



SYMPOSIUM DUBROVNIK 2007

## Concrete Structures - - STIMULATORS OF DEVELOPMENT

Dubrovnik | Croatia | 20-23 May 2007

Topic 3: New materials for concrete structures

# TENSILE BEHAVIOUR OF REINFORCED ULTRA-HIGH PERFORMANCE FIBER REINFORCED CONCRETE ELEMENTS

Dario Redaelli\* & Aurelio Muttoni\*\*

\*PhD Candidate, Ecole Polytechnique Fédérale de Lausanne (EPFL)  
Switzerland

\*\*Professor, Ecole Polytechnique Fédérale de Lausanne (EPFL)  
Switzerland

**Key words:** UHPFC, reinforcement, structural element, tension, tests

**Abstract:** *Ultra-high performance fiber reinforced concrete (UHPFC) is a new class of materials that combine a very strong and dense cementitious matrix with a high fiber content. UHPFC has a very high compressive strength (in the order of 200 MPa) and a relatively large tensile strength (in the order of 10 MPa), along with a strain hardening behavior in tension that ensures that crack openings remain small and provides an enhanced material ductility compared to ordinary concrete. In spite of these advantageous properties, large unreinforced UHPFC members in tension exhibit a brittle behavior at the ultimate limit state, characterized by crack localization and a sudden failure with insufficient structural ductility. To achieve a higher ductility, one possible solution is to add conventional or high-strength reinforcement to the section.*

*The paper presents and discusses some results of an extensive study at the Ecole Polytechnique Fédérale de Lausanne. Tests have been carried out on a series of large-scale unreinforced and reinforced UHPFC specimens, investigating the effect of the amount and type of reinforcement. The specific response of reinforced UHPFC members at cracking is analyzed.*

## 1. INTRODUCTION

Ultra High Performance Fibre Concretes (UHPFC) are a class of cementitious composites made of a strong and compact powder-based matrix and of steel or synthetic fibres as a distributed three dimensional reinforcement. Fibres provide ductility both in tension and in compression and, for a sufficiently large amount (more than 1.5% in volume of fibres) they lead to a strain hardening behaviour in tension.

A test series has been carried out at the Ecole Polytechnique Fédérale de Lausanne (Switzerland) on UHPFC members in tension. Unreinforced members as well as members reinforced with steel rebars were tested. The aim of the present research is to define a simple analytical model to describe the behaviour of reinforced UHPFC ties. To this aim, the analytical equations of cracking are first solved numerically to investigate the role of some physical parameters. This paper introduces the numerical model and presents some considerations about the influence of the stress–crack opening relationship of UHPFC on cracking.

## 2. EXPERIMENTAL BEHAVIOUR

### 2.1 Materials

The UHPFC used for this experimental series is the BSI-Ceracem<sup>®</sup>, by Sika-Eiffage, with an average compressive strength of 190 MPa and approximately 2.4% in volume of 20 mm long steel fibres with a 0.16 mm diameter<sup>1</sup>. The tensile behaviour of BSI-Ceracem<sup>®</sup> is presented in figure 1a, where  $E_c = 60$  GPa and the unloading stiffness at peak  $E_{cu}$  is approximately 5 GPa. The specific behavior of UHPFC in tension is characterized by the strain hardening branch after matrix cracking and by the stress-crack opening relationship.

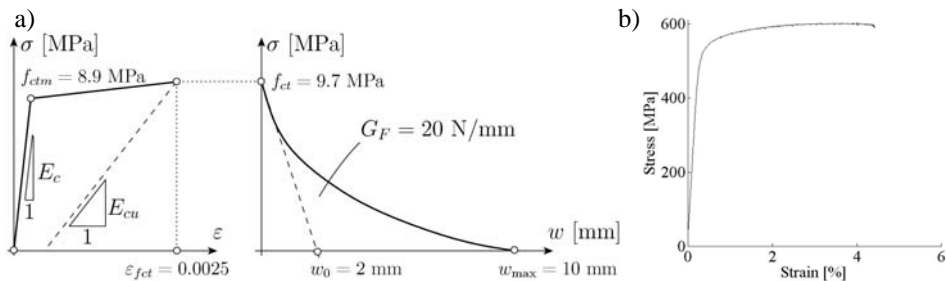


Figure 1: Tensile behaviour of (a) BSI-Ceracem<sup>® 3</sup> and (b) cold-worked steel

### 2.2 Reinforced UHPFC ties

A test series on reinforced UHPFC ties with varying reinforcement grade and ratio was carried out at the EPFL<sup>2, 3</sup>. The specimens have a total length of 1.60 meters and a  $160 \times 160$  mm<sup>2</sup> cross section (figure 2a). They are instrumented with 4 LVDT, placed on different sides on a 1.00 meter measurement length. Two grades of steel were used: hot-rolled steel with a well-defined yielding plateau and cold-worked steel. In this paper, only

specimens with cold-worked steel are investigated. The measured stress-strain diagram for cold-worked steel is given in figure 1b.

Figure 2a shows the measured nominal stress-strain curves, together with the response of the bare bars. The strain is the average measure of the 4 LVDT. The following observation can be made<sup>3, 2</sup>: the interaction between ordinary reinforcement and fibres leads to a very good behaviour in service, with very closely-spaced and thin cracks leading to an important tension stiffening effect. On the contrary, structural response at ultimate is negatively affected: as it can be seen in figure 2a, the maximal strength is larger than the strength of the steel bars alone, but it is reached for a small deformation, closely corresponding to the onset of yielding in steel. Global softening follows the peak, making the structural behaviour potentially brittle and very sensitive to size effects.

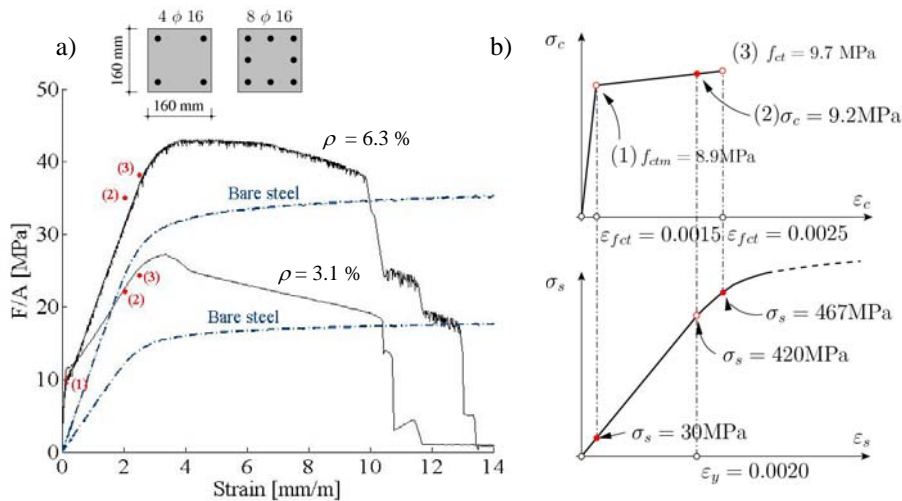


Figure 2: (a) Test results (strains measured on a 1.00 m measurement length) compared with the behaviour of bare steel and with points calculated according to values given in (b)

### 3. ANALYSIS OF UHPFC TIES

This research aims to provide a simple analytical model to describe the different phases of the structural response of reinforced UHPFC ties. The model should predict the force and the strain at peak, as well as the residual post peak strength and the ultimate strain. In this paper, attention is focused on the pre-peak zone. For strains up to  $\epsilon_{fct}$ , stresses increase in concrete and in reinforcement: as a consequence, the structural behaviour can be modeled assuming an homogeneous uncracked behaviour. Figure 2a shows the comparison between the experimental curves and some characteristic points, calculated with the homogeneous section hypothesis, where the accuracy of this hypothesis can be seen. For  $\epsilon > \epsilon_{fct}$ , however, the force in the tie continue to increase, even if UHPFC already experiences crack opening with softening behaviour. A cracked tie model is thus necessary to describe the last portion of the pre-peak structural response.

### 3. Crack modelling

Cracking behaviour in reinforced concrete depends on the mechanical behaviour of the materials and on the bond law:

$$\sigma_s(\varepsilon_s) \quad \sigma_c(\varepsilon_c) \quad \tau(s) \quad (1)$$

where  $\tau(s)$  and  $s = s(x)$  are the bond stress and the local relative displacement, or slip, between steel and concrete. Cracking can be analytically described by a system of three first order differential equations (eq. 2 a-c), representing the equilibrium of a portion  $dx$  of reinforcing bar (a), the steel to concrete stress transfer equilibrium (b) and the relationship between the strains in the two materials and the slip  $s(x)$  (c).

$$\begin{aligned} \text{(a)} \quad & \frac{d\sigma_s(x)}{dx} = -\frac{4}{\phi_s} \cdot \tau[s(x)]; & \text{(b)} \quad & \frac{d\sigma_s(x)}{dx} \cdot A_s = -\frac{d\sigma_c(x)}{dx} \cdot A_c; \\ \text{(c)} \quad & s(x) = \int_0^x [\varepsilon_s(\xi) - \varepsilon_c(\xi)] d\xi \end{aligned} \quad (2)$$

For linear elastic materials, the three equations give the unique second order differential equation of bond<sup>4</sup>:

$$\frac{d^2 s(x)}{dx^2} + \frac{(1+n \cdot \rho) \cdot 4}{\phi_s \cdot E_s} \cdot \tau[s(x)] = 0 \quad (3)$$

where  $n = E_s/E_c$ ,  $\rho = A_s/A_c$  and  $\phi_s$  is the diameter of the reinforcing rebar. If general nonlinear functions are used to express (1), the differential problem cannot be solved in a closed form for all cases. For ordinary reinforced concrete, equation (3) has been analytically solved in different ways<sup>4,5,6,7</sup>. Anyway, a closed form solution is only possible if some hypotheses are made on some of the functions assumed in (1) or, as it was recently done in<sup>7</sup>, on the cinematic equation (2c). Those models provide reliable solutions, because the hypotheses that they admit are based on many experimental evidences and on the sound knowledge about the main aspects of cracking in ordinary reinforced concrete.

For UHPFC, the cracking problem is analytically more complex: the stress in concrete at the crack location cannot be assumed equal to zero, and is a function of the crack opening  $w$  which, in its turn, depends on the stress and strain distribution in concrete and in steel along the introduction length. Moreover, knowledge about bond behaviour in UHPFC is limited, and only few tests are available<sup>3,8</sup>.

### 4. NUMERICAL RESOLUTION

To have a better insight into the effect of the different parameters involved in cracking, equation (3) can be solved in a numerical way. The solution presented in this paper is based on the so-called “shooting technique” for solving two-point boundary value problems<sup>4,9</sup>. The method allows accounting for the stress-crack opening relationship as well as for the non linearity of the materials and of the bond law. In this paper, the numerical method is used to investigate the effect of the  $\sigma(w)$  relationship and of the stiffness of steel on the first post-cracking phase. Figure 3 shows the geometry of the modelled element. The

progressive opening of a single crack in a tie is considered: the introduction length is assumed free to develop until a homogeneous section is reached; the formation of other cracks is neglected. The bond-slip law proposed by MC90 for rebars with improved bond in confined conditions is used for all the calculations.

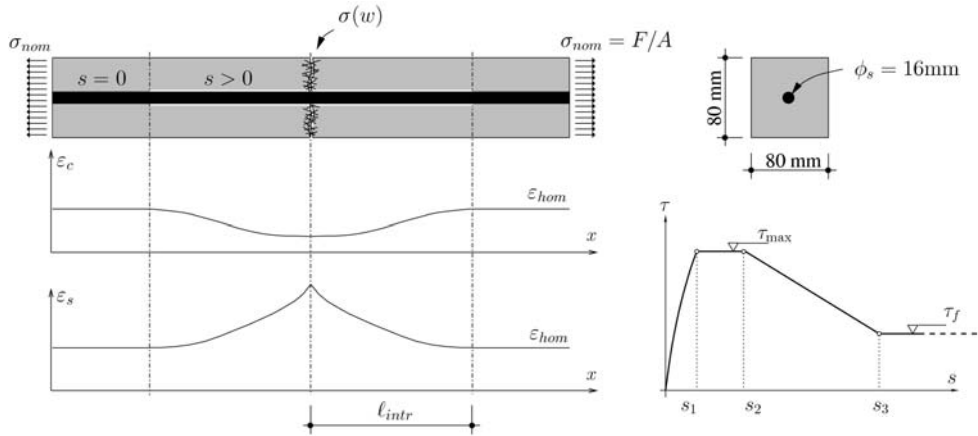


Figure 3: Modelled reinforced tie

#### 4.1 Influence of the slope of the $\sigma(w)$ relationship

The influence of the  $\sigma(w)$  relationship is firstly investigated. Ordinary concrete is compared to fibre reinforced concrete (FRC) and to UHPFC. Linear behaviour is assumed for steel, with  $E_s = 205$  GPa. The  $\sigma(w)$  relationship for ordinary concrete is modelled according to MC90, whereas for FRC values experimentally determined in <sup>10</sup> according to RILEM <sup>11</sup> are used.

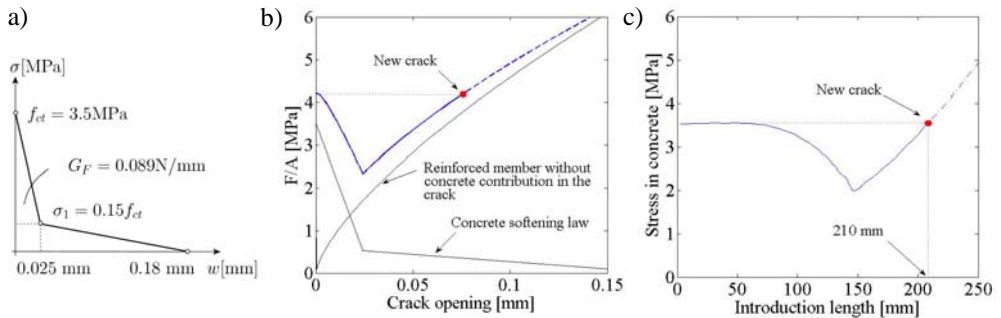


Figure 4: Numerical simulation of the specimen shown in figure 3 for ordinary concrete

Figure 4 shows the  $\sigma(w)$  relationship and the result of the model for ordinary concrete. In figure 4b, the nominal stress in the tie is plotted versus the crack opening. The behaviour of a tensile member without the contribution of concrete in the crack and the  $\sigma(w)$  relationship

are also plotted in the same graph. In figure 4c the stress in concrete at the homogeneous section is plotted versus the introduction length (see figure 3). As it can be seen in figure 4, for ordinary reinforced concrete members the nominal stress (and the stress in concrete at the homogeneous section) decreases immediately after cracking, increasing again as the reinforcement bars are activated. At the point shown in red, the stress in the concrete at the homogeneous section reaches again the cracking stress: another crack can thus create, at a distance from the first one larger than the introduction length indicated in figure 4c. The dotted portions of the curves in figure 4 do not correspond to physical points, because, in reality, the formation of other cracks changes the structural behaviour.

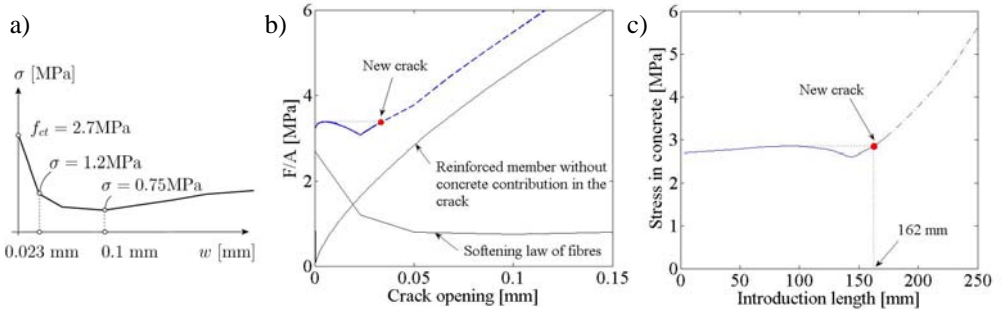


Figure 5 : Numerical simulation for fibre reinforced concrete

FRC (figure 5) shows the same tendency. However, due to the lower initial slope of the stress-crack opening curve, the stress carrying capacity of FRC in the first cracking stage is higher than for ordinary concrete. As a consequence, the nominal stress slightly increases immediately after cracking, up to a local maximum. Other cracks might develop in this phase. Then the nominal stress decreases, although at a lower rate and to a lower level than in ordinary concrete, qualitatively giving the same behaviour as in ordinary concrete.

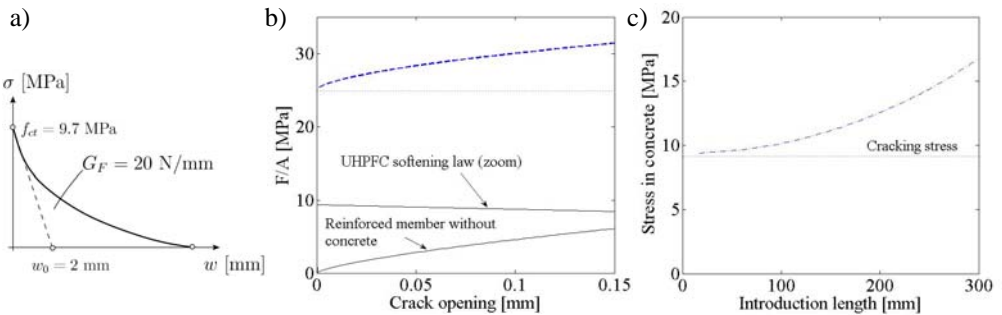


Figure 6 : Numerical simulation for UHPFC

Figure 6 shows the results obtained for UHPFC. For UHPFC, the nominal stress continuously increases after cracking, due to the very low initial slope of the  $\sigma(w)$  relationship. This fact, which is in accordance with experimental evidence (figure 2), makes

the cracking behaviour of UHPFC significantly different from the behaviour of ordinary concrete. The stress in concrete at the homogeneous section is theoretically always higher than the cracking stress, meaning that:

- the criterion ( $\sigma_{c,hom} = f_{ct}$ ) used for ordinary concrete and FRC to determine crack spacing cannot be used for UHPFC, because  $\sigma_{c,hom} > f_{ct}$  in all the first post-cracking phase;
- in a perfectly homogeneous material, the crack spacing would theoretically tend to zero;
- the determination of the crack spacing and of the number of cracks in a reinforced tie should be based on the statistical distribution of the cracking strength of the sections along the element.

As the numerical results show, the slope of the  $\sigma(w)$  relationship is a key parameter governing the cracking behaviour of reinforced UHPFC ties. To model the first cracking phases, a simplified linear  $\sigma(w)$  relationship might be used, if the initial slope of the effective  $\sigma(w)$  law is used. To model the entire cracking process, the effective non linear  $\sigma(w)$  relationship should probably be considered.

#### 4.1 Influence of the tangent stiffness of steel

In the previous calculations, the stiffness of steel was assumed constant and equal to the elastic stiffness. However, softening of the fibres starts when the bars are already yielding (figure 2) and steel has a different tangent stiffness.

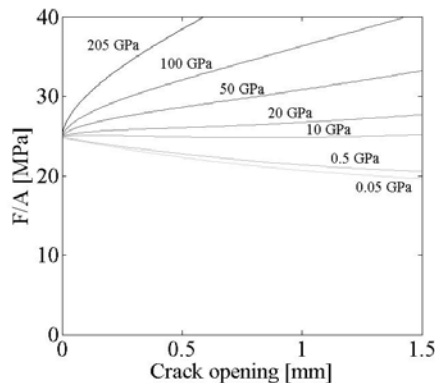


Figure 7 : Effect of the stiffness of steel at cracking

In figure 7 the same nominal stress versus crack opening diagram as in figure 6b is plotted for various values of stiffness. From figure 7 it is clear that the post-cracking response is very sensitive to the value of the stiffness of steel: only for values of the stiffness larger than a certain limit can the nominal stress increase after cracking. For cold worked steel, the tangent stiffness gradually changes from the elastic value to zero while yielding occurs: as a consequence, the effective evolution of the stiffness in steel should be considered to model the post-cracking behaviour.

## 6. CONCLUSIONS

The differential equations of the cracking behaviour of reinforced UHPFC are solved with a numerical technique to gain a better understanding of some governing physical parameters. The numerical study shows the influence of the initial slope of the stress-crack opening relationship on the structural response in the first cracking stages: in UHPFC the force in a reinforced tie continues to increase even during softening of the fibres. As a consequence, the average stress in the tie and the stress in concrete at the uncracked sections increase continuously, and the distance between cracks cannot be determined by simple mechanical consideration as for ordinary reinforced concrete ties. Moreover, the numerical study shows that neither the non linear stress-strain behaviour of reinforcing steel nor the non linear stress-crack opening relationship of UHPFC can be neglected in view of a simplified analytical approach.

## REFERENCES

- [1] Maeder, U., Lallemand-Gamboa, I., Chaignon, J. & Lombard, J.P. 2004. CERACEM a new high performance concrete: characterization and applications *International Symposium on UHPC, Kassel, Germany*: 67-76.
- [2] Redaelli, D. 2006. Testing of reinforced high performance fibre concrete members in tension. *Proceedings of the 6th Int. Ph.D. Symposium in Civil Engineering, Zurich, Switzerland*: 122-123.
- [3] Jungwirth, J. 2006. *Zum Tragverhalten von zugbeanspruchten Bauteilen aus Ultra-Hochleistungs-Faserbeton, Thèse EPFL N°3429, 157pp.* Lausanne, Switzerland.
- [4] FIB. 2000. *Bond of reinforcement in concrete, Bulletin n°10, state-of-art report. Prepared by Task Group Bond models 10, 427pp.* Lausanne, Switzerland.
- [5] Balázs, G. L. 1987. Bond Model with non-linear Bond-slip Law. *Studi e Ricerche, Politecnico di Milano, vol. 9*: 157-180. Milan, Italy.
- [6] Marti, P., Alvarez M., Kaufmann W. & Sigrist V. 1998. Tension chord model for structural concrete. *Structural Engineering International, vol. 8, n° 4*: 287-298. Switzerland.
- [7] Fernández Ruiz, M., Muttoni, A. & Gambarova, P. 2006. Analytical modelling of the pre- and post-yield behaviour of bond in reinforced concrete. *ASCE Journal of Structural Engineering* (to be published). Reston, USA.
- [8] Shionaga, R. 2006. Structural behavior of high performance fiber reinforced concrete in tension and bending, *Proceedings of the 6th Int. Ph.D. Symposium in Civil Engineering, Zurich, Switzerland*: 142-143.
- [9] Fantilli, A. P., Ferretti, D., Iori, I. & Vallini, P. 1999. Behaviour of R/C elements in bending and tension: the problem of minimum reinforcement ratio. In Alberto Carpinteri (Ed.) *Minimum reinforcement in concrete members*: 99-125.ESIS Publication 24.
- [10] Barragán, B. E., Gettu, R., Martín, M. A. & Zerbino R. L. 2003. Uniaxial tension test for steel fibre reinforced concrete – a parametric study. *Cement & Concrete Research, vol. 25*: 767-777.
- [11] RILEM. 2002. TC 162-TDF: Test and Design Methods for Steel Fibre Reinforced Concrete. *Materials and Structures, Vol. 35*: 262-278.