

Cyclic metal plasticity model parameters with limited information: a constrained optimization approach

Albano de Castro e Sousa¹, Ph.D.

Alexander R. Hartloper², MEng, S.M ASCE

Dimitrios G. Lignos³, Ph.D, M. ASCE

ABSTRACT

Tensile test data for structural steels are ubiquitous. Yet, the information monotonic loading provides with respect to the material's characteristics is limited. Notably, features of inelastic response to cyclic loading, such as the Bauschinger effect, can not be determined without testing with a load reversal protocol. This paper aims at addressing this shortcoming by formulating a constrained optimization problem that provides best-fit material parameters to a tensile test, while simultaneously imposing representative cyclic properties for structural steels. Recommendations on constraints are given. Results demonstrate that improvements can be achieved when compared to: (1) direct fits to tensile data and (2) input model parameters from the same steel material but from different batches calibrated to a wide range of strain-based protocols. Given the available data, it is concluded that simpler models with one backstress tend to perform best with the proposed constraints.

Keywords: Inverse problem, structural steel, tensile test, material constitutive modeling, Earthquake loading, cyclic hardening, constrained optimization

INTRODUCTION

¹Post-Doctoral Researcher, École Polytechnique Fédérale de Lausanne. EPFL ENAC IIC RESSLab, GC B3 465, Station 18, 1015 Lausanne, Switzerland. E-mail: albano.sousa@epfl.ch

²Doctoral Assistant, École Polytechnique Fédérale de Lausanne. EPFL ENAC IIC RESSLab, GC B3 514, Station 18, 1015 Lausanne, Switzerland. E-mail: alexander.hartloper@epfl.ch

³Associate Professor, École Polytechnique Fédérale de Lausanne. EPFL ENAC IIC RESSLab, GC B3 485, Station 18, 1015 Lausanne, Switzerland.(corresponding author). E-mail: dimitrios.lignos@epfl.ch

19 Seismic response of steel structures to extreme events is often nonlinear in nature. The
20 sources of the nonlinearity can be classified mainly into two categories: (1) material and
21 (2) geometric. Without accurate material and geometric properties, reliable physics-based
22 modeling of the hysteretic response of a component or sub-assembly can not be established.
23 Consequently, their impact at the system level can be the source of accumulated errors that
24 may lead to misleading conclusions of overall structural performance. Yet, all too often are
25 structural steel’s nonlinearities oversimplified or all together overlooked. In situations where
26 material and geometric nonlinearities strongly interact (*e.g.* open-section beam-columns),
27 it has been shown that such oversimplifications can lead to distinct member failure modes
28 (Hartloper et al. 2019a).

29 Accurate material modeling depends both on the constitutive model and on the ability
30 to obtain its characteristic input parameters. Solving inverse problems, however, are marred
31 with nonuniqueness issues (Cooke and Kanvinde 2015), *i.e.* a different set of parameters can
32 yield the same overall material response for a particular loading history— *c.f.* *e.g.* Fig. 1.
33 This feature can significantly hamper modeling efforts in situations where loading histories
34 differ from the calibrated cases. For earthquake applications this is naturally an issue of
35 concern due to the stochastic character of seismic action. To mitigate this problem, a
36 methodology has been proposed to calibrate parameters to a rich set of protocols that are
37 representative of seismic loading (de Castro e Sousa et al. 2020). While this approach
38 shows promise, obtaining test data can be challenging since it can be costly both in terms
39 of the number of specimens and of specialized testing equipment in order to achieve high
40 compressive strains. Furthermore, there are situations, like the performance assessment of
41 existing steel structures (CEN 2005; ASCE 2017), where collecting the required amount of
42 samples may be challenging. Given that there is a need for accurate material modeling, on
43 one hand, and that the ability to carry out extensive testing campaigns may be restricted,
44 on the other hand, a way to use limited test information, such as monotonic tensile tests, is
45 of particular interest to the research and practice communities.

46 Most of steel's mechanical design characteristics are typically obtained by tensile tests.
47 These tests are both common and standardized (ASTM 2013; CEN 2001), meaning they are
48 relatively economical and the testing methodology is established. Their main shortcoming,
49 however, is that they can not depict how the material would behave under cyclic loading.
50 Importantly, the extent of the Bauschinger effect and isotropic versus kinematic hardening
51 saturation levels are information that cannot be derived solely from monotonic loading. This
52 paper proposes addressing this challenge by determining material parameters from a specific
53 tensile test but that, at the same time, satisfy some *a priori* knowledge pertaining to its
54 cyclic properties. It is envisioned that this *a priori* knowledge should be directly related to
55 the steel grades' general material characteristics, *e.g.* its microstructure (in its grain size and
56 crystalline structure) or its chemical composition. As will be shown more concretely in the
57 methodology and results sections, different steel grades commonly present different ratios of
58 isotropic to kinematic hardening, a behavior believed to be closely related to the material's
59 grain refinement (Bouaziz et al. 2008).

60 The interpretation of material parameters is inextricably tied to the chosen model repre-
61 sentation. Herein, two materials models are used: (1) the Voce-Chaboche (VC) (Voce 1948;
62 Chaboche et al. 1979) and (2) an Updated Voce-Chaboche (UVC) (Hartloper et al. 2020).
63 Both are part of the rate independent, J_2 plasticity framework, but differ on their hardening
64 rules - the isotropic, in particular. The first is a widely used material model in the context
65 of nonlinear analysis of steel structures (*c.f. e.g.* Fell et al. (2009), Araújo et al. (2017),
66 Elkady and Lignos (2018)). Among its most important features is its ability to adequately
67 capture cyclic loading behavior with exponentially saturating hardening laws. The second
68 builds upon the VC by adjusting the material response to more accurately capture the dis-
69 continuous yielding phenomenon present in mild steels (Lubliner 2008). Both models aim at
70 representing physical mechanisms that lead to work hardening - *e.g.* the Bauschinger effect
71 through the kinematic hardening representation. However, the interpretation of material pa-
72 rameters through these models should only be made if they can be determined in a reliable

73 and consistent manner.

74 The approaches made in de Castro e Sousa et al. (2020) and Hartloper et al. (2019b)
75 are focused toward consistency of the parameters obtained for the VC and UVC models, re-
76 spectively. Therein, unconstrained optimization procedures, based on Newton Trust-Region
77 (NTR) framework, are presented that are able to navigate the non-convex parameter search
78 space both efficiently and reliably, given a diverse set of strain-based load protocols. The
79 parameters obtained for each of these models, with those methodologies, are considered to
80 be intrinsic to the material since they are both reliable and physically motivated. While
81 differences between steel production batches are to be expected, the hypothesis is made that
82 certain behavioral aspects are directly transferable between materials of the same grade;
83 that is to say that, specific material parameters might be different but, for example, ratios
84 of isotropic to kinematic hardening should remain largely of the same magnitude for a given
85 steel grade.

86 This paper concerns itself with answering mainly one question: can parameter optimiza-
87 tion of a single tensile test, holding constant characteristic material ratios for cyclic loading,
88 return satisfactory stress-strain responses for a wide range of cyclic load protocols? There is
89 a pressing need for an answer to this question. Consider, for example, Fig. 1 which depicts
90 test results for two load protocols (Fig. 1a /1c- monotonic; Fig. 1b/1d - cyclic variable
91 amplitude)¹. On top of the test results, two uniaxial VC model responses for the tests'
92 strain histories are also depicted, whose parameters are determined with (1) a wide range
93 of cyclic load protocols and (2) naively with only the tensile test data. It can be seen that,
94 although the parameters perform similarly in the tensile case, the tensile fitting's response
95 deviates largely from the variable amplitude response. Naive tensile-only calibrations do not
96 perform well when simulating cyclic loading because monotonic tensile tests do not contain
97 information regarding the Bauschinger effect nor cyclic hardening, therefore, it can be taken
98 for granted that naively using a tensile test to calibrate a cyclic material model is an inad-

¹load protocol designations can be consulted in de Castro e Sousa et al. (2020)

99 equate approach. Since it has been shown that even small differences in a material model
100 can have a disproportionate effect on the overall member behavior and influence simulated
101 geometric instabilities (Hartloper et al. 2019a), this aspect may be crucial in the accuracy
102 of structural responses in the nonlinear regime.

103 To address the problem stated above, an approach is proposed in this paper that makes
104 use of (1) material performance metrics and (2) a constrained optimization procedure using
105 the developed metrics in order to solve for the corresponding model parameters . Their effi-
106 cacy is evaluated by using appropriate error metrics. Additionally, comparisons are made to
107 material model performance using parameters from known databases for nominally identical
108 materials. It is shown that the proposed method outperforms a “naive” optimization, and
109 is on-or-above par with the use of relevant parameters from databases, for the case-study
110 addressed herein. Details for the approach are given in the Methodology section. After,
111 results of the application of the methodology to a number of materials are presented and a
112 discussion of those results is given in the subsequent section. The paper concludes with a
113 brief section summarizing the main outcomes of this research.

114 **METHODOLOGY**

115 A solution to the problem of determining a set of constitutive model parameters repre-
116 sentative for cyclic loading using only a tensile test is proposed in this section. The workflow
117 of the proposed methodology is shown in Fig. 2 applied to structural steel materials. The
118 workflow consists of three stages for a given steel material: Stage 1: determine the *a priori*
119 cyclic hardening characteristics of the material for a set constitutive model, Stage 2: use this
120 *a priori* information to inform the model calibration using only a tensile test, and Stage 3:
121 validate the results.

122 In Stage 1, the metrics to quantify the behavior of steel materials subjected to cyclic
123 loading, and the associated constraints, are proposed. An existing database of uniaxial
124 cyclic tests, denoted as the Root dataset, is used to calculate the bounds in these constraints
125 for selected materials through established calibration methods (Hartloper et al. 2019b; de

126 Castro e Sousa et al. 2020). In Stage 2, the constraints are used with tensile tests to form a
127 constrained optimization problem using this limited data. The solution to this minimization
128 problem yields the constitutive model parameters that match a tensile test with cyclic loading
129 information embedded. In Stage 3, an assessment of the methodology is made by comparing
130 the predicted steel material response using the tensile-only parameters with uniaxial coupon
131 tests from the Root dataset in addition to a separate dataset (denoted the Validation dataset)
132 to avoid bias in the results.

133 **Database of Steel Materials**

134 As shown in Fig. 2, data regarding the cyclic and tensile behavior of steel materials is
135 required at each step of the process, therefore, a database of such information is a necessary
136 starting point. An existing database of uniaxial tensile tests on structural steel materials, in
137 combination with several uniaxial cyclic tests conducted on round-bar coupons, is utilized in
138 this paper (Grigoriou and Lignos 2017; Suzuki 2018; Hartloper et al. 2020). This database
139 contains the original data used in the authors' research in the domain of inverse problems
140 and is thus coined the Root dataset. A further description of the database, including the
141 definitions of the cyclic load protocols used, is provided in de Castro e Sousa et al. (2020).
142 In brief, the strain-based load protocols are cataloged numerically into 10 different histories
143 that are deemed to be representative of strain histories in steel frame structural components
144 when subjected to earthquake loading. In total, nine sets of different steel materials (some
145 materials having multiple samples) are included in the database, as summarized Table 1.

146 The Root dataset (Table 1) is largely comprised of low-carbon (or mild) low-alloy struc-
147 tural steels that are commonly used in Europe (*e.g.* S355J2 and S355J2+N), North America
148 (*e.g.* A992 Gr. 50 and A500 Gr. B), and Japan (*e.g.* BCR295 and BCP325) (CEN 2005;
149 ASTM 2016; ASTM 2018). These mild steel materials are the focus of this study. Other
150 materials included in Table 1 are high-strength low-alloy steels (S460NL and S690QL) (CEN
151 2019b; CEN 2019a), and a high-performance steel that is manufactured with a thermome-
152 chanically controlled process and grain refinement (HYP400) (Suzuki et al. 2008) that are

153 of emerging interest in structural earthquake engineering. However, only a single set of tests
 154 is available for each of these materials, and therefore, the hardening characteristics of these
 155 materials cannot be reliably established at this time. Informative results for these materials
 156 are included for use in future work that may expand on the present study.

157 **Definition of the Constitutive Models**

158 Implementation of the VC and UVC models has already been extensively described in the
 159 literature (Hartloper et al. 2019b; de Castro e Sousa et al. 2020), therefore, only pertinent
 160 details are covered herein for brevity. The model implementation outlined in this section is
 161 specific to uniaxial stress states owing to the nature of the data included in the previously
 162 described database.

163 Both the VC and UVC models use a von Mises yield condition, ϕ^{VM} ,

$$164 \quad \phi^{VM} := |\sigma - \alpha| - (\sigma_{y,0} + K) \leq 0, \quad (1)$$

165 where σ is the uniaxial stress, α is the backstress defining the kinematic hardening compo-
 166 nent, $\sigma_{y,0}$ is the initial yield stress, and K is the isotropic hardening. The isotropic hardening
 167 rule defined in the UVC model is

$$168 \quad K = Q_\infty (1 - \exp[-b\dot{\varepsilon}_{eq}^p]) - D_\infty (1 - \exp[-a\dot{\varepsilon}_{eq}^p]), \quad (2)$$

169 where $\dot{\varepsilon}_{eq}^p = |\dot{\varepsilon}^p|$ is the equivalent plastic strain, Q_∞ is the magnitude of isotropic hardening
 170 at model saturation, b is the associated rate parameter, D_∞ and a are parameters to account
 171 for discontinuous yielding in structural steels. The over-dot indicates the derivative with
 172 respect to time. The parameter D_∞ imposes an initial reduction in the yield surface that
 173 saturates quickly when the rate parameter a is properly chosen. Isotropic hardening in the
 174 classic VC model is attributed to Voce (1948), and is simply recovered by removing the D_∞
 175 term from Eqn. 2. Removal of this term is the only difference in the hardening rules between
 176 the VC and UVC models.

177 Both material models employ the nonlinear kinematic hardening rule proposed by Chaboche
 178 et al. (1979) based on the work of Armstrong and Fredrick (1966). The Chaboche hardening
 179 rule is $\alpha = \sum_k^{N_k} \alpha_k$, where α_k are backstress components, $k = 1, 2, \dots, N_k$. Each component
 180 is defined by the Armstrong-Frederick rule for uniaxial loading,

$$181 \quad \dot{\alpha}_k = \text{sign}[\sigma - \alpha] C_k \dot{\varepsilon}_{eq}^p - \gamma_k \alpha_k \dot{\varepsilon}_{eq}^p, \quad (3)$$

182 where C_k is the magnitude of contribution to the total backstress for component k , and γ_k is
 183 the corresponding rate parameter. In this paper, the backstress rate parameters are defined
 184 such that $\gamma_1 > \gamma_2 > \dots > \gamma_{N_k}$.

185 Two constraints on the set of UVC parameters are required for modeling structural steels
 186 (Hartloper et al. 2019b); they are denoted g_1 and g_2 , and are respectively defined as

$$187 \quad g_1 := -Q_\infty b - \sum_k C_k + D_\infty a \leq 0, \quad (4)$$

$$188 \quad g_2 := Q_\infty b^2 + \sum_k C_k \gamma_k - D_\infty a^2 \leq 0. \quad (5)$$

190 These two constraints are sufficient conditions for the set of UVC model parameters to lead to
 191 a non-softening stress-strain response, therefore, they are considered an essential component
 192 of the UVC model for structural steels. Constraints (4) and (5) do not apply to the VC
 193 model.

194 Considering the uniaxial stress state, the addition of the elastic modulus, E , completes
 195 the set of parameters. The set of parameters for the VC and UVC models are respectively

$$196 \quad \mathbf{x}^{VC} = [E, \sigma_{y,0}, Q_\infty, b, C_1, \gamma_1, \dots, C_{N_k}, \gamma_{N_k}],$$

197 and

$$198 \quad \mathbf{x}^{UVC} = [E, \sigma_{y,0}, Q_\infty, b, D_\infty, a, C_1, \gamma_1, \dots, C_{N_k}, \gamma_{N_k}].$$

199 **Definition of the Inverse Problem**

200 Following de Castro e Sousa et al. (2020), an objective function is defined that represents
 201 the error in the model prediction based on the squared strain energy. The objective function
 202 $f(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ is

$$203 \quad f(\mathbf{x}) = \sum_{j=1}^{N_T} \frac{\int_0^{\varepsilon_j^*} (\sigma_j^{model}(\varepsilon_j; \mathbf{x}) - \sigma_j^{test})^2 d\varepsilon^*}{\int_0^{\varepsilon_j^*} d\varepsilon^*}, \quad (6)$$

204 where $\sigma_j^{model}(\varepsilon_j; \mathbf{x})$ is the predicted model stress given a strain history ε_j and parameters \mathbf{x} ,
 205 σ_j^{test} is the stress recorded in the uniaxial test j , and ε^* is the accumulated strain

$$206 \quad \varepsilon_j^* = \int_0^{t_j} |\dot{\varepsilon}_j| d\tau. \quad (7)$$

207 Note that Eqn. 6 is defined for N_T load histories allowing for multiple cyclic tests to be used
 208 in the calibration simultaneously.

209 We seek the set of parameters that minimizes the defined objective function, *i.e.*, the
 210 inverse problem of parameter calibration is posed as the constrained minimization problem

$$211 \quad \begin{aligned} & \text{minimize} && f(\mathbf{x}) && (8a) \\ & \mathbf{x} \in \mathbb{R}^n && \end{aligned}$$

$$212 \quad \text{subject to} \quad \mathbf{g}(\mathbf{x}) \leq \mathbf{0}, \quad (8b)$$

214 where $\mathbf{g}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a vector valued function of m constraints given n parameters in \mathbf{x} .
 215 These constraints are an essential component of the proposed methodology as they embed
 216 characteristics of the material subjected to cyclic loading into just a single tensile test.

217 **Definition of the Constraints to Embed Cyclic Loading Information**

218 Metrics are proposed to characterize the behavior of structural steels subjected to cyclic
 219 loading. These metrics should be derived from tests that are representative of the expected
 220 loading conditions of the material. In the context of the current paper, the load protocols
 221 should be consistent with the expected inelastic strain demands in steel components under

222 earthquake loading. Constraints on these metrics are then defined to embed this information
 223 into the calibration process when only a tensile test is used.

224 *Material Hardening Metrics*

225 The metrics defined herein are consistent with those used in de Castro e Sousa et al.
 226 (2020) to quantify the isotropic and kinematic hardening found in structural steels. Given
 227 a set of parameters for either the UVC or VC models, the following metrics are defined in
 228 terms of the equivalent plastic strain assuming monotonic loading from an initially strain
 229 free state. This assumption is necessary to define metrics independent of the strain history
 230 for the kinematic hardening component because the time-integration of Eqn. 3 is not a
 231 monotonically increasing function under cyclic loading. Where applicable, the metrics for
 232 the VC model are calculated neglecting D_∞ and a .

233 The total increase in stress due to combined isotropic and kinematic hardening is de-
 234 scribed by the metric

$$235 \quad \sigma_{hard}(\varepsilon_{eq}^p) = Q_\infty (1 - \exp[-b\varepsilon_{eq}^p]) + \sum_k C_k/\gamma_k (1 - \exp[-\gamma_k\varepsilon_{eq}^p]), \quad (9)$$

236 and the ratio of isotropic-to-total hardening is defined by the metric

$$237 \quad \rho_{iso}(\varepsilon_{eq}^p) = \frac{Q_\infty (1 - \exp[-b\varepsilon_{eq}^p])}{\sigma_{hard}(\varepsilon_{eq}^p)}. \quad (10)$$

238 A value of $\rho_{iso} = 1$ indicates that there is only isotropic hardening present in the material,
 239 and $\rho_{iso} = 0$ indicates purely kinematic hardening (*e.g.*, in high-strength steels). The total
 240 stress at a particular value of ε_{eq}^p is defined by

$$241 \quad \sigma_{total}(\varepsilon_{eq}^p) = \sigma_{y,0} + \sigma_{hard}(\varepsilon_{eq}^p) - D_\infty (1 - \exp[-a\varepsilon_{eq}^p]), \quad (11)$$

242 and the ratio of total stress to the initial yield stress is defined as

$$243 \quad \rho_{yield}(\varepsilon_{eq}^p) = \frac{\sigma_{total}(\varepsilon_{eq}^p)}{\sigma_{y,0}}. \quad (12)$$

244 A value of $\rho_{yield} = 2$ would indicate that the stress at the chosen value of ε_{eq}^p is twice the initial
 245 yield stress. In this paper, all the metrics are defined at their saturation value, $\varepsilon_{eq}^p \rightarrow \infty$,
 246 this is indicated by the *sat* superscript, *e.g.*, $\rho_{iso}^{sat} = Q_\infty / (Q_\infty + \sum_k C_k / \gamma_k)$.

247 A further metric is introduced relating the rates of kinematic and isotropic hardening.
 248 This metric quantifies the rates at which the different hardening components saturate as-
 249 suming monotonic loading, and is defined as

$$250 \quad \rho_{\gamma_1 b} = \frac{\gamma_1}{b}. \quad (13)$$

251 If $\rho_{\gamma_1 b} > 1$ this indicates that the kinematic hardening saturates at a lower equivalent plastic
 252 strain than the isotropic hardening under monotonic loading. A metric is proposed to define
 253 the rate of saturation for various kinematic hardening components if more than one backstress
 254 is specified in the constitutive model:

$$255 \quad \rho_{\gamma_1 \gamma_2} = \frac{\gamma_1}{\gamma_2}. \quad (14)$$

256 This constraint is necessary to prevent duplication of the backstress rate terms, for instance,
 257 if $\gamma_1 = \gamma_2$, then essentially the two-backstress model reduces to a one-backstress model. This
 258 issue arises because there is not enough information in a single tension test to differentiate
 259 the kinematic hardening components since a near-perfect fit can be obtained with a single
 260 component (see Figs. 1a and 1c). Whereas, this same issue only appears to occur after four
 261 backstresses are included when the full set of load protocols are used in the calibration effort
 262 (de Castro e Sousa et al. 2020).

263 An additional metric is proposed for the UVC model that specifies the ratio of the initial

264 reduction in the yield surface to the total hardening at saturation. More specifically,

$$265 \quad \rho_D(\varepsilon_{eq}^p) = \frac{D_\infty (1 - \exp[-a\varepsilon_{eq}^p])}{Q_\infty (1 - \exp[-b\varepsilon_{eq}^p]) + \sum_k C_k/\gamma_k (1 - \exp[-\gamma_k \varepsilon_{eq}^p])}. \quad (15)$$

266 Another possible constraint is to specify the ratio between D_∞ and Q_∞ , however, this con-
 267 straint is not valid for steels with very low isotropic hardening (*e.g.*, high strength steels).
 268 This is also common in the case of cold-forming, such as in the corners of press-bent products.

269 *Constraints on the Hardening Metrics*

270 Constraints are imposed on each of the metrics described in the previous section for use
 271 in the constrained minimization problem. All the constraints are posed as bound constraints
 272 in the standard form specified in (8b), therefore, two constraints are defined for the lower and
 273 upper bound of each metric, g^{inf} and g^{sup} , respectively. Each of the following constraints
 274 consists of a metric, ρ , and a constant that specifies the lower/upper bound of the metric,
 275 $c^{inf/sup}$.

276 In total, there are ten constraints specified assuming a maximum of two backstresses in
 277 the constitutive model:

$$278 \quad g_{yield}^{inf} := c_{yield}^{inf} - \rho_{yield}^{sat}; \quad g_{yield}^{sup} := \rho_{yield}^{sat} - c_{yield}^{sup}; \quad (16a)$$

$$279 \quad g_{iso}^{inf} := c_{iso}^{inf} - \rho_{iso}^{sat}; \quad g_{iso}^{sup} := \rho_{iso}^{sat} - c_{iso}^{sup}; \quad (16b)$$

$$280 \quad g_{\gamma_1 b}^{inf} := c_{\gamma_1 b}^{inf} - \rho_{\gamma_1 b}; \quad g_{\gamma_1 b}^{sup} := \rho_{\gamma_1 b} - c_{\gamma_1 b}^{sup}; \quad (16c)$$

$$281 \quad g_{\gamma_1 \gamma_2}^{inf} := c_{\gamma_1 \gamma_2}^{inf} - \rho_{\gamma_1 \gamma_2}; \quad g_{\gamma_1 \gamma_2}^{sup} := \rho_{\gamma_1 \gamma_2} - c_{\gamma_1 \gamma_2}^{sup}; \quad (16d)$$

$$282 \quad g_D^{inf} := c_D^{inf} - \rho_D^{sat}; \quad g_D^{sup} := \rho_D^{sat} - c_D^{sup}; \quad (16e)$$

283

284 The constants in each of (16a)–(16e) depend on the constitutive model (*e.g.*, VC or UVC,
 285 including the number of backstresses assumed), and the equivalent plastic strain used in
 286 computing the corresponding metrics (*e.g.*, at saturation). Both these factors should be
 287 consistent with the calibration at hand when computing the constants.

288 **Proposed Calibration Procedure**

289 The optimization problem (8a) subjected to (8b) is solved with the NITRO algorithm
290 (Byrd et al. 1999) as implemented in Scipy (Virtanen et al. 2020). The NITRO algorithm
291 requires an initial starting point, and is an iterative method where the stopping point for a
292 local minimum is determined by the first- and second-order necessary conditions (i.e., the
293 norm of the objective function’s gradient is less than a specified tolerance, and the objective
294 function’s Hessian is positive definite). Gradients and Hessians required in this algorithm are
295 computed using algorithmic differentiation, as previous studies have demonstrated the effi-
296 cacy of this approach when combined with gradient-based optimization methods (Hartloper
297 et al. 2019b; de Castro e Sousa et al. 2020).

298 The use of a minimization algorithm requires an initial starting point. Two starting
299 points are used in this study to assess if similar optimal parameters are obtained from
300 different initial conditions. The first starting point, denoted as SP1, is feasible with respect
301 to the constraint set. To generate SP1, an initial set of parameters is chosen that represents
302 an elastic perfectly-plastic material as assumed in de Castro e Sousa et al. (2020); this set is
303 then modified according to Algorithm 1 so that the initial point is feasible with respect to the
304 constraints. The basis for this algorithm is to set the parameters such that the constraints
305 are set to their mean of the upper and lower bound values. The second starting point,
306 SP2, is a potentially infeasible point that is just the set of parameters leading to a nearly
307 perfectly-plastic response (SP1 prior to applying Algorithm 1).

308 With reference to Stage 2 of the workflow shown in Fig. 2, the proposed procedure for
309 calibration of the VC and UVC model parameters using only a tensile test is as follows:

310 **1. Specify the constraints:**

- 311 (a) VC model: The constraint vector, $g(x)$, is composed of constraints (16a)–(16c)
312 regardless of the model and number of backstresses. Constraints defined in (16d)
313 are added if two backstresses are considered.

314 (b) UVC model: The constraint vector, $g(x)$, is composed in the same manner as the
 315 VC model for either one or two backstresses. Additionally, g_1 and g_2 are added
 316 to ensure non-softening of the material.

317 **2. Set the material cyclic hardening metric constraint bounds:** The bounds on
 318 the constraints are defined using the minima and maxima of relevant metrics from prior
 319 calibrations on similar materials using relevant cyclic loading histories. The metrics
 320 used to establish the bounds should match the employed material model and number
 321 of backstresses for the constraints to be meaningful.

322 (a) VC model:

- 323 • One backstress: $c_{yield}^{inf}, c_{yield}^{sup}, c_{iso}^{inf}, c_{iso}^{sup}, c_{\gamma_1 b}^{inf}, c_{\gamma_1 b}^{sup}$,
- 324 • Two backstresses: VC model one backstress, additionally $c_{\gamma_1 \gamma_2}^{inf}, c_{\gamma_1 \gamma_2}^{sup}$

325 (b) UVC model:

- 326 • One backstress: VC model one backstress, additionally c_D^{inf}, c_D^{sup}
- 327 • Two backstresses: VC model two backstresses, additionally c_D^{inf}, c_D^{sup}

328 **3. Select the starting point:** The starting point is intended to represent a material with
 329 nominal elastic and yield properties, and a nearly-perfectly plastic behavior. Nominal
 330 values for the elastic modulus and yield stress are selected as $E_n = 200$ GPa and
 331 $f_{y,n} = 355$ MPa for all materials in this study.

332 (a) VC model:

- 333 • One backstress: $x_{SP2}^{VC} = [E_n, f_{y,n}, 1, 1, 1, 1]$
- 334 • Two backstresses: $x_{SP2}^{VC} = [E_n, f_{y,n}, 1, 1, 1, 1, 1, 1]$

335 (b) UVC model: Note that the initial value for “ a ” is set to 200 to approximately
 336 reach 99 % saturation of D_∞ at 2 % strain (typical plateau strain in mild steels).

- 337 • One backstress: $x_{SP2}^{UVC} = [E_n, f_{y,n}, 1, 1, 1, 200, 1, 1]$

338

- Two backstresses: $x_{SP2}^{UVC} = [E_n, f_{y,n}, 1, 1, 1, 200, 1, 1, 1, 1]$

339

4. **(Optional) Make the starting point feasible:** If desired, Algorithm 1 can be applied to $x_{SP2}^{VC} / x_{SP2}^{UVC}$ so that the starting point satisfies the hardening metric constraint bounds. The basis of this algorithm is to set the material properties to the average of the relevant bounds. The feasible starting point is denoted $x_{SP1}^{VC} / x_{SP1}^{UVC}$.

340

341

342

343

5. **Determine the optimal set of parameters:** The minimization, Problem (8), is solved using the NITRO algorithm starting from SP1 or SP2 with a tolerance on the convergence criteria of 10^{-8} and a maximum of 600 iterations. Note that the solution point typically converges to a local minima within 200 iterations for the steel materials examined in this paper (on average, approximately 5 minutes on a desktop computer with a 4 GHz processor), therefore, the 600 iterations is deemed to be adequate.

344

345

346

347

348

Algorithm 1 Generate a feasible starting point with respect to $g_{yield}, g_{iso}, g_{\gamma_1 b}, g_{\gamma_1 \gamma_2}$.

- 1: $\rho_{yield,avg}^{sat} \leftarrow 0.5 \times (c_{yield}^{inf} + c_{yield}^{sup})$
 - 2: $\rho_{iso,avg}^{sat} \leftarrow 0.5 \times (c_{iso}^{inf} + c_{iso}^{sup})$
 - 3: $\rho_{\gamma_1 b,avg}^{sat} \leftarrow 0.5 \times (c_{\gamma_1 b}^{inf} + c_{\gamma_1 b}^{sup})$
 - 4: $\rho_{\gamma_1 \gamma_2,avg} \leftarrow 0.5 \times (c_{\gamma_1 \gamma_2}^{inf} + c_{\gamma_1 \gamma_2}^{sup})$
 - 5: $\gamma_{2,0} \leftarrow \gamma_{1,0} / \rho_{\gamma_1 \gamma_2,avg}$
 - 6: $C_{1,0} \leftarrow -\gamma_{1,0} \sigma_{y,0,0} (-1 + \rho_{iso,avg}) (-1 + \rho_{yield,avg})$
 - 7: $Q_{\infty,0} \leftarrow C_{1,0} \rho_{iso,avg} / (\gamma_{1,0} (-1 + \rho_{iso,avg}))$
 - 8: $b_0 \leftarrow \gamma_{1,0} / \rho_{\gamma_1 b,avg}$
-

349

Metrics Used for Procedure Evaluation

350

351

352

Referring to Stage 3 of the proposed workflow shown in Fig. 2, the normalized form of the objective function is used to assess the accuracy of the proposed calibration procedure using only a tensile test. The normalized error metric is

353

$$\bar{\varphi} = \sqrt{\frac{f(\mathbf{x})}{E_{total}}}, \quad (17)$$

354 where the total normalized strain energy of all the considered tests in a data set is

$$355 \quad E_{total} = \sum_{j=1}^{N_T} \frac{\int_0^{\epsilon_j^*} (\sigma_j^{test})^2 d\epsilon^*}{\int_0^{\epsilon_j^*} d\epsilon^*}. \quad (18)$$

356 The metric $\bar{\varphi}$ represents the error in strain energy calculated over the entire data set (*i.e.*,
357 tensile and cyclic tests), normalized by the total strain energy of all the tests.

358 Consistency between sets of parameters is evaluated using the ξ_2 metric defined in de
359 Castro e Sousa et al. (2020). This metric is computed between two sets of parameters \mathbf{x}_{base}
360 and \mathbf{x}_{sample} as

$$361 \quad \xi_2 = \frac{(\mathbf{x}_{sample} - \mathbf{x}_{base})^T \nabla_{xx} f(\mathbf{x}_{base}) (\mathbf{x}_{sample} - \mathbf{x}_{base})}{E_{total}}, \quad (19)$$

362 where $\nabla_{xx} f(\mathbf{x}_{base})$ is the Hessian of the objective function for a given data set evaluated
363 at the local minimum of $f(\mathbf{x})$, \mathbf{x}_{base} . The ξ_2 metric represents the normalized increase in
364 the minimum value of the objective function when choosing \mathbf{x}_{sample} instead of \mathbf{x}_{base} . If this
365 increase is insignificant, then the two sets of parameters are said to be consistent.

366 Validation of the Results

367 The main objective of this paper is to answer the question of whether calibrations of single
368 tensile tests, constrained by cyclic material characteristics, will yield *satisfactory* responses
369 for a wide range of cyclic protocols. Here, the proposed tensile-only calibration procedure
370 is satisfactory if the results out-perform, in the $\bar{\varphi}$ -sense for all LPs, parameters derived
371 for other materials using all available load protocols. The satisfactory condition defined
372 above reflects the common situation where a modeler selects input model parameters from
373 a nominally identical steel grade that have been calibrated in a prior study for an entirely
374 different production batch. For instance, parameters from de Castro e Sousa et al. (2020) for
375 a hot-rolled HEB500 section of S355J2 ($f_y = 355$ MPa) steel may be selected to model an
376 IPE300 section from a different production heat of the same steel grade. In this hypothetical
377 situation, we would like to demonstrate that, on average, there is less error in the $\bar{\varphi}$ -sense by
378 using the proposed tensile-only calibration with a supplied tensile test than using alternative

379 parameters from the same steel grade.

380 Since the cyclic characteristics are obtained *a priori*, validation of the proposed method-
381 ology is done using data sets of S355J2 steel that are separate from the database summarized
382 in Table 1 to avoid bias in the results. The characteristics are derived from the Root dataset
383 (see Fig. 2) and used to analyze steel materials from another set, coined the Validation
384 dataset. The Validation dataset consists of tests conducted on round-bar coupons of S355J2
385 steel (with same group of ten uniaxial cyclic load protocols) sampled from two alternative
386 sources: (1) the web of an IPE300 hot rolled profile and (2) the flange of the same IPE300.
387 The proposed calibration procedure described above is subsequently applied to estimate the
388 model parameters using a single tensile test. Comparisons are then made to parameters that
389 would be obtained if all available load protocols were used, as well as using the parameters
390 from Tables 2 and 6 for S355 steels calibrated from the Root dataset.

391 **RESULTS**

392 The methodology proposed in the previous section is applied to the database of steel
393 materials summarized in Table 1. Results for the VC material model are presented first,
394 followed by the UVC model. In all cases, discussion of the results is delayed to the Discussion
395 section.

396 **Voce-Chaboche Model Results**

397 *Constraint Bounds*

398 Bounds on the metrics used to form the constraints defined in (16a)–(16d), *i.e.*, the c
399 values, are calculated using the parameters obtained for mild structural steels. As mentioned
400 in the Methodology section, only the S355J2(+N), A992 Gr. 50, and BCP325 steels are
401 utilized for the tensile-only calibration because these materials are consistent in terms of
402 their nominal chemical compositions and yield/tensile stresses (see Table 1) (CEN 2005;
403 ASTM 2016). The other materials included in Table 1 are excluded from the metric bounds,
404 and the tensile-only calibration is not conducted. The materials are excluded on the basis
405 that: (1) they are high-strength low-alloy steels (*e.g.*, S460NL and S690QL), (2) are cold

406 worked from the corners of hollow structural sections (*e.g.*, A500 Gr. B and BCR295), or
407 (3) were manufactured with a thermomechanically controlled process and grain refinement
408 (*e.g.*, HYP400). However, parameters and hardening metrics using all available protocols
409 are provided for each material in the Root dataset for use in future research.

410 Parameters for the VC model with one and two backstresses computed using the method-
411 ology in de Castro e Sousa et al. (2020) are recorded for all the Root materials in Table 2.
412 The material hardening metrics computed for the subset of mild steels are summarized in
413 Table 3, and a summary of the metrics can be viewed graphically in Fig. 3. Most importantly,
414 observe that the computed metrics in Table 3 depend on the number of backstresses included
415 in the model, therefore, a separate set of bounds are used for either one or two backstresses.
416 Notice that there is an increase in the ratio of kinematic hardening when two backstresses
417 are included in the model, this is because the kinematic hardening is more accurately repre-
418 sented by two backstress components, thus increasing its relevance and magnitude. The total
419 hardening, σ_{total}^{sat} , is also greater when two backstresses are included. The reason for this is
420 that γ_1 in the one-backstress model is greater than γ_2 in the two-backstress model, therefore,
421 the two-backstress model saturates at a much greater plastic strain than the one-backstress
422 model. This issue could be addressed by evaluating this metric at a finite equivalent plastic
423 strain value, but for simplicity, this is not taken into account herein. As for the initial yield
424 stress, comparing the $\sigma_{y,0}$ values in Table 2 with the nominal yield stresses in Table 1, the
425 VC model values are close to, but consistently lower than, the nominal yield stress values.
426 This is a known issue with the VC model for structural steels, and is the motivation behind
427 the development of the UVC model (Hartloper et al. 2019b).

428 Low coefficient of variation (CoV) values for the metrics imply consistency across the
429 steel materials, thereby providing an indication of the link between the physical material
430 properties and the proposed hardening metrics. Results in Fig. 3 suggest that a reasonably
431 low variance in all metrics is found for the VC one-backstress model, while this variance is
432 significantly larger for the ρ_{yield}^{sat} and $\rho_{\gamma_1 b}$ metrics of the two-backstress model. Furthermore,

433 the metric $\rho_{\gamma_1\gamma_2}$ applicable to the VC two-backstress model does not appear to be reliable
 434 given that the CoV is slightly lower than 90 %. Recall, however, that this constraint should
 435 be considered to avoid duplication of the γ_k parameters when two backstresses are used in
 436 the tensile-only calibration.

437 The constraint bounds applicable to mild steels contain the minima and maxima of the
 438 metrics in Table 3 for S355J2(+N), A992 Gr. 50, and BCP325 steels. For the VC one-
 439 backstress model, these bounds are: $1.5 \leq \rho_{yield}^{sat} \leq 2.5$, $0.35 \leq \rho_{iso}^{sat} \leq 0.50$, and $2.25 \leq \rho_{\gamma_1b} \leq$
 440 3.25 . Similarly, the bounds used in this study for the VC model and two backstresses are:
 441 $1.75 \leq \rho_{yield}^{sat} \leq 2.3$, $0.15 \leq \rho_{iso}^{sat} \leq 0.30$, $8.5 \leq \rho_{\gamma_1b} \leq 25.0$, and $13 \leq \rho_{\gamma_1\gamma_2} \leq 92$.

442 *Tensile-Only Calibration Results*

443 Bounds on the metrics are used to establish the constraint vector $\mathbf{g}(\mathbf{x})$ used in the
 444 proposed calibration procedure (Stage 2, Fig. 2) for the subset of *Root* data. Results for the
 445 tensile-only calibration of the seven selected steel materials are provided in Table 4. The
 446 normalized error metric values, $\bar{\varphi}$, in Table 4 are evaluated using the parameters in this table
 447 along with all the available load protocols for each steel material. Similarly, the values of
 448 the ξ_2 metric are calculated using Eqn. 19 with \mathbf{x}_{base} taken from Table 2 for the full data
 449 calibration (the base), and \mathbf{x}_{sample} taken from Table 4 for the tensile-only calibration (the
 450 sample). The results in Table 4 suggest that there is no benefit in this approach to use two
 451 versus only one backstress in terms of the value of the error function—in fact it is worse to
 452 use two backstresses. Also, if one takes the threshold value of $\xi_2 = 5\%$ discussed in de Castro
 453 e Sousa et al. (2020), these results show that the tensile only parameters are inconsistent
 454 with those retrieved from a calibration a full set of load protocols. The implications of these
 455 findings are further expounded in the Discussion section.

456 The influence of the two starting points is shown in Fig. 4 for the S355J2+N 50 mm plate
 457 data set, and similar results hold for the other data sets although they are not shown herein.
 458 The convergence criteria at each iteration of the NITRO algorithm is shown in Fig. 4a,
 459 and the results imply that the starting point does not have a major influence on the rate of

460 convergence for the algorithm. Furthermore, the convergence criteria of 10^{-8} for Problem (8)
461 are satisfied in all but one case (S355J2+N 25 mm plate). In this case, the exit convergence
462 criterion value was about 10^{-5} after 600 iterations. The plot of the objective function value
463 at each iteration, Fig. 4b, indicates that the same level of error is reached regardless of the
464 starting point. The reason is that the model parameters also converge to the same value for
465 both SP1 and SP2, as shown in Fig. 4c that tracks the model parameters at each iteration.
466 Therefore, the chosen starting points do not appear to be influential on the obtained solution
467 point in this calibration procedure.

468 *Validation*

469 Data sets of uniaxial coupon tests conducted on samples from the flanges and web of an
470 IPE300 profile of S355J2 steel are used as the *Validation* set for our proposed tensile-only
471 calibration. The results of the validation are summarized in Table 5 via the $\bar{\varphi}$ metric. Values
472 of the $\bar{\varphi}$ metric in this table are computed using all available load protocols for the IPE300
473 flange/web, and parameters that are either: (1) calibrated using the methodology in this
474 paper using only the tensile test of the IPE300 flange or web data sets (tensile-only), or
475 (2) calibrated using all available load protocols of other S355J2(+N) steel materials. For
476 instance, the error computed across the entire IPE300 flange dataset using the parameters
477 from the IPE300 tensile-only calibration is $\bar{\varphi} = 10.53 \%$, while the error computed using the
478 parameters from the S355J2+N (50 mm plate) dataset is $\bar{\varphi} = 11.25 \%$. The implications of
479 these results will be discussed shortly.

480 Figure 5 is provided to compare the level of fit between different options for selected
481 tests from the IPE300 flange data set. Material model predictions in these figures are com-
482 puted with the parameters from the VC 1-backstress model tensile-only calibration using
483 the IPE300 flange (“Tensile”), and the VC 2-backstress model parameters calibrated to the
484 S355J2 HEB500 flange data (“HEB500-Flange”). For the case shown in Fig. 5, the pro-
485 posed tensile-only calibration leads to results that are in closer agreement with the test data
486 regardless of the load protocol. Moreover, a stark contrast is apparent between the poor

487 results of the “naive” approach, where no information is used to guide the tensile-only cali-
488 bration (see Fig. 1b/1d), and the reasonably good results shown in Fig. 5c when the proposed
489 methodology is employed.

490 Updated Voce-Chaboche Model Results

491 *Constraint Bounds*

492 The same materials and methodology as discussed above for the VC model is applied to
493 the UVC model to derive appropriate bounds of the model’s material performance metrics.
494 UVC material model parameters are provided in Table 6 for one and two backstresses, and
495 the hardening metrics are summarized in Table 7. A summary of the metrics can also be
496 viewed in graphical form in Fig. 3.

497 Many observations regarding the link between the calibration results and the physical
498 material properties made for the VC model also apply to the UVC model. However, a quick
499 review of the obtained metrics shows a significant dispersion in the ratios in the material
500 metrics ρ_{iso}^{sat} and ρ_D^{sat} when only one backstress is included in the UVC model (see Figs. 3b
501 and 3e). The later metric essentially measures the decay in isotropic hardening stress with
502 respect to the other hardening components of the model, and the accuracy of this parameter
503 is crucial to the UVC functioning as intended since it is the main differentiator with respect
504 to the VC model. Furthermore, considering that these two metrics cannot be defined with
505 precision, the initial hypothesis that these metrics are an intrinsic property of the material
506 cannot be made with any degree of confidence when only one backstress is included in the
507 UVC model. It follows, then, that it is pointless to pursue analyses with one backstress using
508 the UVC model. This is not the case, however, for the two backstress condition, where ρ_D^{sat}
509 and ρ_{iso}^{sat} are defined within a narrow CoV range.

510 Following the above argument, the recommended bounds used in this study to assess
511 tensile-only calibrations for the UVC model are based solely on a two-backstress model. For
512 the steels in question (S355J2(+N), A992 Gr. 50, and BCP325) these are: $1.8 \leq \rho_{yield}^{sat} \leq 2.1$,
513 $0.25 \leq \rho_{iso}^{sat} \leq 0.30$, $13.0 \leq \rho_{\gamma_1 b} \leq 20.0$, $15.0 \leq \rho_{\gamma_1 \gamma_2} \leq 30.0$, and $0.2 \leq \rho_D \leq 0.3$.

Tensile-Only Calibration and Validation Results

In contrast with the VC model, UVC tensile-only parameters are not presented herein. The reason for this decision follows from the results in Table 8 for the validation of the UVC two-backstress model. This table indicates that, in terms of the error metric $\bar{\varphi}$, the performance of the tensile-only parameters is significantly worse than using the parameters from a different steel from a database derived with full load protocols (*i.e.*, using any of the two-backstress parameter sets from Table 6). For example, the error in the IPE300 web data set is around $\bar{\varphi} = 25\%$ when the proposed tensile-only calibration procedure is used with the UVC model, while the error is around $\bar{\varphi} = 10\text{--}18\%$ using the parameters calibrated from other data sets. Poor results of the UVC tensile-only calibration are apparent in Fig. 6: despite matching the tensile test, the other cyclic load protocols are not accurately represented by the model predictions. These results demonstrate that UVC tensile-only calibrations, even with two backstresses, are inadequate with the methodology and data presented herein, and should not be used when parameters from a material with a similar microstructure are available.

DISCUSSION

The methodology presented in this paper assumes a strong link between physical material response and the parameters used in its representation by a material constitutive model. This assumption is crucial in order to obtain accurate cyclic model responses by performing a constrained optimization with limited information (in this case a tensile-only test of a material sample). From the results presented in the previous section, it was shown that while precise performance metrics can be found for the material models under analysis (VC with one backstresses, and UVC with two backstresses), when these metrics are used in the constrained optimization with tensile-only test data, the results are not universally satisfactory. In fact, the poor performance of the UVC under these conditions leads to the inescapable conclusion that it should not be used for calibrations under limited information. This is not the case for the VC model—some nuances notwithstanding .

541 The results depicted in Table 4 for the VC show persistently that tensile-only calibrations
542 with two backstresses underperform, in error-metric terms ($\bar{\varphi}$), the model with a single
543 backstress. One key observation in this Table is that the second backstresses parameters are
544 close to zero, meaning that the model response is close to a single-backstress condition. A
545 probable cause for this result is the significant dispersion in the $\rho_{\gamma_1\gamma_2}$ performance metric in
546 Table 3 and depicted in Fig. 3d, since it is the only metric that relates both backstresses.
547 Furthermore, the consistency of the solutions, as measured by ξ_2 metric with respect to
548 the parameters calibrated with all available load protocols, is significantly worse than the
549 one-backstress case. Moreover, the results from the validation dataset shown in Table 5
550 show overwhelmingly that the tensile-only two-backstress VC, underperforms models with
551 parameters unrelated to the specific sample, *i.e.* taken from another (Root) dataset. All-
552 in-all, these findings seem to indicate that there just isn't enough information in a tensile
553 test to accurately provide two-backstress VC model parameters and that, under conditions
554 in which the modeler is faced with a single tensile-only test, it is preferable to use known
555 database parameters rather than attempting to calibrate a two-backstress VC model. This
556 conclusion, however, does not hold for the one-backstress VC model.

557 The one-backstress VC model performance, shown in Tables 4 and 5, suggests that error
558 levels are typically smaller than the two-backstress tensile-only VC model. Moreover, in the
559 validation cases the one-backstress model performs at error levels on or above par with respect
560 to known two-backstress database parameters established with many more load protocols
561 (the recommended VC calibrations strategy of de Castro e Sousa et al. (2020)). These results
562 seem to indicate that a limited information calibration procedure with a one-backstress VC
563 model is, in fact, a suitable approach to modeling material response when only monotonic
564 test data is available. Indeed it could be said, also with respect to the UVC results, that the
565 simplest model is the one that performs best under limited information conditions.

566 Why, then, should one find the possibility of using a single test result to calibrate a
567 material model worthwhile? The main advantage of using the constrained optimization

568 procedure advocated in this paper, instead of using known database parameters, is that
569 it diminishes the uncertainty related to the steel making process. Consider, for instance,
570 using database parameters for S355J2 taken from an HEB500 flange and applying them
571 to an S355J2 IPE300 flange. The results in Table 5 show that an error of $\bar{\varphi} = 16.91\%$ is
572 obtained, whereas if the tensile-only IPE300 flange test is used, one obtains an error $\bar{\varphi} =$
573 10.53% . This suggests that, although the materials are “*theoretically*” the same, variations
574 in manufacturing conditions (*e.g.* differences between production batches or profile rolling
575 schemes) can affect a material’s cyclic response and that the methodology presented herein
576 can mitigate this effect.

577 The facts above notwithstanding, if a modeler can have access to a material’s cyclic
578 response using multiple load protocols, the modeler should not hesitate in using them (de
579 Castro e Sousa et al. 2020). The accuracy of the parameters obtained using multiple load
580 protocols is substantially better than the alternatives (constrained optimization or estab-
581 lished material databases). This can also be seen in Table 4, where mid-single-digit errors
582 are obtained if a two-backstress solution with all available load protocols is used. Moreover,
583 it should be noted that, although satisfactory results are obtained using the one-backstress
584 VC model tensile-only method, the values of $\xi_2 > 5\%$ in Table 4 indicate that the parameters
585 are inconsistent with those obtainable from more protocols – *c.f.* de Castro e Sousa et al.
586 (2020) for details on the 5% bound. In other words, the proposed method is acceptable **only**
587 in situations where limited information is available on the material at hand, but it is not
588 without its shortcomings.

589 Another disadvantage of this methodology is the fact that best-fit is only achieved with
590 a one-backstress solution for the VC model. The feature restricts the possible shapes of
591 the hardening material, which in certain applications may reveal crucial. Here, care should
592 be taken on the range of plastic strains to which the model is to be applied. For instance,
593 hardening slopes play a significant role in localization problems and, as such, this approach is
594 likely not suitable in studying cases like ultra low cycle fatigue of materials where significant

595 plastic strains ranges coupled with damage occurs. It should be stated, in fact, that the most
596 suitable cases to use in this approach would be cases where nominal strains (as opposed to
597 strain riser regions, such as sharp geometry changes or welds) are fairly bounded (between
598 $-5/+5$ % plastic strains, the range of most cyclic calibration tests).

599 Lastly, it should be noted that the best-fit constrained optimization method presented
600 herein is geared toward obtaining better cyclic material properties than otherwise obtained
601 with a tensile test or from existing databases, which may not directly translate itself to a
602 "best-fit" component behavior. The material will be better characterized (as shown above),
603 but how this relates to a better hysteretic response on the component side is a different
604 issue. Component behavior depends not only on material characteristics, but also on how
605 these interact with other features like its slenderness, geometric imperfections or residual
606 stresses and demand histories. As such, one can only point out to the component modeler
607 that some care should be taken with regards to the sensitivity of the component response to
608 material parameters, because cases can arise where different failure modes can be triggered
609 as a function of the material response (Hartloper et al. 2019a).

610 **CONCLUSIONS**

611 This paper presented a methodology to obtain material model parameters under limited
612 information conditions. The models discussed herein are the Voce-Chaboche (VC) and an
613 Updated Voce-Chaboche (UVC) in the context of structural steel applications. Limited in-
614 formation conditions can arise when one intends to study the behavior of steel structures
615 under large amplitude cyclic loading, but knowledge of the employed steel material is re-
616 stricted to a single tensile test. A procedure is proposed herein that allows the calibration
617 of material parameters taking into account its cyclic properties with a single monotonic
618 test using a constrained optimization scheme. This procedure has been incorporated into
619 the open-source Python library, RESSPyLab (de Castro e Sousa et al. 2019), which is made
620 publicly available. The following conclusions can be made from this work:

- 621 • The definition of adequate material cyclic-performance metrics is an essential part

622 in the constrained optimization procedure. Metrics are proposed herein that relate
623 typical ratios of isotropic to kinematic hardening and their rates in order to inform the
624 constrained optimization procedure;

- 625 • The number of backstresses used in the kinematic part of the models plays a significant
626 role in the accuracy of tensile-only parameter estimates;
- 627 • The UVC model does not provide satisfactory results in parameter estimates in the
628 tensile-only constrained optimization, regardless of the number of backstresses em-
629 ployed;
- 630 • The VC model with two backstresses does not provide satisfactory estimates of material
631 response for tensile-only calibrations with the constraints presented herein. More data
632 can however motivate different constraints;
- 633 • The VC model with one backstress, however, yields satisfactory results and is the
634 recommended material model if a modeler is faced with a situation where tensile-only
635 data is available.

636 It should be underscored that this methodology is a best-fit approximation in limited
637 information cases. Whenever possible, material testing taking into account multiple load
638 protocols should, nevertheless, be used.

639 **DATA AVAILABILITY**

640 Some or all data, models, or code generated or used during the study are available in
641 a repository or online in accordance with funder data retention policies (de Castro e Sousa
642 et al. 2019).

643 **ACKNOWLEDGMENTS**

644 The authors recognize and are grateful for the financial support of this study by the
645 Nippon Steel Corporation, internal grants from École Polytechnique Fédérale de Lausanne

646 (EPFL), as well as an exploratory grant from EPFL’s School of Environmental, Archi-
647 tectural and Civil Engineering, and the Swiss National Science Foundation (Project No.
648 200021_188476). Any opinions, findings, and conclusions or recommendations expressed in
649 this paper are those of the authors and do not necessarily reflect the views of sponsors.

650 REFERENCES

- 651 Araújo, M., Macedo, L., and Castro, J. M. (2017). “Evaluation of the rotation capacity limits
652 of steel members defined in EC8-3.” *Journal of Constructional Steel Research*, 135, 11–29.
- 653 Armstrong, P. and Fredrick, C. (1966). “A Mathematical Representation of the Multiaxial
654 Bauschinger Effect.” *Report No. RD/B/N 731*, Central Electricity Generating Board.
- 655 ASCE (2017). “ASCE 41-17 - Seismic Evaluation and retrofit of existing buildings.” Ameri-
656 can Society of Civil Engineers (ASCE).
- 657 ASTM (2013). *E8/E8M - 13a - Standard Test Methods for Tension Testing of Metallic*
658 *Materials 1*. ASTM International, West Conshohocken, PA, USA.
- 659 ASTM (2016). “Standard Specification for General Requirements for Rolled Structural Steel
660 Bars, Plates, Shapes, and Sheet Piling.” *A6/A6M-16a*, ASTM International, West Con-
661 shohocken, PA, USA.
- 662 ASTM (2018). “Standard Terminology Relating to Steel, Stainless Steel, Related Alloys, and
663 Ferroalloys.” *A941-18*, ASTM International, West Conshohocken, PA, USA.
- 664 Bouaziz, O., Aouafi, A., and Allain, S. (2008). “Effect of grain refinement on the mechanical
665 behaviour of ferritic steels: Evolution of isotropic hardening and kinematic hardening.”
666 *Materials Science Forum*, 584-586 PA, 605–609.
- 667 Byrd, R. H., Hribar, M. E., and Nocedal, J. (1999). “An Interior Point Algorithm for Large-
668 Scale Nonlinear Programming.” *SIAM Journal on Optimization*, 9(4), 877–900.
- 669 CEN (2001). “EN 10002-1:2001 - Metallic materials Tensile testing Part 1: method of test
670 at ambient temperature.” European Commission for Standardization (CEN).
- 671 CEN (2005). “EN 1998-3: Eurocode 8: Design of structures for earthquake resistance - Part

672 3: Assesment and retrofitting of buildings.” European Comission for Standardization
673 (CEN).

674 CEN (2005). “Hot rolled products of structural steels - Part 2: Technical delivery conditions
675 for non-alloy structural steels.” *NF EN 10025-2*, European Comission for Standardization
676 (CEN), Brussels, Belgium.

677 CEN (2019a). “Hot rolled products of structural steels - Part 3: Technical delivery conditions
678 for normalized/normalized rolled weldable fine grain structural steels.” *NF EN 10025-3*,
679 European Comission for Standardization (CEN), Brussels, Belgium.

680 CEN (2019b). “Hot rolled products of structural steels - Part 6: Technical delivery conditions
681 for flat products of high yield strength structural steels in the quenched and tempered
682 condition.” *NF EN 10025-6*, European Comission for Standardization (CEN), Brussels,
683 Belgium.

684 Chaboche, J., Dang Van, K., and Codier, G. (1979). “Modelization of the Strain Memory
685 Effect on the Cyclic Hardening of 316 Stainless Steel.” *Structural mechanics in reactor
686 technology - SMiRT 5*, L11.

687 Cooke, R. J. and Kanvinde, A. M. (2015). “Constitutive parameter calibration for structural
688 steel: Non-uniqueness and loss of accuracy.” *Journal of Constructional Steel Research*,
689 114, 394–404.

690 de Castro e Sousa, A., Hartloper, A. R., and Lignos, D. G. (2019). “RESSPyLab version 1.0
691 - Python tools for structural steel research, <<https://pypi.org/project/RESSPyLab/>>.

692 de Castro e Sousa, A., Suzuki, Y., and Lignos, D. (2020). “Consistency in Solving the
693 Inverse Problem of the Voce-Chaboche Constitutive Model for Plastic Straining.” *Journal
694 of Engineering Mechanics*, 146(9), 1–16.

695 Elkady, A. and Lignos, D. G. (2018). “Improved Seismic Design and Nonlinear Modeling
696 Recommendations for Wide-Flange Steel Columns.” *Journal of Structural Engineering*,
697 144(9), 04018162.

698 Fell, B. V., Kanvinde, A. M., Deierlein, G. G., and Myers, A. T. (2009). “Experimental

699 investigation of inelastic cyclic buckling and fracture of steel braces.” *Journal of Structural*
700 *Engineering*, 135(1), 19–32.

701 Grigoriou, V. and Lignos, D. G. (2017). “Characterization of the cyclic hardening properties
702 of european steels.” *Report No. (unpublished)*, EPFL - RESSLab.

703 Hartloper, A., de Castro e Sousa, A., and Lignos, D. G. (2019a). “Sensitivity of Simu-
704 lated Steel Column Instabilities to Plasticity Model Assumptions.” *Proceedings of the*
705 *12th Canadian Conference on Earthquake Engineering*, Quebec City, QC, Canada, 8.

706 Hartloper, A. R., de Castro e Sousa, A., and Lignos, D. G. (2019b). “A Nonlinear
707 Isotropic/Kinematic Hardening Model for Materials with Discontinuous Yielding.” *Re-*
708 *port No. 271062*, Resilient Steel Structures Laboratory, École Polytechnique Fédérale de
709 Lausanne (EPFL), Lausanne, Switzerland.

710 Hartloper, A. R., de Castro e Sousa, A., and Lignos, D. G. (2020). “Constitutive Modeling
711 of Structural Steels: A Nonlinear Isotropic/Kinematic Hardening Material Model and its
712 Calibration.” *Journal of Structural Engineering*, (under review).

713 Lubliner, J. (2008). *Plasticity Theory*. Dover Publications, Mineola, N.Y., U.S.A.

714 Suzuki, T., Yoshida, Y., Shimura, Y., Suzuki, Y., Kubota, S., and Nagata, M. (2008).
715 “Development of Building Structural Steel with High Yield Ratio and High Yield Point
716 Leading to Innovative Steel Structural System.” *Report No. 97*, Nippon Steel, Tokyo,
717 Japan.

718 Suzuki, Y. (2018). “Earthquake-induced collapse of steel moment resisting frames with con-
719 ventional and high performance steel columns.” Ph.D. thesis, McGill University, Montreal,
720 QC, Canada.

721 Virtanen, P., Gommers, R., Oliphant, T. E., Haberland, M., Reddy, T., Cournapeau, D.,
722 Burovski, E., Peterson, P., Weckesser, W., Bright, J., van der Walt, S. J., Brett, M.,
723 Wilson, J., Millman, K. J., Mayorov, N., Nelson, A. R. J., Jones, E., Kern, R., Larson,
724 E., Carey, C. J., Polat, Í., Feng, Y., Moore, E. W., VanderPlas, J., Laxalde, D., Perktold,
725 J., Cimrman, R., Henriksen, I., Quintero, E. A., Harris, C. R., Archibald, A. M., Ribeiro,

726 A. H., Pedregosa, F., van Mulbregt, P., and SciPy 1.0 Contributors (2020). “SciPy 1.0:
727 Fundamental Algorithms for Scientific Computing in Python.” *Nature Methods*, 17, 261–
728 272.

729 Voce, E. (1948). “The relationship between stress and strain for homogenous deformation.”
730 *Journal of Institute of Metals*, 74, 537–562.

NOTATION

The following symbols are used in this paper:

- a = isotropic saturation rate parameter associated with D_∞ ;
- b = isotropic saturation rate parameter associated with Q_∞ ;
- C_k = kinematic stress parameter for the k -th backstress;
- c = constant in hardening metric constraint;
- D_∞ = isotropic differential stress at saturation for discontinuous yielding;
- E = Young's modulus;
- E_{total} = total normalized strain energy in stress-strain data;
- $f(\mathbf{x})$ = objective function in minimization problem with vector variable \mathbf{x} ;
- g = constraint on hardening metric;
- Q_∞ = isotropic differential stress at saturation;
- γ_k = kinematic saturation rate parameter for the k -th backstress;
- ε = uniaxial true strain;
- ε^* = error-metric strain;
- $\dot{\varepsilon}_{eq}^p$ = equivalent plastic strain;
- ξ_2 = consistency distance metric;
- ρ = constitutive model hardening metric;
- σ = uniaxial true stress;
- $\sigma_{y,0}$ = initial yield stress;
- φ = error function;
- $\bar{\varphi}$ = normalized error function;
- ϕ^{VM} = Von Mises yield criterion;
- $\nabla_{\mathbf{x}}f(\mathbf{x})$ = gradient of function f with respect to vector variable \mathbf{x} ;
- $\nabla_{\mathbf{x}\mathbf{x}}f(\mathbf{x})$ = hessian of function f with respect to vector variable \mathbf{x} ;

733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748

List of Tables

1	Overview of the Root material database.	33
2	Voce-Chaboche material model parameters with one and two backstresses obtained using the full data sets.	34
3	Voce-Chaboche hardening metrics with one and two backstresses obtained using the full data sets.	35
4	Voce-Chaboche tensile test only calibration with one and two backstresses using the NITRO algorithm and SP1 for mild steels.	36
5	Results from the validation of the tensile test only calibration methodology with one and two backstresses, using the VC model for mild steels.	37
6	Updated Voce-Chaboche full data set calibration with one and two back- stresses for mild steels.	38
7	Metrics for the Updated Voce-Chaboche full data set calibration with one and two backstresses.	39
8	Results from the validation of the tensile test only calibration methodology with two backstresses for the UVC model.	40

TABLE 1: Overview of the Root material database.

ID	Material	Source	Tested By ^a	Load Protocols	$f_{y,n}$ ^b [MPa]	$f_{u,n}$ ^c [MPa]
1	S355J2+N	50 mm plate	1	1–10	355	470–630
2	S355J2+N	25 mm plate	1	1–10	355	470–630
3	S355J2	HEB500 flange	3	1,2,5,6,9	355	470–630
4	S355J2	HEB500 web	3	1,1–3,5–9	355	470–630
5	S460NL	25 mm plate	1	1,3–7,9,10	440	540–720
6	S690QL	25 mm plate	1	1,3–7,9,10	690	770–940
7	A992 Gr.50	W14X82 web	2	1,2,5,6,9	345	450 ^d
8	A992 Gr.50	W14X82 flange	2	1,2,5,6,9	345	450 ^d
9	A500 Gr.B	HSS305X16	2	1,2,5,6,9	315	400 ^d
10	BCP325	22 mm plate	2	1,2,5,6,9	325	490–610
11	BCR295	HSS350X22	2	1,2,5,6,9	295	400–550
12	HYP400	27 mm plate	2	1,2,5,6,9	400	490–640

^a 1: Grigoriou and Lignos (2017), 2: Suzuki (2018), 3: Hartloper et al. (2020).

^b: nominal yield stress, ^c: nominal tensile stress, ^d: minimum value.

TABLE 2: Voce-Chaboche material model parameters with one and two backstresses obtained using the full data sets.

ID	$\bar{\varphi}$ [%]	E [GPa]	$\sigma_{y,0}$ [MPa]	Q_∞	b [MPa]	C_1	γ_1	C_2 [MPa]	γ_2
1	8.21	177.54	296.62	123.26	7.17	6501.83	27.90	-	-
	6.31	184.98	270.96	107.22	5.97	14327.30	115.12	1771.06	7.56
2	8.49	182.11	285.67	125.79	11.80	6801.09	29.78	-	-
	6.57	191.52	265.29	104.45	11.63	12997.99	99.51	1560.41	7.35
3	6.02	186.13	272.57	132.55	9.10	3738.39	28.95	-	-
	4.63	191.85	245.50	119.77	8.67	14019.86	205.39	1247.05	4.45
4	7.38	193.82	275.65	104.97	14.29	8437.43	62.41	-	-
	6.86	200.39	265.46	103.29	11.18	11996.11	104.79	627.33	1.14
5	8.02	172.85	376.44	106.09	11.08	6857.43	23.44	-	-
	6.18	185.86	358.93	68.04	10.02	14202.40	104.22	2259.11	8.01
6	7.56	184.88	607.47	0.50	0.58	15648.17	60.98	-	-
	7.29	184.70	602.64	0.48	0.55	16107.81	78.81	937.51	6.32
7	7.78	194.55	328.83	101.76	16.73	5159.90	38.80	-	-
	6.23	207.32	299.86	88.30	13.96	14878.98	166.00	1424.51	8.57
8	7.69	178.64	324.68	127.20	11.89	3943.98	30.59	-	-
	5.75	189.95	292.75	110.54	9.71	15142.58	170.54	1117.53	5.86
9	7.72	180.57	321.68	36.64	0.94	3995.53	24.59	-	-
	5.43	191.39	286.87	24.08	0.84	11934.29	160.01	1585.62	7.71
10	6.67	173.87	325.05	103.34	8.75	5945.26	32.19	-	-
	4.88	177.92	306.09	94.16	5.81	11613.11	122.00	1744.03	8.29
11	7.75	173.46	355.97	0.56	0.60	5135.19	44.95	-	-
	6.24	177.88	329.13	0.57	0.61	12773.06	196.86	1746.22	11.96
12	7.02	187.13	401.29	44.20	9.64	5758.71	38.19	-	-
	5.22	189.35	376.22	29.12	6.41	13711.66	139.70	1147.15	4.59

TABLE 3: Voce-Chaboche hardening metrics with one and two backstresses obtained using the full data sets.

ID	One Backstress			Two Backstresses			
	ρ_{yield}^{sat}	ρ_{iso}^{sat}	$\rho_{\gamma_1 b}$	ρ_{yield}^{sat}	ρ_{iso}^{sat}	$\rho_{\gamma_1 b}$	$\rho_{\gamma_1 \gamma_2}$
1	2.20	0.35	3.89	2.72	0.23	19.27	15.22
2	2.24	0.36	2.52	2.69	0.23	8.56	13.55
3	1.96	0.51	3.18	2.91	0.26	23.70	46.13
4	1.87	0.44	4.37	3.90	0.13	9.37	92.16
7	1.71	0.43	2.32	2.15	0.26	11.89	19.37
8	1.79	0.50	2.57	2.33	0.28	17.56	29.12
10	1.89	0.36	3.68	2.31	0.24	21.00	14.72
Avg.	1.95	0.42	3.22	2.72	0.23	15.91	32.90
StD.	0.19	0.06	0.73	0.54	0.05	5.52	26.46
CoV.[%]	9.49	14.76	22.61	20.05	19.44	34.72	80.44

TABLE 4: Voce-Chaboche tensile test only calibration with one and two backstresses using the NITRO algorithm and SP1 for mild steels.

ID	$\bar{\varphi}$ [%]	ξ_2 [%]	E [GPa]	$\sigma_{y,0}$ [MPa]	Q_∞ [MPa]	b	C_1 [MPa]	γ_1	C_2 [MPa]	γ_2
1	12.4	13.5	206.38	298.28	201.10	7.10	4692.07	23.07	-	-
	14.31	37.0	199.38	308.79	60.68	1.79	5183.37	15.21	0.01	1.17
2	12.4	14.4	200.40	299.99	193.89	8.15	4861.80	25.00	-	-
	13.65	44.0	200.87	311.37	82.02	1.96	5380.87	16.67	0.04	1.28
3	9.5	10.9	220.12	300.02	157.51	3.07	2018.60	6.90	-	-
	18.05	20.9	200.00	289.05	55.60	0.41	3081.98	9.85	0.16	0.76
4	11.5	7.5	204.86	290.20	118.20	6.23	3078.12	14.02	-	-
	19.01	37.5	200.00	291.16	77.46	1.09	3666.21	12.24	0.69	0.94
7	8.68	4.6	200.95	356.51	104.04	6.90	2999.91	15.53	-	-
	14.97	7.4	200.00	357.10	60.84	0.54	3619.24	13.25	0.91	1.02
8	10.1	9.3	200.15	339.64	114.86	5.61	2690.49	12.61	-	-
	19.45	13.0	200.00	331.53	78.09	0.51	3656.42	12.51	0.81	0.96
10	10.5	12.1	212.62	353.76	114.24	6.21	2965.99	13.98	-	-
	14.30	10.2	200.00	352.67	81.40	0.51	3662.68	12.33	1.33	0.95

TABLE 5: Results from the validation of the tensile test only calibration methodology with one and two backstresses, using the VC model for mild steels.

Calibration Data Set	Material	Load Protocols	Model	IPE300 flange $\bar{\varphi}, [\%]$	IPE300 web $\bar{\varphi}, [\%]$
Validation	S355J2 (IPE300 flange)	Tensile only	VC, 1bk	10.53	-
Validation	S355J2 (IPE300 web)	Tensile only	VC, 1bk	-	11.82
Validation	S355J2 (IPE300 flange)	Tensile only	VC, 2bk	18.31	-
Validation	S355J2 (IPE300 web)	Tensile only	VC, 2bk	-	19.92
Validation	S355J2 (IPE300 flange)	All available	VC, 2bk	5.46	6.94
Validation	S355J2 (IPE300 web)	All available	VC, 2bk	6.03	6.26
Root	S355J2+N (50 mm plate)	All available	VC, 2bk	11.25	11.86
Root	S355J2+N (25 mm plate)	All available	VC, 2bk	10.71	11.50
Root	S355J2 (HEB500 flange)	All available	VC, 2bk	16.91	18.36
Root	S355J2 (HEB500 web)	All available	VC, 2bk	11.99	13.64

TABLE 6: Updated Voce-Chaboche full data set calibration with one and two backstresses for mild steels.

ID	$\bar{\varphi}$ [%]	E [GPa]	$\sigma_{y,0}$ [MPa]	Q_∞ [MPa]	b	D_∞ [MPa]	a	C_1 [MPa]	γ_1	C_2 [MPa]	γ_2
1	8.18	177.51	327.15	128.25	7.86	37.28	203.33	6572.34	28.32	-	-
	6.33	197.41	338.8	134.34	14.71	133.75	229.25	26242.00	199.04	2445.30	11.66
2	8.43	182.96	330.42	146.77	13.72	67.20	133.57	6962.12	30.65	-	-
	6.53	185.97	332.18	120.48	8.14	93.15	261.75	21102.00	173.60	2300.60	10.42
3	5.81	186.17	304.96	153.96	12.13	59.91	98.68	4044.49	32.44	-	-
	3.96	192.13	315.04	138.01	11.36	96.16	223.66	18587.84	257.31	1351.98	6.52
4	7.00	193.91	341.22	454.07	34.70	419.75	56.92	8907.58	66.38	-	-
	6.61	199.68	334.94	139.32	14.07	120.33	274.73	28528.03	315.17	2569.46	24.68
7	7.43	194.62	376.35	623.86	38.46	574.87	51.64	5690.09	44.29	-	-
	5.21	210.72	378.83	122.63	19.74	143.49	248.14	31638.00	277.32	1548.60	9.04
8	7.53	178.75	360.64	195.99	20.52	113.60	74.36	4425.80	36.83	-	-
	4.76	191.02	373.72	141.47	15.20	135.95	211.16	25621.00	235.12	942.18	3.16
10	6.51	173.95	360.85	123.00	12.50	60.74	127.27	6193.34	34.44	-	-
	3.89	178.61	368.03	112.25	10.78	105.95	221.92	20104.00	200.43	2203.00	11.76

TABLE 7: Metrics for the Updated Voce-Chaboche full data set calibration with one and two backstresses.

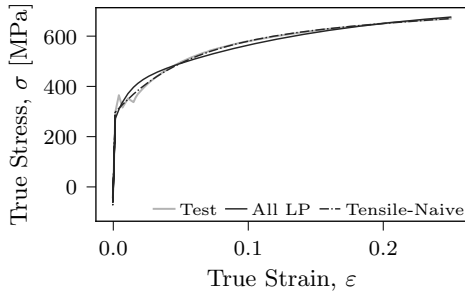
ID	One Backstress				Two Backstresses				
	ρ_{yield}^{sat}	ρ_{iso}^{sat}	$\rho_{\gamma_1 b}^{sat}$	ρ_D^{sat}	ρ_{yield}^{sat}	ρ_{iso}^{sat}	$\rho_{\gamma_1 b}^{sat}$	ρ_D^{sat}	ρ_{γ_1/γ_2}
1	1.99	0.36	3.60	0.10	2.11	0.26	21.33	0.20	16.67
2	1.93	0.39	2.23	0.18	2.01	0.28	13.53	0.28	17.08
3	1.72	0.55	2.68	0.22	2.02	0.33	22.65	0.23	39.49
4	1.49	0.77	1.91	0.71	1.64	0.42	22.39	0.36	12.77
7	1.47	0.83	1.15	0.76	1.70	0.30	14.05	0.35	30.66
8	1.56	0.62	1.80	0.36	2.10	0.26	15.47	0.25	74.44
10	1.67	0.41	2.76	0.20	1.80	0.28	18.60	0.26	17.05
Avg.	1.69	0.56	2.30	0.36	1.91	0.30	18.29	0.28	29.74
StD.	0.19	0.17	0.73	0.25	0.18	0.05	3.66	0.06	20.26
CoV. [%]	11.28	31.00	31.87	68.39	9.43	17.20	20.01	20.04	68.14

TABLE 8: Results from the validation of the tensile test only calibration methodology with two backstresses for the UVC model.

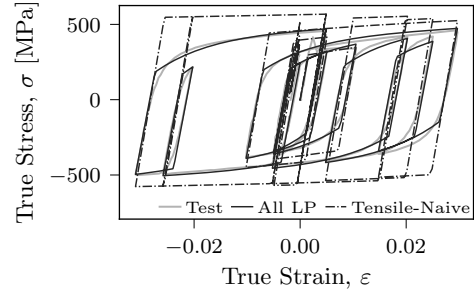
Calibration Data Set	Material	Load Protocols	Model	IPE300 flange $\bar{\varphi},[\%]$	IPE300 web $\bar{\varphi},[\%]$
Validation	S355J2 (IPE300 flange)	Tensile only	UVC, 2bk	19.80	-
Validation	S355J2 (IPE300 web)	Tensile only	UVC, 2bk	-	24.69
Validation	S355J2 (IPE300 flange)	All available	UVC, 2bk	4.94	6.35
Validation	S355J2 (IPE300 web)	All available	UVC, 2bk	5.59	5.42
Root	S355J2+N (50 mm plate)	All available	UVC, 2bk	10.86	11.31
Root	S355J2+N (25 mm plate)	All available	UVC, 2bk	10.19	10.69
Root	S355J2 (HEB500 flange)	All available	UVC, 2bk	16.42	17.86
Root	S355J2 (HEB500 web)	All available	UVC, 2bk	10.40	12.19

List of Figures

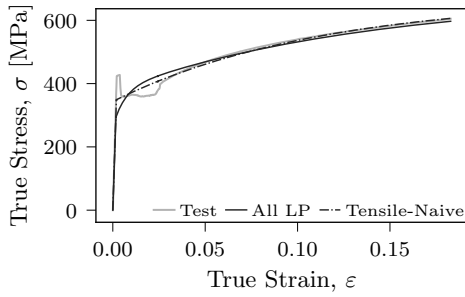
750	1	Comparison of stress-strain using the S355J2+N 50 mm plate and A992 Gr. 50	
751		W14X82 flange data sets: test data (Test); VC model 2 backstresses, all load	
752		protocols used in calibration (All LP); and naive calibration using only a	
753		tensile test (Tensile-Naive).	42
754	2	Workflow of proposed methodology applied to structural steel materials. . .	43
755	3	Relevant model metrics for $\sigma_y \approx 355$ MPa materials based on parameters	
756		calibrated with all available load protocols.	44
757	3	Relevant model metrics for $\sigma_y \approx 355$ MPa materials based on parameters	
758		calibrated with all available load protocols. (Cont.)	45
759	4	Evolution of the constrained optimization solution based on the different start-	
760		ing points using the S355J2+N 50 mm data set.	46
761	5	Comparison of stress-strain using the S355J2 IPE300 flange data set: test data	
762		(Test), VC 2Back Root dataset calibration (HEB500-Flange), and proposed	
763		tensile-only calibration with the VC model (Tensile).	47
764	6	Comparison of stress-strain using the S355J2 IPE300 flange data set: test data	
765		(Test), UVC 2Back Root dataset calibration (HEB500-Flange), and proposed	
766		tensile-only calibration with the UVC model (Tensile).	48



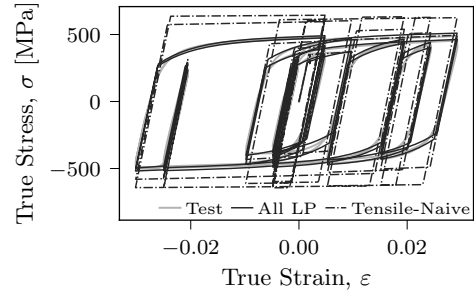
(a) S355J2+N, tensile (Load Protocol 1)



(b) S355J2+N, variable amplitude (Load Protocol 9)



(c) A992Gr.50, tensile (Load Protocol 1)



(d) A992Gr.50, variable amplitude (Load Protocol 9)

FIG. 1: Comparison of stress-strain using the S355J2+N 50 mm plate and A992 Gr. 50 W14X82 flange data sets: test data (Test); VC model 2 backstresses, all load protocols used in calibration (All LP); and naive calibration using only a tensile test (Tensile-Naive).

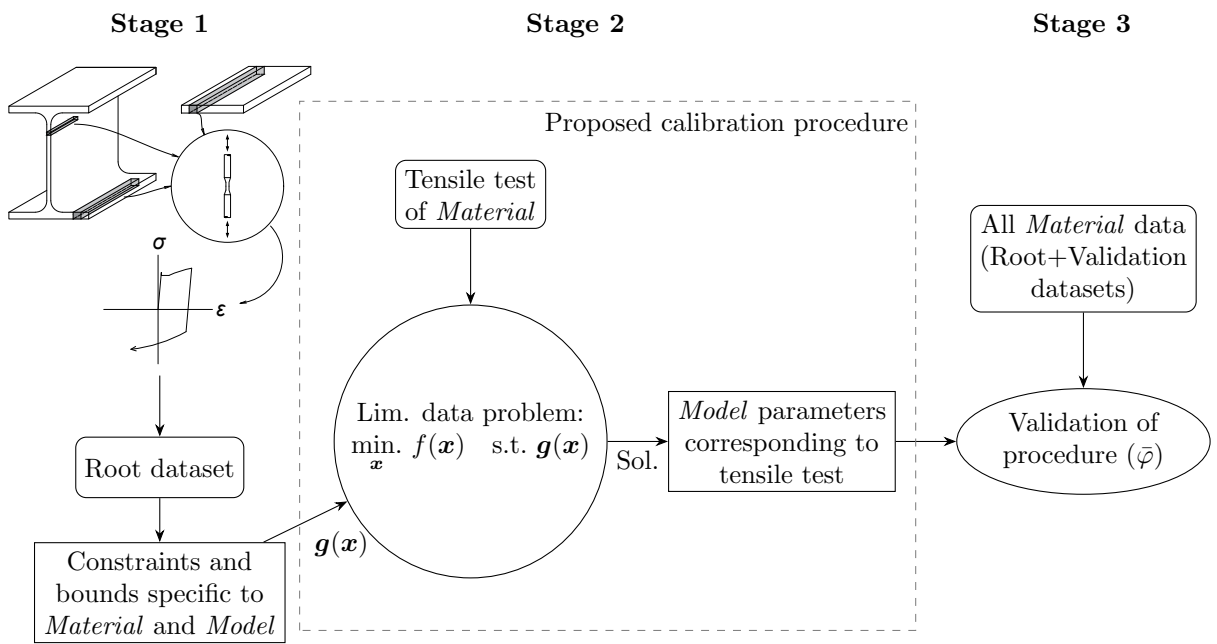
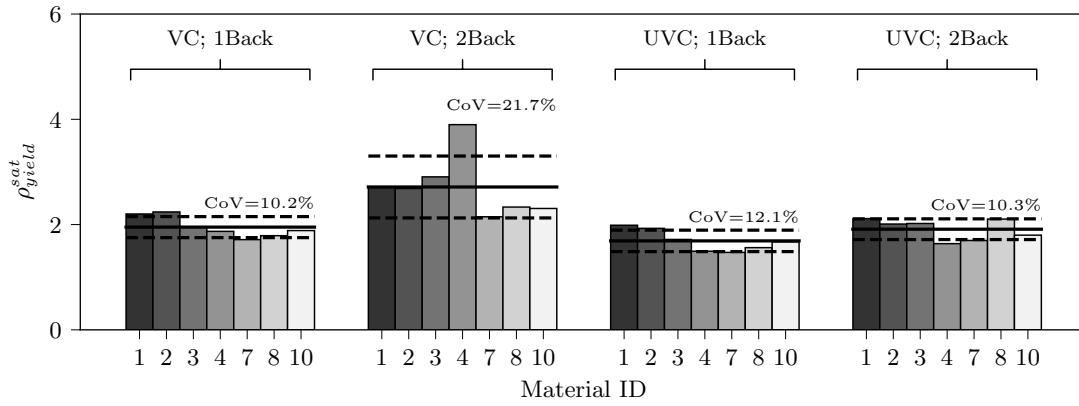
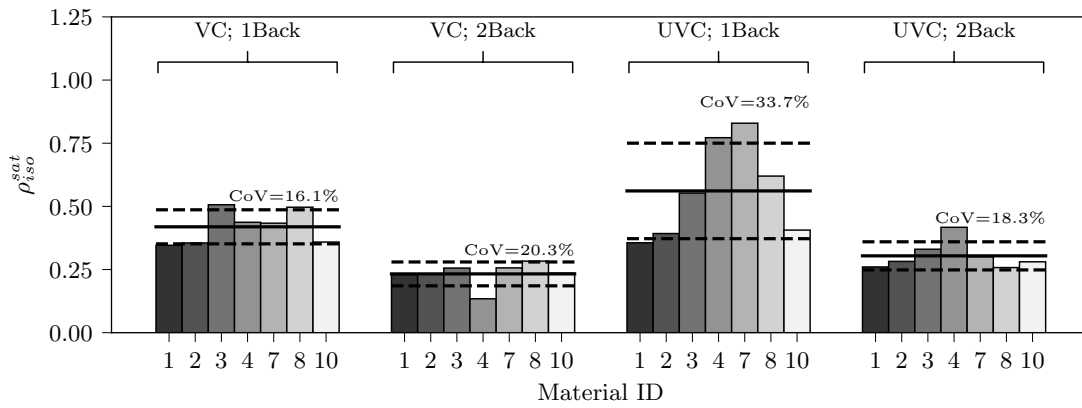


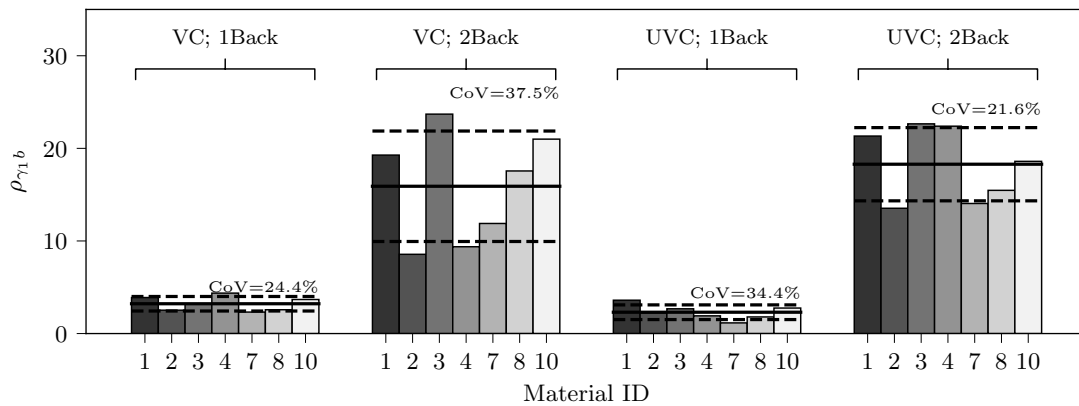
FIG. 2: Workflow of proposed methodology applied to structural steel materials.



(a)

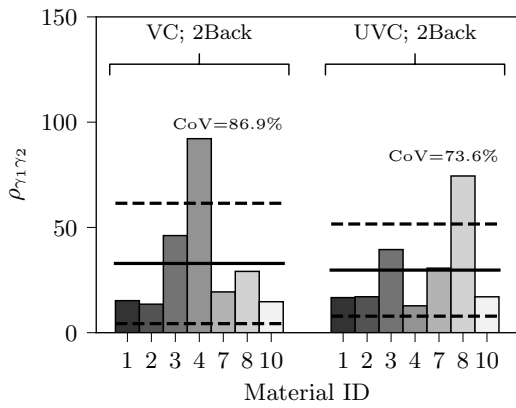


(b)

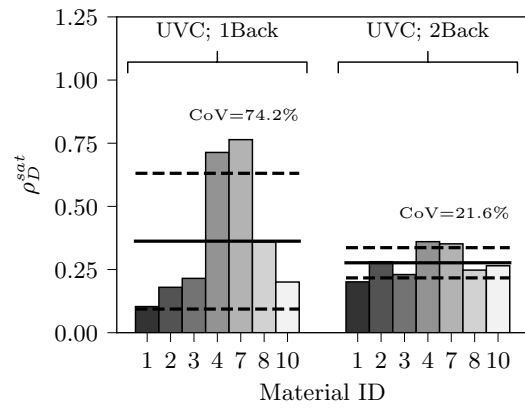


(c)

FIG. 3: Relevant model metrics for $\sigma_y \approx 355$ MPa materials based on parameters calibrated with all available load protocols.

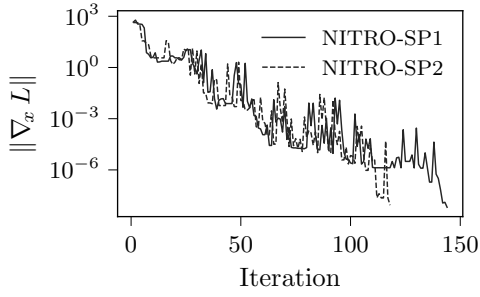


(d) (cont.)

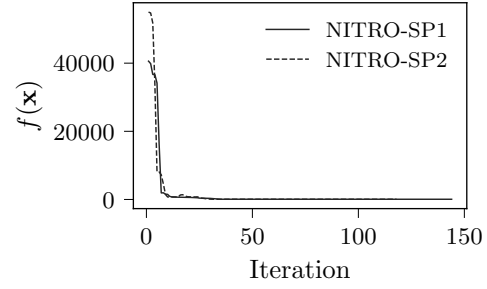


(e)

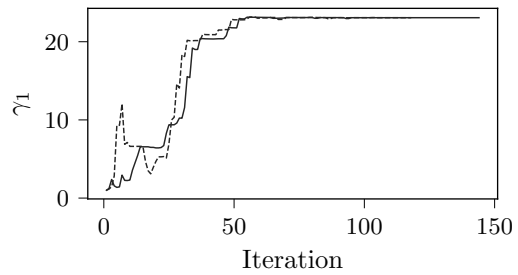
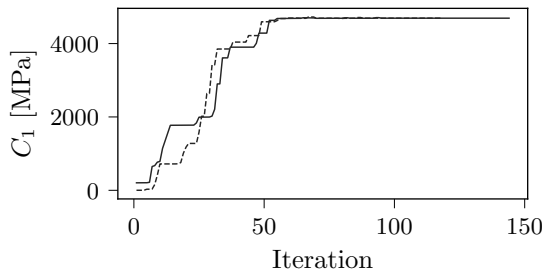
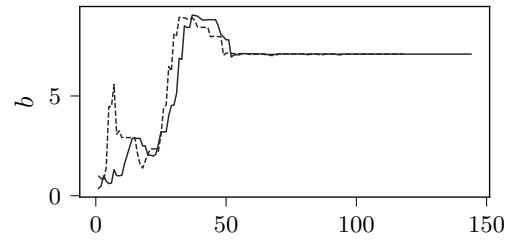
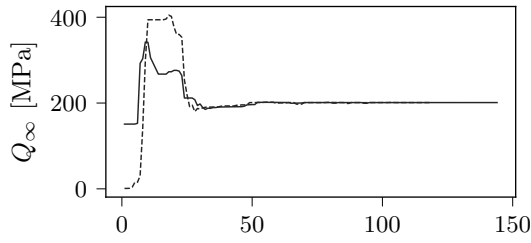
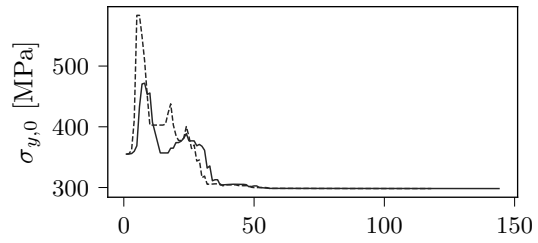
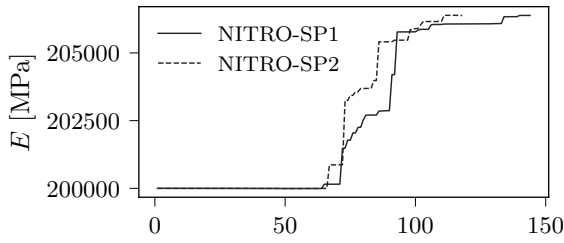
FIG. 3: Relevant model metrics for $\sigma_y \approx 355$ MPa materials based on parameters calibrated with all available load protocols. (Cont.)



(a) Convergence criteria

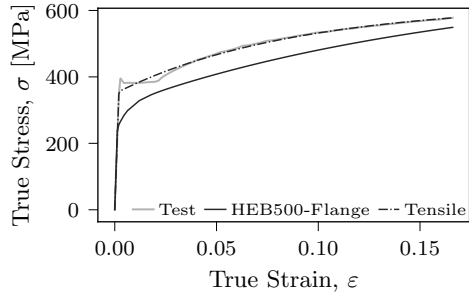


(b) Objective function

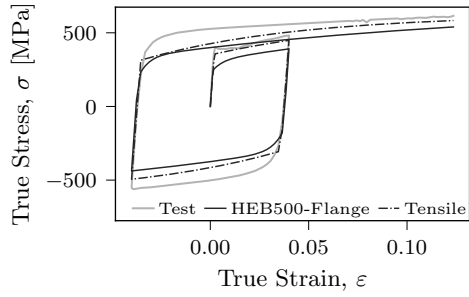


(c) Parameter values

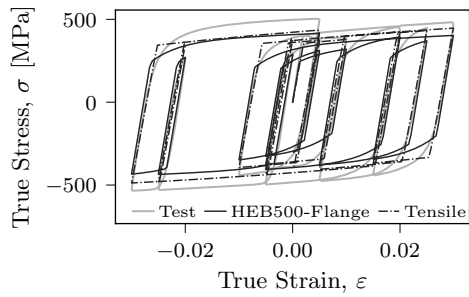
FIG. 4: Evolution of the constrained optimization solution based on the different starting points using the S355J2+N 50 mm data set.



(a) VC, Load Protocol 1

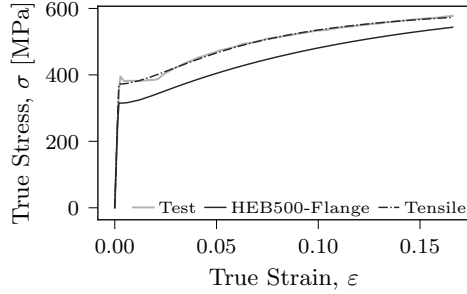


(b) VC, Load Protocol 3

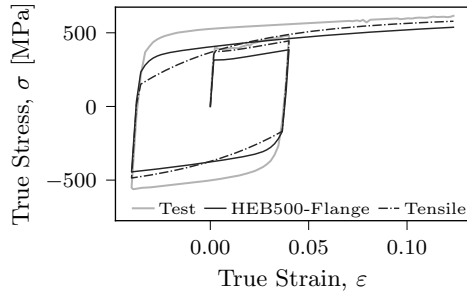


(c) VC, Load Protocol 9

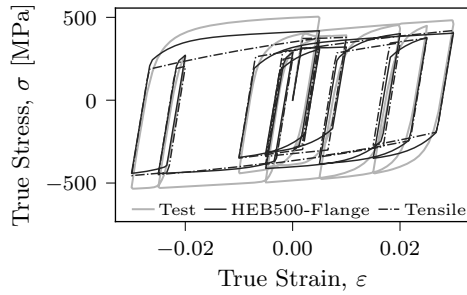
FIG. 5: Comparison of stress-strain using the S355J2 IPE300 flange data set: test data (Test), VC 2Back Root dataset calibration (HEB500-Flange), and proposed tensile-only calibration with the VC model (Tensile).



(a) UVC, Load Protocol 1



(b) UVC, Load Protocol 3



(c) UVC, Load Protocol 9

FIG. 6: Comparison of stress-strain using the S355J2 IPE300 flange data set: test data (Test), UVC 2Back Root dataset calibration (HEB500-Flange), and proposed tensile-only calibration with the UVC model (Tensile).