as ongoing development of simplified techniques, such as the ones that can be found in the above-mentioned software, are contributing to addressing this issue.

An alternative path for overcoming these limitations is to resort to statistical methods based on mathematical expressions known as metamodels, surrogate models or emulators. Their use in the field of building performance is fairly recent yet increasing. The mathematical expression often takes the form of a multi-linear regression [7, 8, 9] that is then used for energy prediction at the building-scale.

This paper proposes a multi-criteria decision-support workflow that distinguishes itself through its generative, multi-variant approach as well as its underlying predictive performance evaluation conceived for the neighborhood scale. Three performance criteria are evaluated: (i) the daylight potential, quantified by the spatial daylight autonomy, (ii) the passive solar potential, quantified by the annual energy need for heating and cooling, and (iii) the active solar potential, quantified by the annual energy production. The decision to use these metrics follows from a previous study [10].

To sidestep the computational cost and required information linked to daylight and thermal (physics-based) simulations, particularly important at the neighborhood scale, the first two criteria are evaluated through predictive mathematical functions, taking as inputs a set of parameters related to the geometry and irradiation level of the buildings.

This paper introduces the proposed workflow before focusing on the development and testing of the predictive functions.

OVERVIEW OF PROPOSED WORKFLOW

The proposed workflow was implemented as a prototype that was tested by architects and urban designers during workshops conducted in the fall of 2015 and further detailed in [11]. The workflow and the tools used in its implementation are presented in Figure 1. Screenshots of the prototype, which was named Urban SOLar Visual Explorer (UrbanSOLve), are shown in Figure 2.

The workflow follows a generative approach in which design alternatives are automatically and randomly sampled from the space of possible designs, defined through user-inputs, and passed to the performance assessment engine.

Users must first model the existing context (optional) and parcel of land on which to ‘build’ in the 3D modeller Rhino [12]. Using the custom interface, coded in C# and packaged as a Grasshopper [13] component, they then select each building’s typology – simple volume, L-shaped or courtyard – and define its position as a center or corner point on the parcel (see Figure 2, left). Minimum and maximum values for each design variable (building dimensions) and constraint (e.g. density) are set before launching the automated generation of design variants. This phase is done by the plugin that randomly samples the solution space defined by the above user-inputs. For each variant, an irradiation simulation is executed via DIVA-for-Grasshopper [14] and a set of geometry- and irradiation-based parameters are computed. These serve as inputs to the underlying performance assessment engine, i.e. the predictive mathematical functions or metamodels, which provide an estimate for the passive solar and daylight potential. These functions are described in the next section. The third criterion, the active solar potential, is not presented in this paper, as its estimation follows a different approach (simple calculation based on the irradiation data).

Results are presented to the user in various complementary formats. First, the irradiation map (see Figure 2, right) of a user-selected design variant can be seen along with its geometrical and performance information, displayed in the Grasshopper remote control panel (green window). Then, the 3D Rhino model of each variant is automatically saved in a folder, along with an Excel file listing the performance results [15]. Finally, the relative performance of the variants is presented in three 2D graphs using the Rhino viewports. One example graph is shown below the irradiation map in Figure 2.

DEVELOPMENT OF PREDICTIVE FUNCTIONS

Developing a metamodel consists in the following steps. First, a dataset linking a series of inputs (or predictors, $\boldsymbol{x}$) and outputs (or responses, y) is acquired by sampling the design decision space, through the available yet expensive simulation [16]. A choice of metamodel form is then selected and fitted to the data. The concept of fitting means that we attempt to learn a mapping $y=f(\boldsymbol{x})$ that emulates the black box which is the simulation, but that veils its physics [16]. The dataset of samples is thus used to construct an approximation of the function $f(\boldsymbol{x})$ that represents the simulation and that can then be used to cheaply predict the output for a given set of new inputs.

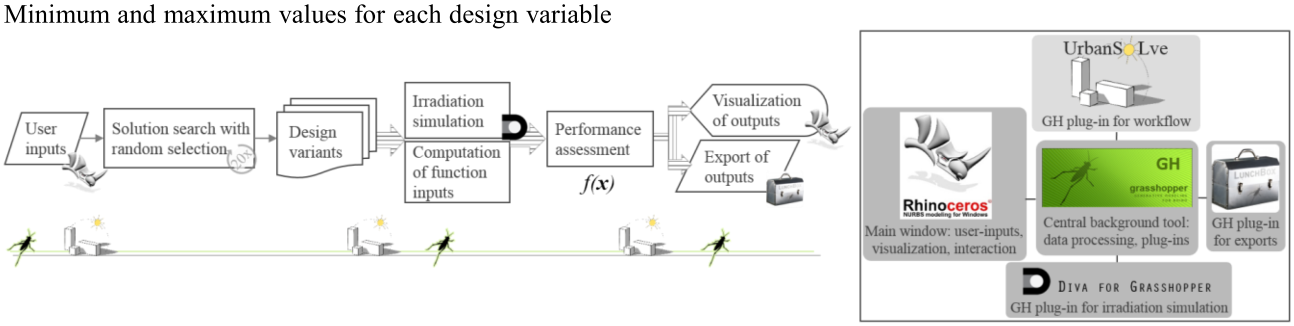

Figure 1: Proposed workflow and tools used in the implementation. Rhino [12], Grasshopper [13], DIVA-for-Grasshopper [14], LunchBox [15] and custom plug-in UrbanSOLve coded in VisualStudio C#.

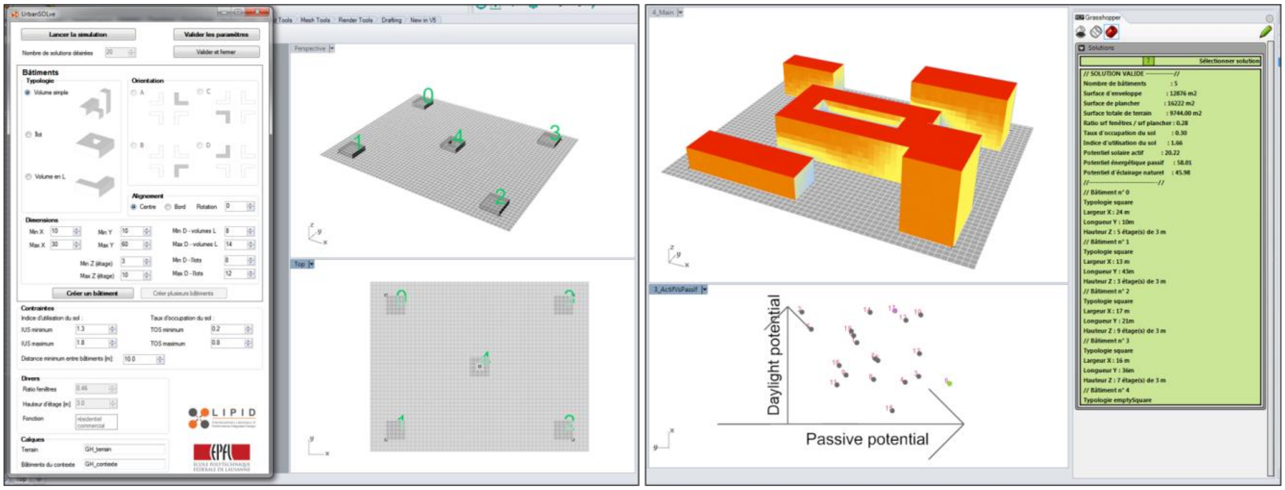

Figure 2: Prototype screenshots. Left: custom interface and Rhino window for user-inputs. Right: visualization of outputs (irradiation map and performance graph) and Grasshopper remote control pane.

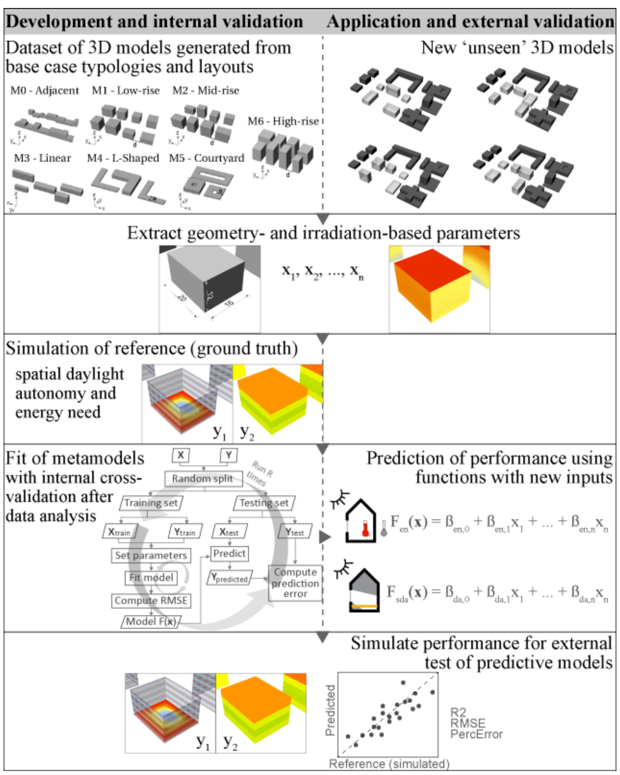

Figure 3: Development and application of the metamodels, from the data generation to the performance prediction.

The development and application phases specific to this work are illustrated in Figure 3. Two metamodels were developed: the 'energy' function, predicting the energy need for heating and cooling and representing the passive solar potential, and the 'daylight' function, predicting the ground-floor spatial daylight autonomy and standing for the daylight potential.

Modeling of neighborhood design variants

Our reference dataset was acquired through parametric modeling and simulation of neighborhood design variants, starting from seven base case designs distinct in terms of building typology, number and layout. An example variant for each base case can be seen in Figure 3 (top left). M0 to M2 were provided by an architecture and urban design firm with whom we collaborated (Urbaplan), while M3 to M5 were inspired by student projects analyzed in the context of a collaborative study [17]. M6 was later added to cover the tower typology.

Variants were generated by varying design parameters with which designers typically play during early-phase neighborhood designs and that affect solar access and can feasibly be varied parametrically. This choice of design variables, as well as the choice of the list of metamodel inputs introduced later on, was supported by various sources including masterplans e.g. [18], studies e.g. [19-22], and communication with the above-mentioned local urban design firm. The height, depth, and width of each building were thus taken as variables along with the grid orientation, while the window-to-wall ratio and simulation settings (e.g. U-value, building function) were kept constant to reduce the degrees of freedom and the level of detail of the required input information.

Ranges were defined for each variable and base case design, as listed in Table 1. For M0-M2, these values were provided by the urban firm with whom we collaborated. For M3-M5, similar intervals were defined, while M6 was parametrized so as to lead to high-rise buildings. Variants were generated, for M0-M2 and M6, by randomly sampling from the possible variable values, while a 3-level Box-Behnken sampling was used for M3-M5 [23].

Table 1: Variables ranges, constraints (when present) and constant parameters for each base case design M. See Figure 3 for definition of x, y, z, a and d. Dist.: distance, b/w: between, bldg(s): building(s).

Case name	M0	M1	M2	M3	M	M5	M6
Variable				Min:Step:Max			
Width (x) [m]	8:1:15	10:2:20	10:2:20	fixed (75-122)	fixed (54)	fixed (28-71)	15:2:30
Length (y) [m]	6:1:24	12:2:24	12:2:24	6:6:18 & 8:7:22	fixed (54)	fixed (28)	15:2:30
Height (z) [story]	2:1:4	3:1:6	4:1:8	1:2:5 & 2:3:8	1:2:5 & 2:3:8	1:2:5 & 2:3:8	10:1:20
Depth (a) [m]	-	-	-	-	6:6:18 & 8:7:22	6:5:16 & 8:5:18	-
Constant/Constraint				Conditions/Value			
Dist. b/w bldgs (d) [m]	=0	6≤d≤20	6≤d≤20	-	-	-	6≤d≤20
Number of bldgs (n)	8≤n/side≤14	4/side	4/side	5	3	3	4/side
Min plot ratio	0.9	0.9	0.9	-	-	-	-
Min bldg footprint [m^2]	50	200	200	-	-	-	-

Windows were modeled parametrically from fixed intervals (e.g. window base to floor) leading to a window-to-wall ratio of around 0.46 in all cases. Each variant was assigned two grid orientations: the initial one (0°) and a 90° rotation, both resulting in facades aligned along the four cardinal directions.

Computation of inputs

In view of obtaining mathematical functions that can predict the performance from a set of simple, early-phase available inputs, two series of potential performance indicators were computed for each design variant. First, 10 geometry-based values – e.g. plot ratio, compactness, window-to-floor ratio – capturing morphological characteristics, and second, 14 irradiation-based values such as the mean envelope irradiation, quantifying the level of solar exposure while accounting for the inter-building effect in terms of shading. Each value was computed over the whole neighborhood design, using Rhino, Grasshopper and DIVA-for-Grasshopper [14].

Simulation of reference values

For each neighborhood variant, an EnergyPlus [24] file was generated through Archsim [25], a Grasshopper plug-in, and processed in MATLAB [26] to adjust the simulation settings. An office function was assigned, with average insulation levels (U-value = 1.3W/m^{2}K), double low e argon glazing and blinds activated when the incident irradiation exceeds 180 W/m^2. Each floor was modeled as a thermal zone and an ideal loads air system was assumed. Simulations were done using the weather file of Geneva, Switzerland.

Due to its high computational cost, the daylight simulation was conducted only for the ground floor of each building (representing a conservative and possibly worst-case scenario), at work-plane height (0.8m) using DIVA-for-Grasshopper, which is based on Radiance/Daysim [27, 28].

The outputs, or values to be predicted by the metamodels, were then computed: the floor area normalized energy need (kWh/m^{2}year) and the ground-floor spatial daylight autonomy (%).

Data analysis and metamodel fitting

The following steps were done separately for the energy and daylight datasets. Each was examined for possible anomalies and correlations between inputs and outputs as well as among inputs, which respectively help identify the relevant and redundant inputs to use when fitting the metamodels. To further reduce the initial extensive list of 24 potential inputs, stepwise linear regression was applied to help identify the inputs that most improve the fit [29].

Before fitting the final functions to be implemented in the workflow, the iterative procedure illustrated in the flowchart of Fig. 3 was executed to further verify that our metamodel choice – linear regression – appeared adequate. The data were split into a training and testing set of equal size. Multiple linear regression was applied on the training data to estimate a metamodel of the form:

$$y = f(\boldsymbol{x}) + \epsilon = \beta_0 + \sum_{i=1}^{P} \beta_i x_i + \epsilon \qquad (1)$$

where ϵ is the approximation error, β_i one of the unknown coefficients corresponding to x_i in the list of P geometry- and irradiation-based inputs.

The equation obtained was applied on the testing set and the predicted responses compared to the simulation values. This represents what we refer to as the ‘internal’ testing phase. To quantify the predictive capabilities of the metamodels and their prediction error, the coefficient of determination (R^2) and the root mean square error (RMSE) were computed, with:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - f(x_i))^2}{N}} \qquad (2)$$

where N is the number of samples in the testing set, y the simulation value, and *f(x)* the predicted value. According to these fit and error metrics, adjustments can be done before fitting the final metamodels from the entire dataset, representing the version implemented in the workflow described above.

Following this first validation, an 'external' test was made by applying the final metamodels on a set of new unseen cases: 30 neighborhood designs generated in the context of the above-mentioned workshops, further detailed in [11]. Part of these designs were developed in preparation for the workshops as trial cases. The remaining ones were developed by the participants, using their usual methods and tools (e.g. scale model), and then following their experience with the prototype introduced earlier. It must therefore be noted that not all designs represent variants that could have been generated by the prototype in its current status; for some it would have been 'impossible' due to the format of the user-inputs and sampling and assessment engine. These aspects were restricted to loosely match the reference dataset used for deriving the metamodels (e.g. in terms of typology), to stay in line with the prediction capabilities.

Based on the same parcel of land, but consisting of different building typologies, layouts and densities, the newly acquired designs were simulated using the same settings and tools as when generating the initial training data. The resulting spatial daylight autonomy and energy need were compared to the predictions made by the respective metamodels. Results from the internal and external tests are presented in the next section, along with the metamodels structure.

RESULTS

Structure of metamodels

The coefficients appearing in the final energy and daylight metamodels are given in Table 2. These represent the β's in equation (1) associated to each input x_i. The energy and daylight functions respectively contain 9 and 12 inputs, counting the constant term. It is important to note that the inputs appearing in each function, as well as their coefficient's value, depend on the reference dataset (e.g. range spanned by each input). As such, parameters that one might expect to be influential, such as the window-to-floor ratio, present in the initial inputs list, do not appear in the metamodels because they did not influence the response as much as other parameters present. In the same line of thought, cautious interpretation is required not to confuse correlation for causation when using statistical methods. Moreover, the relative influence level of the inputs cannot be assessed through these coefficients, as they are not normalized and thus dependent on the units.

Table 2: Model coefficients

Parameter	Energy	Daylight
Constant (β_0)	85.19	120.91
Plot ratio	-0.98	10.38
Site coverage		-147.83
Floor area / envelope area	-10.58	-15.62
Roof area / envelope area	20.60	-1.88
North façade area / envelope area		45.25
East façade area / envelope area		42.93
Mean height		-1.25
Mean irradiation on envelopes	-0.049	
Mean irradiation on roofs		-0.051
Mean irradiation on façades		0.062
Mean irradiation on north facades	-0.047	0.028
Mean irradiation on east facades	-0.014	
Envelope irradiation / floor area	0.025	0.023
East façade irradiation / floor area	0.041	

Internal testing

Figure 4 shows the predicted versus simulated energy and daylight values, over 25 runs of the iterative internal training-testing phase. The correlation level, quantified by the displayed R^2 values, indicates a high prediction accuracy. This is also supported by the RMSE, expressed in the respective units of the data. These results can be explained by the fact that, despite the random data splitting, designs present in the testing set may not be so different than the ones in the training set. Indeed, it is likely that designs from all seven base case series are included in both subsets, set apart by (possibly small) variations in their geometry- and irradiation-based parameters. For this reason, it is essential to conduct the second, external test on completely unrelated designs.

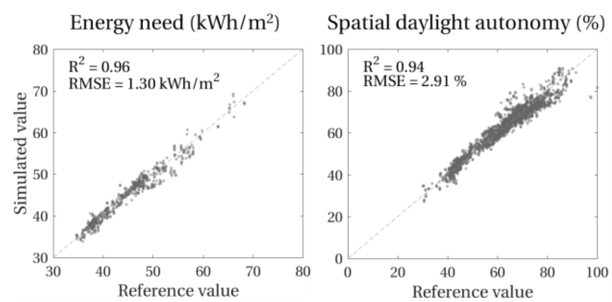

Figure 4: Predicted versus simulated energy and daylight values for the training dataset, for 25 runs of the train-test iterative fitting process.

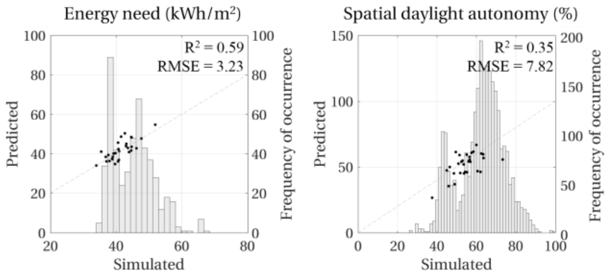

Figure 5: Predicted versus simulated energy and daylight values for the external testing dataset, overlaid on the reference dataset histogram (simulated values).

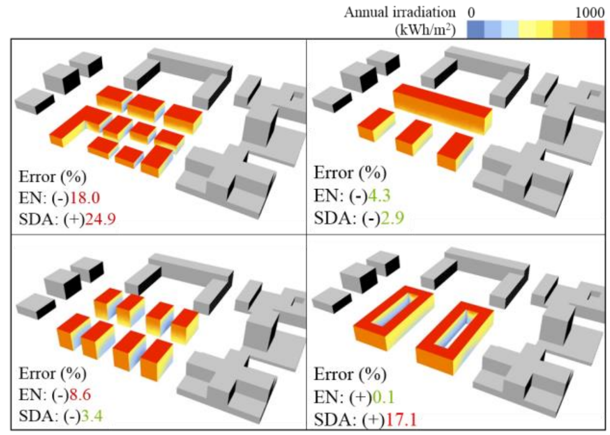

Figure 6: Irradiation map of four designs part of the testing dataset, along with the prediction error of the energy (EN) and daylight (SDA) metamodels. (+/-) mean under/over prediction.

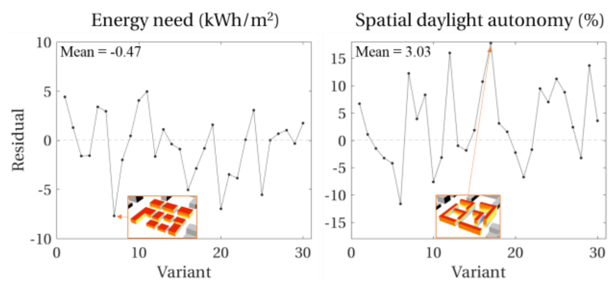

Figure 7: Residuals plot showing the difference between the simulated and predicted energy and daylight values for each design variant in the testing dataset. The variant with the highest error for each performance criterion is displayed (irradiation map).

External testing

Figure 5 shows the histogram of the simulated energy need and spatial daylight autonomy for the reference (internal training) dataset, over which the predicted versus simulated values for the external testing data have been plotted (dots). Both the predicted and simulated testing values fall within the range of the reference dataset. The observed correlation and RMSE values are respectively lower and higher than for the internal testing. These results are to be expected since the new cases are different from the reference cases in terms of the buildings layout and parcel of land on which they were designed (size and aspect ratio). They also present mixed building typologies within a same variant, as can be seen in the top left example shown in Figure 6. The RMSE's represent 16% and 17% of the energy and daylight simulated data ranges respectively, coming short from the 10% threshold defined in [16] to qualify a reasonable global predictive model. To improve the metamodels, multiple avenues are being explored including expanding the reference dataset with diverse design variants and investigating other function forms (e.g. including interaction terms) and more powerful but complex fitting techniques (e.g. Gaussian Processed).

Figure 6 shows four examples of the (external) testing design variants in the form of an annual irradiation map located in the fixed surrounding context. The relative errors (in %) vary across the designs and performance criteria. The residuals – difference between simulated and predicted values – are plotted in Figure 7. The mean is near the desired zero value for the energy metric, however the daylight metamodel presents a bias towards under-prediction. In both cases some outliers can be seen, information that can help us identify the type of designs that should be added to the reference dataset in order to improve the generalization potential of the metamodels.

Indeed, as highlighted earlier, some of these designs could not have been produced by the prototype in its current status. This fact partly explains the errors observed. Improving the metamodels will allow adding flexibility in the user-inputs, all constituents of the prototype being inter-dependent.

CONCLUSION AND FUTURE WORK

This paper introduces a design decision-support workflow that was implemented as a Rhino- and Grasshopper-based prototype. Its novel aspects were highlighted: a multithreaded design variant generation and assessment engine addressed for the neighborhood scale and respectively based on random solution space sampling and predictive functions.

Results show that the prediction error increases for variants with design characteristics (e.g. buildings dimensions, typology mix and layout) further from those from which the predictive functions were built. However, with an absolute mean error of 1.1% and 5.4% for the passive solar and daylight metamodels respectively, these represent in their current status a promising assessment method for comparing early-design alternatives in terms of their solar potential.

The proposed workflow built around this performance assessment method provides the means for rapidly exploring a user-defined solution space. To enhance its performance-based guidance features, improvements are to be brought to increase design flexibility and the metamodels' validity boundaries. Future work will thus include further development of the interface in terms of the design variables included and building typologies offered, leading to more realistic architectural design. To ensure that the metamodels keep up with these improvements, their application will be expanded by enlarging the training dataset and looking at the effect of using different simulation settings (e.g. residential function). More complex yet powerful fitting techniques such as Gaussian Processes are also being investigated.

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