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Characterizing time intermittency in non-diffusive fast ion transport through plasma turbulence

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Abstract

Turbulent fast ion transport has been investigated in astrophysical, laboratory and fusion plasmas. When gyro- and drift-averaging across plasma structures manifest as non-markovian and non-local effects, this results in generally non-diffusive transport. The intermittency of the formation of these plasma structures, such as blobs, can potentially be reflected in the transport of the fast ions as well. In the TORPEX basic plasma device, a toroidal beam of suprathermal Li-6 ions is injected into electrostatic plasma turbulence. Conditional sampling techniques confirm turbulent $E \times B$-drifts as physical driving mechanism of radial fast ion transport, which features sub-, super- or quasi-diffusive regimes depending on the fast ion energy and propagation time. To address the question of how far local intermittency is associated with each regime, we analyze characteristics of time-intermittency on an extensive set of local fast ion time-series across all observed regimes. Modeling the time-average fast ion profiles as the result of a meandering smaller instantaneous beam allows us to predict the skewness of such time-series based on their time-average. Comparisons with the skewness of simple two-valued time-series can yield relative indications towards certain transport regimes in our specific system, based on the differences in the size of the instantaneous fast ion beam.

Keywords: plasma turbulence, suprathermal ions, non-diffusive transport, intermittency, time series analysis

(Some figures may appear in colour only in the online journal)

1. Introduction

The transport of suprathermal ions has been investigated across various domains in plasma physics ranging from astrophysical plasmas and cosmic rays [1], over solar energetic particles [2] to laboratory [3] and fusion plasmas [4]. In the latter, they can originate e.g. from neutral beam injection or fusion reactions, and their confinement is thus crucial to the heating performance of any fusion device [5]. Depending on the specific setting, different mechanisms can impact fast particle transport, such as orbit losses, strong drifts, coupling to Alfvén eigenmodes [6] or interaction with plasma turbulence [7, 8]. While turbulent transport is expected to play a major role for fusion-born ions in ITER or DEMO plasmas, it may more strongly affect those at moderate energies or impact beam ions [7, 9–11]. Furthermore, turbulence remains a major contributor to the spreading of fast particles in space plasmas [12, 13] and thus central to space weather.

The transport of suprathermal ions through plasma turbulence has therefore been central to research on the TORoidal Plasma EXperiment (TORPEX) [14, 15]. A toroidal beam of Li-6 ions is injected at energies of 30–70 eV into a helical magnetic geometry featuring a cold plasma with strong electrostatic turbulence [3]. Due to the intrinsically non-local and non-markovian effects of gyro- and drift-averaging, the spreading of the time-average fast ion beam along the major radius of TORPEX was identified as either sub-, super- or quasi-diffusive depending on the fast ion injection energy [3, 16]. Initial measurements of local fast ion currents revealed strong intermittency during superdiffusion,
as the skewness of the acquired time-series distinctly exceeded background levels [17]. This motivated further investigation, as inferring the global non-diffusive transport regime from purely local measurements would be particularly useful in systems with intrinsically limited diagnostic access as e.g. with satellites in the magnetosphere [2, 18].

In this paper, we further address this question by elaborating on results presented in [19], based on an extensive three-dimensional set of fast ion time-series, which show distinct instances of time intermittency across all observed transport regimes. In section 2 we briefly introduce the fast ion experimental set-up in TORPEX, as well as methods for quantifying intermittency in the obtained fast ion time-series. The main experimental results are presented in section 3 in the form of time-average fast ion profiles and skewness profiles in various fast ion transport regimes. Conditional sampling (CS) is used on time-resolved Langmuir-probe (LP) measurements [20] to directly illustrate the effects of turbulent $E \times B$-drifts as fast ion transport mechanism [17]. In section 4, we briefly review an analytical model developed in [21], that predicts the skewness of local fast ion time-series by modeling how the time-average fast ion profile is constructed through the motion of a smaller instantaneous fast ion beam, as described in detail in [19]. In section 5 we use the established picture of the meandering instantaneous fast ion beam, to motivate how intermittency can be quantified by the skewness of a simple, two-valued time-series. Finally, section 6 summarizes our findings and discuss current and further investigations.

2. Experimental methods

2.1. The TORPEX device

TORPEX features a toroidal vacuum vessel with minor radius $a = 0.2$ m and major radius $R_0 = 1$ m as depicted in figure 1 [14, 15, 23]. In the simple magnetized torus (SMT) configuration, a helical magnetic field structure (purple) is set up through a combination of toroidal field coils (light brown) yielding $B_0 = 74$ mT on-axis and vertical field coils (green) with $B_z \lesssim 2$ mT. Other magnetic geometries including x-points and snowflakes can be achieved through an internal toroidal conductor, but have not been used in this investigation [24, 25]. In the SMT configuration, a plasma is generated on the high field side (HFS) of the device cross-section through the bottom injection of micro-waves in the electron cyclotron (EC) frequency range at 150 W of power. Both EC and upper hybrid resonances contribute to produce a hydrogen plasma with a temperature $T_e \approx 5$ eV and $T_i < 1$ eV for electrons and ions respectively and densities of $n_e = n_i \approx 10^{16}$ m$^{-3}$ [14, 26]. The present temperature and density gradients drive an interchange-mode propagating upward with $k_z \approx 35$ rad m$^{-1}$ and $k_\parallel \approx 0$ along with strong electrostatic plasma turbulence [27–29]. Intermittently, coherent plasma filaments termed ‘blobs’ detach from the mode near the center of the cross-section and propagate outward towards the low field side (LFS) [20, 23, 30]. The highly reproducible TORPEX plasmas and their structures have been characterized through a variety of diagnostics, such as a fast-framing camera [31], a triple-LP [32], and foremost through the LP-arrays of the HEXagonal Turbulence Imaging...
Probe (Upgrade), HEXITP-U [22, 23]. The two present arrays each cover the majority of the poloidal cross section with up to 94 probes at a grid constant of 3.5 cm and are used to acquire measurements of floating potential or of ion-saturation current, as a proxy for plasma density, at opposite toroidal locations (see figure 1) with a time-resolution of 4 μs.

2.2. Fast ion experiments

A fast ion beam is injected by extracting ions via thermal emission from a positively biased Li-6 ion source, and accelerating them through a negatively biased grid to supra-thermal energies of between $E = 30 \text{ eV}$ and $E = 70 \text{ eV}$. With a vertical injection angle of $+0.1 \text{ rad}$ w.r.t. the magnetic field-line, this results in Larmor radii of approximately 5 mm and 8 mm respectively [3]. Due to the magnetic field pitch, curvature and gradient, the fast ions generally drift upwards [5]. Fluctuations in the ambient plasma potential result in turbulent $E \times B$-drifts, which spread the fast ion beam in the poloidal $R$-$Z$-plane as it propagates [17]. At a toroidal source-detector distance $D$, the beam is detected with a dedicated set of back-to-back gridded energy analyzers (GEAs), biased to filter out thermal ions and electrons [33]. While amplifying the GEA signals, the back-side signal is subtracted from the front-side to reduce noise from the plasma, electronics or surrounding equipment. Since the local density of the fast ions can only reach 0.5% of the background plasma density at the very most, they are effectively treated as tracer particles. No appreciable effect of the fast ions on the background turbulence has been revealed e.g. through the electrostatic measurements.

In previous studies [3], lock-in detection was used to determine the time-average poloidal profile of the fast ion beam and its horizontal variance $\sigma_R^2$ at different $E$ and $D$. In conjunction with these measurements, fast ions were traced numerically in simulations of the TORPEX plasma from the global Braginskii solver (GBS) fluid code [16, 34], to quantify the growth of $\sigma_R^2$ as a function of fast ion propagation time $t$.

The spreading of the fast ion beam along $R$ was thus quantified as generally non-diffusive, i.e. $\sigma_R^2 \sim t^\nu$ with $\nu = 1$. Before they can measurably interact with the surrounding plasma, fast ions experience an initial ballistic transport ($\nu = 2$), for approximately one gyro-period. Then, transport transitions towards superdiffusion for fast ions with $E = 30 \text{ eV}$, whereas an injection energy of $E = 70 \text{ eV}$ results in subdiffusive behavior ($\nu < 1$) due to increased gyro- and drift-averaging over the responsible plasma structures [3, 16]. Once the superdiffusive fast ion beam has attained a $\sigma_R$ comparable to the scale of the radial plasma density and temperature gradients after 5–6 gyro-periods, the transport thus features strong asymmetry and transitions gradually towards quasi-diffusion ($\nu \approx 1$) [3].

Since we present measurements at $D \geq 126 \text{ cm}$ in the following, we access the subdiffusive transport regime for $E = 70 \text{ eV}$ and the super- to quasi-diffusive regime for $E = 30 \text{ eV}$. Measurements at $E = 50 \text{ eV}$ correspond to an intermediate, and therefore subdiffusive case.

2.3. Quantifying intermittency

To quantify the local time-intermittency of the fast ion transport, time-series (see figure 2) are acquired from the GEA-signal at a frequency of 250 kHz. The acquired current is scaled by the area of the circular aperture of the detector (diameter $d = 8 \text{ mm}$), and fast ion time-series $J(t)$ thus presented in units of current density, which is consistent with

![Figure 2. Segments of local time-series of fast ion current density $J(t)$](image)

The on-phases during the ion source modulation are indicated in red, off-phases in black. Note the distinct peaks during the on-phase, for which the inset pdfs are given. The mean of the fast ion signal $\langle J \rangle = \text{mA m}^{-2}$ and on-phase skewness $\gamma_J$ are given. All selected time-series were acquired at $D = 171 \text{ cm}$. Panels (a) and (b) show the time-series of 30 eV ions with the highest $\mu_J$ and $\gamma_J$ respectively.

In (c) and (d), the same is shown for 70 eV. Their poloidal locations $(R, Z)$ are shown in (cm) on the right and are indicated also in figures 3, 5. ((b), (d) Reprinted with permission from [19], Copyright (2019) by the American Physical Society.)
previous studies [3, 15]. The bias of the fast ion source alternates between on- and off-phases at a frequency of 23 Hz and we therefore distinguish these phases also in the acquired GEA time-series. Statistical quantities corresponding to the on-phase, which contains the fast ion signal as well as noise, are denoted with subscript \( S \), while quantities related to the pure background noise are indicated by the subscript \( N \), and obtained from the off-phases. To find e.g. the mean of the pure fast ion signal, the off-phase mean is then subtracted from the on-phase mean, i.e. \( \mu_S = \mu_S - \mu_N \).

To quantify the intermittency of the time-series, the Fisher–Pearson coefficient of skewness (see [35], henceforth \( \gamma \)) was chosen [17], as it is dominantly affected by the presence of outliers (peaks) w.r.t. the mean of the signal, through the use of its 3rd central moment \( (\gamma_S > \gamma_N) \) was found during early superdiffusion \((E = 30\,\text{eV}, D = 40\,\text{cm})\), but not during early subdiffusion \((E = 70\,\text{eV}, D = 40\,\text{cm})\), prompting these more in-depth investigations of the prevalence of such time-intermittency across different transport regimes.

Furthermore, a noise reduced skewness (\( \gamma_f \)), corresponding to the appropriate statistic for the pure fast ion signal has been considered for statistically independent signal and noise [19]

\[
\gamma_f = \frac{\gamma_S \sigma_S^3 - \gamma_N \sigma_N^3}{(\sigma_S^3 - \sigma_N^3)^{1/2}},
\]

where \( \sigma_S \) and \( \sigma_N \) denote the standard deviation of the time-series during on- and off-phases respectively. However, it is clear that \( \gamma_f \) quickly diverges when the signal-to-noise ratio is low, so that the on-phase skewness \( \gamma_S \) remains the more useful statistic, when compared to noise values of \( \gamma_N \leq 2.2 \) globally.

### 3. Results

#### 3.1. Fast ion profiles

An extensive set of fast ion time-series has been obtained at different injection energies \( E = \{30, 50, 70\} \,\text{eV} \) and source-detector distances \( D \geq 126 \,\text{cm} \). Fast ion mean-profiles \( J(R, Z) \) were constructed at each \( D \) and \( E \) by subtracting the off-phase time-average from the on-phase value of the corresponding set of time-series and interpolating between the measurement locations in the \( R-Z \)-plane. To analyze the prevalence of intermittency in each case, the on-phase skewness for each time-series was likewise computed and skewness-profiles \( \gamma_S(R, Z) \) then interpolated. Differences in the quality and lifetime of different fast ion sources lead to variations in total emitted currents between profiles. To account for the resulting changes in the measured skewness and mean, we normalize both the mean-profile and skewness profile towards an average total fast ion current \( I_{\text{ave}} = 2.85 \,\mu\text{A} \) by mapping

\[
J \mapsto cJ,
\]

\[
\gamma_S \mapsto \frac{c^3 \sigma_S^3 \gamma_S + (1 - c^3) \sigma_N^3 \gamma_N}{(c^2 \sigma_S^3 + (1 - c^3) \sigma_N^3)^{1/2}},
\]

where we have used the normalization factor \( c = I_{\text{ave}} / I \) and again assumed statistically independent signal and noise [19]. These normalizations lead to the mean-profiles as presented in figures 3–5(a)–(c) and skewness-profiles in figures 3–5(d)–(f).

The mean-profiles confirm the location and spread of the fast ion beam expected from earlier studies and numerical simulations to within \( \approx 20\% \). Remaining deviations may be due to small systematic offsets in injection angle or energy.

Regarding the skewness profiles, we locally find \( \gamma_S \) above background level across all transport regimes. Generally speaking, skewness is most prevalent towards the LFS of each profile, where the contribution from background noise is less important, due to the decay of plasma density fluctuations. Furthermore, skewness is never peaking near the maximum of

![Figure 3. Normalized mean- and skewness-profiles for 30 eV ions. Note the larger spread and reduced vertical drift in the mean compared to higher energies, as well as the lower peak current density and higher maximum skewness. The gray circles in (c) indicate the different detector positions in this profile. Experimental resolution was lowest here, due to the larger size of the profile. The green crosses indicate the positions at which the time-series samples in figures 3(a), (b) were acquired. (c), (f) Reprinted with permission from [19]. Copyright (2019) by the American Physical Society.)](image-url)
the mean-profile, where any peaks in the time-series become less distinct w.r.t. the higher mean. Comparing the highest measured skewness across different profiles (see figure 6) at varying $D$, it becomes apparent that intermittency tends to increase for longer fast ion propagation times. Likewise, we find higher maximum values of skewness in the super- to quasidiffusive regime ($E = 30$ eV), compared to the subdiffusive case.

3.2. Conditional sampling

The prevalence of skewness in most of the acquired profiles reflects to varying degrees the presence of distinct, intermittent peaks in the fast ion signal, as seen in figure 2. For instance, peaks above the level of $2\sigma_n$ in the time-series for 30 eV and 70 eV ions of figures 2(a) and (c) are characterized by an average duration of 14 $\mu$s and 10 $\mu$s respectively, as well as average waiting times of 302 and 192 $\mu$s between them, all with approximately exponential distributions. For both energies, this would be indicative of a more concentrated instantaneous fast ion beam, that is being displaced towards the detector at these instances. To illustrate the physical mechanism and possible extent of such displacements, we use the technique of CS [20]. Specifically, CS is employed to characterize the behavior of the fast-ion beam during the propagation of different structures formed by the plasma turbulence [17]. After choosing a reference probe on HEXTIP, plasma structures are identified by local plasma density fluctuations $\Delta n(t) = n(t) - \langle n \rangle$, where $n$ is the local plasma density and $\langle n \rangle$ its time-average. By imposing an absolute threshold condition on $\Delta n$, a conditional sample of peaks is identified on the reference probe signal. A time-window of $\pm 120$ $\mu$s is then identified around each peak. For each time-step (4 $\mu$s) within the time-windows, an average over all time-windows in the sample is computed. This conditional average is performed over the same time-windows in the fast ion

Figure 4. Normalized mean- and skewness-profiles for 50 eV ions. Spread and drift lie between the values for the other fast ion energies, as expected. The highest skewness values do so too, except for the furthest profile. This exception is most likely due to unaccounted drops of the injected fast ion current when the ion-source is depleting. For ongoing investigations, a real-time detection circuit for the injected fast ion current has therefore been added.

Figure 5. Normalized mean- and skewness-profiles for 70 eV ions. Note the reduced spread and larger vertical drift in the mean density profiles. Skewness levels are lower compared to other cases, but with peak values of $\approx$3 still appreciably above the background values. The green crosses refer to the positions at which the time-series samples in figures 2(c), (d) were acquired. (Reprinted with permission from [19]. Copyright (2019) by the American Physical Society.)
time-series. By repeating the same procedure for each fast ion time-series in a profile, a fast ion CS-profile is obtained analogous to [17]. Analogously, the simultaneously acquired plasma density time-series of all HEXTIP probes provide the CS plasma profile.

Figures 7(a), (b) shows the CS-profiles obtained at $D = 171$ cm. Structures with large positive fluctuations ($\Delta n > 3.5 \times 10^{15} \text{m}^{-3}$) deform the CS-profile the most and shift its center-of-mass down and towards the LFS. These blobs clearly dominate transport events near the peak of the skewness-profile and are preceded and followed by negative density fluctuations from the interchange-mode, as shown in the CS plasma profile in figure 8(a). Figures 7(c), (d) shows the corresponding CS-profiles for structures with large negative fluctuations ($\Delta n < -1.8 \times 10^{15} \text{m}^{-3}$). These deviate the CS-profiles upward and toward the HFS, i.e. opposite to the deviations by positive fluctuations. As expected from previous studies [17], this behavior is still in excellent agreement with $E \times B$-drifts around alternating dipoles in the different plasma structures as outlined in figure 8(a). In our current studies, we have additionally obtained the CS floating potential profile in figure 8(b), using the extended 2nd LP-array of HEXTIP-U. These measurements now directly confirm this dipole pattern in the CS plasma structures.

The center-of-mass of the fast ion CS-profile is deflected by up to $\approx 2$ cm for 30 eV ions, and only $\approx 1.2$ cm for 70 eV ions. This is consistent with both the stronger gyro- and drift-averaging as well as shorter propagation times for fast ions of higher energies. From the standpoint of anomalous diffusion, these trends indicate that transport in the regions of highest skewness in figures 3, 5 is dominated by the intermittent generation of large plasma structures. The stronger average $E \times B$-motion for lower energies can be interpreted as a stronger prevalence of large ‘jumps’, consistent with a higher transport exponent.

4. Analytical modeling

Strong intermittent peaks in the fast ion time-series (see figure 2) consistently indicate that the fast ion beam at these instances is much more concentrated than the time-average beam, especially for lower ion energies. The physical origin and extent of the displacements of this instantaneous fast ion beam have been quantified in detail by CS analysis. We have therefore developed an analytical model [21] for the statistics of time-series, where the time-average current density profile $J(R, Z) = J(R)$ is constructed through the motion of a smaller (and approximately rigid) instantaneous profile $j(R)$.

As illustrated in figure 9, the statistics of this motion are contained in a displacement PDF $f(R)$, so that the time-average profile at a particular measurement position $R_0$ is given by the convolution of $f$ and $j$. While the time-average $J(R_0)$ corresponds to the 1st moment of a time-series taken at $R_0$, this directly generalizes to higher orders, such that the $q$th order non-central moments $J_q(R_0)$ are given by

$$J_q(R_0) = \int (R - R_0)^q j(R) dR dZ.$$  \hspace{1cm} (4)
Figure 8. (a) Full CS plasma profile for positive density fluctuations on HEXTIP, similar to figure 7(a), with the same contour levels. Due to the interchange mode structure, negative fluctuations are prominent between positive ones. We superimpose a suspected floating potential structure (orange and purple) consistent with the displacements of the fast ion CS-profiles in figure 7 through $E \times B$-drifts (magenta). Note the strong agreement with the actual CS profile of floating potential fluctuations $\Delta V_t = V_f(t) - \langle V_f \rangle$ in (b). This profile was constructed using the second HEXTIP array. The CS plasma density contours from the first are superimposed after projecting along magnetic field-lines.

Figure 9. Schematic of quantities in the analytical model, comprising the mean profile $J(R)$ and smaller instantaneous profile $j(R)$, which is deflected rigidly through the displacement PDF $f(R)$ towards a particular measurement position $R_0$, where the detector aperture is centered. We also indicate the key parameters of the model, being the peak instantaneous current density ($j_p$), and the widths ($\sigma_j$, $\sigma_p$) of the instantaneous and time-average profile respectively.

Quantities such as skewness (or kurtosis) of local time-series have been modeled quite successfully by adapting this method to simulations [21] as well as the experimental time-series data [19], as described next.

4.1. Subdiffusive case

To find a simple analytical expression for $J_p$, we firstly consider the case where $f(R)$ and $j(R)$ represent symmetric 2D Gaussians (with zero mean), and therefore so does $J(R)$. Let $\sigma_j^2$, $\sigma_j^2$ and $\sigma_j^2 = \sigma_j^2 + \sigma_p^2$ thus denote their respective variances. It can be shown that in this case one finds [19]

$$J_q(R) = q^{-1}q^1\rho(q^2-2\rho) J(R) \times \gamma_s \sigma_q$$

with $\gamma_s = [1 - \rho^2(1 - q^{-3})]^{-1}$; (5)

where we have defined the width-ratio $\rho = \sigma_j/\sigma_p < 1$ as well as the peak instantaneous current density $\sigma_p$. If these two parameters of our system are determined, one can predict any moment of the local time-series, and therefore their skewness, solely from their local time-average value $J(R)$. To assess how consistently this method applies to the time-series in different subdiffusive profiles, we keep both $\rho$ and $\sigma_p$ as free parameters and perform a least-square-fit of the predicted 2nd and 3rd central moments $J_{\{2,3\}}$ (equations (6), (7), rhs) against their experimentally measured values (equations (6), (7), lhs) [19]

$$\sigma_j^2 - \sigma_j^2 \approx J_{2,2} = J_2 - J^2, \quad \gamma_s^2 \sigma_j^2 - \gamma_s^2 \sigma_j^2 \approx J_{3,3} = J_3 - 3JJ_2 + 2J^3.$$

By then combining equations (6) and (7), we arrive at an expression for the predicted on-phase skewness [19]

$$\gamma_s \approx \frac{J_{3,3} - \gamma_s^2 \sigma_j^2}{J_{2,2} + \sigma_j^2/2},$$

which is shown in comparison to measured values in figure 10. Predictions agree with measurements usually to within $\pm 20\%$, as shown in figure 10. Stronger deviations in the measurements can be caused by momentary fluctuations in the injected fast ion current as discussed before in section 3, figure 4. The fitted value of $J_p \approx 15$ mA m$^{-2}$ also agrees well with peaks in the measured time-series, pending noise (see figures 2(c), (d)). If we consider the known total fast ion current $I$ from integrating the mean-profile, we hence find $\sigma_j = \sqrt{\frac{I}{2\sigma_p}} \approx 5$ mm, which corresponds well to results from simulations [21]. However, the fitted parameter $\rho$ then underestimates the true width of the mean-profiles by a factor of $\approx 2$. This is due to the fact that the non-Gaussian features of the subdiffusive fast ion mean profiles cannot be appropriately modeled given the above assumptions. This limitation has an even greater impact in the case of the more
Figure 11. Predicted on-phase skewness $\gamma_5$ at $D = \{126, 146, 171\}$ cm (a)–(c) for $E = 30$ eV, i.e. during super- to quasi-diffusion, compared against measurements. The total fast ion currents estimated by integrating the mean-profiles are $I = \{2.35, 2.5, 3.2\}$ $\mu$A. (Reprinted with permission from [19], Copyright (2019) by the American Physical Society.)

5. Skewness of binary time-series

Complementary to the preceding section, we can exploit differences in the instantaneous beam profile $j(R)$ between heavy-tailed super- and quasi-diffusive profiles, so that a different approach is required.

4.2. Super- to quasi-diffusive case

Since the mean-profiles for 30 eV ions are much more spread out than in the 70 eV case, we assume $\rho \ll 1$. This allows us to perform a Taylor expansion of $f(R)$ over $R_0$ in equation (4) and we find (to 2nd order) [19]

$$J_q(R) \approx J(R) \int j(R) \rho \, dR \, dZ + \mathcal{O}\left(\frac{\sigma_j^2}{2}(\partial_R^2 + \partial_Z^2)J\right).$$

No further assumptions on the shape of $f(R)$ are required. From this we can again derive simple expressions to predict the central moments $J_{c,q}$ (see [19, 21])

$$J_{c,2} = \frac{1}{2}j_p J - J^2,$$

$$J_{c,3} = \frac{1}{3}j_p J - \frac{3}{2}j_p J^2 + 2J^3.$$  

This allows us to fit the 2nd and 3rd central moments against experimental data, and obtain predictions for the on-phase skewness again according to equation (8). There is again good agreement between predictions and measurements, as shown in figure 11.

The remaining fitted parameter of e.g. $j_p \approx 14$ mA m$^{-2}$ for $D = 171$ cm appears realistic when comparing to the peaks in figures 2(a), (b).

It should be emphasized that the presented model makes no intrinsic assumptions on the given transport regime. The prevalence of intermittency in each profile is quite consistently predicted from the local mean-profile and different justifiable assumptions on $j(R)$ and $f(R)$. Since the assumption of $\rho \ll 1$ is less valid in the subdiffusive case, skewness predictions based on the series-expansion become significantly more inconsistent than the presented results for approximately Gaussian distributions.

5. Skewness of binary time-series

Figure 12. A two-valued (binary) time-series, consisting of zeros and a unique peak-value $j_p$. The skewness $\gamma_{SB}$ for a given mean $j_p$ is given by equation (12).

different transport regimes, to distinguish the corresponding time-series relative to one another. Simulations indicate, fast ions in the super- to quasi-diffusive case often form a more concentrated instantaneous fast ion beam than in the sub-diffusive cases, due to their smaller Larmor radii.

Let us suppose that the instantaneous fast ion beam was extremely concentrated ($\sigma_j \to 0$) and hence much smaller than our detector aperture. Depending on $f(R)$, the beam would either be detected fully, or not at all, resulting exclusively in the two detected values $j_p$ or 0, i.e. a time-series that appears binary in this sense (figure 12).

The skewness $\gamma_{SB}$ of such a binary time-series with $M$ entries (indexed $i$) containing $N$ peaks of value $j_p$ and hence a mean of $\mu_j = \frac{j_p N}{M}$ is given by

$$\gamma_{SB} = \frac{1}{M^2\sigma_i^2}(\mu_i - \mu_j)^3 \left(\frac{1}{M^2\sigma_i^2}(\mu_i - \mu_j)^2\right)^2$$

$$= \frac{N}{M}(\mu_i - \mu_j)^3 \left(\frac{1}{N}(\mu_i - \mu_j)^2\right)^2 = \frac{\mu_j^3}{\nu_j} \left(\frac{1}{\nu_j}(\mu_i - \mu_j)^2\right)^2 = \frac{1}{\nu_j}.$$  

(12)

Including contributions from noise, we find analogously to equation (8) for the binary on-phase skewness $\gamma_{SB}$ that

$$\gamma_{SB} = \frac{\mu_j^3}{\nu_j} \left(1 - \frac{\mu_j^2}{\nu_j}\right) + \sigma_i^2 \gamma_{N}$$

$$= \frac{\mu_j^3}{\nu_j} \left(1 - \frac{\mu_j^2}{\nu_j}\right) + \sigma_i^2 \gamma_{N}.$$  

(13)

In our experiments, the diameter of the detector $d = 8$ mm is similar to the sizes of the typical fast ion Larmor-radii. A simultaneous detection of the full instantaneous fast ion beam is therefore uncommon, as can be seen from $j_p < \frac{1}{2\sigma_i}$ and $d \approx 2\sigma_i$, especially in the subdiffusive case. However, since $\gamma_5$ is dominated by outliers, we can still compare it against $\gamma_{SB}$. We quantify how well a given time-series is approximated by a binary equivalent through the relative error $\epsilon = \frac{2\sigma_i - \gamma_{SB}}{\gamma_5}$. Figure 13 shows $\epsilon$ computed on the experimental time-series, plotted against $\gamma_5$ and $\mu_j$.

It becomes clear that the value of $\epsilon$ at the same $\gamma_5$ and $\mu_j$ is systematically lower for time-series of 30 eV ions. This indicates that a consistently high fraction of the instantaneous beam is detected more frequently since the instantaneous
beam remains (on average) more concentrated at 30 eV. Hence its intermittency is approximated more closely by that of a binary time-series, or even underestimated at a given choice of \( j_p \). The larger widths of the instantaneous fast ion beam in the 70 eV case make the binary approximation much less suitable and lead often to over-estimates in skewness \( \gamma_\text{SB} \) at the same \( \gamma \) and \( \mu_f \). Therefore we indirectly find a systematically higher \( \gamma \) in the subdiffusive regime, compared to super- to quasi-diffusion. It should be noted that this observation can become less consistent at shorter \( D \), when Larmor-oscillations are still more distinct in the instantaneous and time-average fast ion beam widths.

**6. Conclusions**

Detailed studies on the non-diffusive transport of fast ions by electrostatic plasma turbulence have been carried out on the TORPEX device and through fluid-tracer simulations using the GBS code. The local time-intermittency of this transport process has been quantified by measurements of skewness on an extensive set of fast ion time series. Instances of intermittency distinctly above noise level were found in both the super- to quasi-diffusive and subdiffusive transport regime [19]. The distinct peaks in the fast ion time-series are generated through the intermittent motion of a concentrated instantaneous fast ion beam.

An analytical model has been developed to describe the moments of time-series generated from a meandering particle beam and adapted to our experiments [21]. Using this model, we consistently predict the skewness of the time-series based on their time-averages [19]. Hence the mechanism for the generation of local time-intermittency in our system is determined to be qualitatively independent from any global transport regime. Continued studies using CS confirm turbulent \( E \times B \)-drifts as the physical transport mechanism across all regimes, and indicate how intermittent transport is generated and how strongly the fast ion beam is affected on average.

All the above findings lead us to consider simple two-valued time-series as a model for an extremely concentrated, meandering fast ion beam. The skewness characteristic of such time-series yields a stronger over-estimate of the measured skewness in subdiffusion compared to other regimes. This reflects the (on average) larger size of the instantaneous fast ion beam there, and may in some cases serve as an indirect and relative indicator of the transport regime.

We therefore conclude that locally measured time-intermittency is no decisive indication of any particular non-diffusive transport regime in a general setting. As our case illustrates, the generation and prevalence of intermittency results from the interplay of a range of parameters and merits analysis from a variety of stand-points. Depending on each system and accessible information, different methods can provide complementary findings to establish a better physical picture of the origin and role of intermittency. If successful, one can therefore nonetheless attempt to predict its prevalence and indirectly find relative indications towards global transport.

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**References**


[33] Plyushchev G et al 2006 Fast ion source and detector for investigating the interaction of turbulence with suprathermal ions in a low temperature toroidal plasma Rev. Sci. Instrum. 77 10F503


[36] She Z S, Jackson E and Orszag S A 1990 Intermittent vortex structures in homogeneous isotropic turbulence Nature 344 226