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Single product, finite horizon, periodic review inventory model with working capital requirements and short-term debt

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ABSTRACT

In this paper, we build on a single product, finite horizon, periodic review inventory management setting and include key financial aspects such as working capital constraints, payment delays and multiple sources of financing. We numerically solve for the optimal working capital target and the order-up-to level using an embedded Nelder and Mead optimization, and we perform sensitivity analysis on cash flows and short-term debt levels. Our numerical experiments show that when access to short-term debt is granted, the expected cash flows are indeed fairly insensitive to varying short-term debt premiums. However, when short-term debt becomes prohibitive or when downstream payment delays increase, the required working capital target inflates rapidly.

1. Introduction

During the last decade, efforts to connect the financial and operational sides of a firm have focused on reducing transaction costs by implementing automatic payments in enterprise resource planning systems or leveraging new IT platforms to enable reverse factoring. When applying these measures to a supplier–retailer–buyer setting, the buyer may benefit from lower procurement costs and extended payment terms, while the supplier can decrease both invoicing costs and accounts receivable [1]. Although these tools have contributed to the management of ever more complex supply chains, our insight into the fundamental trade-offs between operational and financial considerations in deciding working capital targets and negotiating credit lines and acceptable payment delays is, however, still not sufficiently well developed.

Even in simple settings, working capital management can prove to be difficult due to its complex cost structure and the existence of payment delays and lead times. The standard definition of working capital is inventory plus cash plus accounts receivable minus accounts payable. Each of these components has different associated costs, i.e. inventory has storage and financial costs, cash has opportunity cost, and loans have their associated interest rates. Given this complexity, how do firms deal with it in practice? When companies as diverse as CVS Caremark, a large US pharmaceutical retailer, and Deutsche Post World Net, a logistics service provider, speak of their ambitions to “improve working capital management,” they typically have only massive “reductions in working capital” in mind. Deutsche Post World Net, for example, targeted a 700 million Euro net working capital reduction in its 2007 roadmap to value statement. And indeed, there is evidence that some firms are keeping unnecessarily high levels of working capital. Ernst and Young [14] found that the top 2000 largest companies in the US and Europe have an aggregate total of up to US$ 1.1 trillion in cash unnecessarily tied up in working capital—equivalent to 7% of their sales.

Yet, reducing working capital is not a panacea, and significant costs due to operational disruptions and delayed product introductions have been recorded [19]. In the context of the 2008/2009 financial crisis, during which many companies struggled as a result of a lack of credit and insufficient working capital, this has become strikingly self-evident. Some firms were forced to accept longer payment terms from their customers, which in turn worsened their working capital position [23]. Other companies were faced with tight or unavailable bank credit. Suppliers already suffering from somewhat unfavorable payment terms compared to retailers were particularly impacted. As a result, many firms were forced to halt their operations and, in some cases, starve the whole supply chain as various business reports and news headlines highlighted [3,14,13]. The cost of such operational disruptions likely well outweighs a reduced financing cost. Therefore, improved integration of financial and operational considerations has been advocated by practitioners and academics alike.

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Nevertheless, modeling financial and operational decisions together has theoretical and implementational complexities that need to be carefully considered. Indeed, the relatively small number of academic studies that explicitly take both operational and financial decisions into account simultaneously appears to be a direct consequence of the irrelevance principle developed by Modigliani and Miller [29]. Modigliani and Miller's results call for the complete separation of the financing of a project and its operations. But how sensitive are operations and revenues to restrictions in working capital, limited access to short-term debt and unfavorable payment terms, i.e. to settings when we depart from the assumptions underlying the work of Modigliani and Miller?

Our paper is exploratory in nature and analyzes how, in a single product, finite horizon, periodic review inventory setting, a firm can consider working capital targets and the use of short-term debt jointly, taking into account a spread in interest rates and payment delays. We focus on answering the following questions:

- Are expected profit levels sensitive to working capital constraints and short-term debt access?
- What is the impact of upstream and downstream payment delays on the relation between working capital constraints, short-term debt and expected profit levels?
- Are expected profits, inventory ordering decisions, working capital targets and short-term debt levels sensitive to key product parameters such as product margin and demand volatility?

To examine these questions, our inventory management model encompasses both financial and operational decisions. We explicitly include the trade-offs between working capital constraints, access to short-term debt, interest rate premiums, payment delays and lead time. We discuss the changes in profitability for special cases and comment on the difficulty of devising an optimal inventory control policy. To overcome this difficulty, we use an inventory management simulation model with an embedded optimization algorithm to solve the problem. The remainder of this paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we describe the modeling settings and our mathematical model. In Section 4, we discuss our implementation method, as well as the managerial implications and our main observations from the numerical analysis. In Section 5, we summarize our main results and comment more generally on the relevance of this research area. All proofs are in the Appendix.

2. Literature review

To study the effects of working capital constraints on operational and financial decisions when short-term debt is included, two streams of literature are particularly relevant: capacitated inventory models and financial supply chain models.

Capacitated inventory models provide the foundation for working capital constrained inventory-control studies. Building on the seminal works by Clark and Scarf [9] and Federgruen and Zipkin [15], this body of literature includes random capacity or random yields. For example, Ciarallo et al. [8] find optimal ordering quantities for a single product with random demand and random capacity. In their study, the probability distribution of the random capacity is not derived from the dynamics of the operations or finances, but it is assumed to have a general distribution. With the same setting and assuming an order-up-to level policy, Güllü [17] shows that the stochastic process of the post-production inventory position is analogous to a queueing system. Using this analogy, the author shows how to obtain the optimal base stock level for specific cases, and derives performance measures for the inventory model that are accepted results in queuing theory. Incorporating random yields into the random capacity models, Wang and Gerchak [36] study a firm that minimizes its discounted expected costs and find that the optimal policy is of the reorder-point type. DeCroix and Arreola-Risa [12] extend the analysis to multiple products sharing a finite resource and prove that the modified base stock policy is optimal. They explicitly describe the optimal policy for the case of homogenous products and develop heuristics for the case of heterogenous products. More recently, Iida [20] considers a non-stationary periodic-review inventory model. Acknowledging that the optimal ordering policy is of the order-up-to level type, the author develops lower and upper bounds for the optimal policy, and shows the convergence between the bounds of the finite and the infinite horizons. In addition, the author provides a good review of capacitated inventory models for further reference. In summary, the established capacitated inventory models demonstrate the importance of stochastic constraints in inventory decisions and provide the mathematical tools to solve specific settings. Nonetheless, the analytical approaches to these models are not sufficient when financial constraints are considered. Specifically, working capital constraints cannot be modeled using a general distribution since its level fluctuates with the changes in cash and inventory on hand at every period. Moreover, payment delays, which affect the level of available cash and therefore the level of working capital, need to be included in the model for a complete picture of the financial dynamics. However this rich setting cannot be modeled nor solved with the tools presented in the literature mentioned above.

Previous studies that include financial constraints in order to evaluate their effect on operations are focused on specific areas such as economic order quantity (EOQ) models with trade credit, newsvendor models with debt access, and multi-period models with budget constraints. These studies describe the effect of trade credit financing in supply chain management by enriching the classic EOQ model. Haley and Higgins [18] and Goyal [16] include trade credit in the EOQ model by assuming that payments are not made immediately. They obtain the optimal policy for this setting and find that, for certain parameter conditions, the financing decisions and the inventory policy decisions remain independent. Chung [6] extends previous models by taking into account discounted cash flows. Jaggi and Aggarwal [21] further enrich the model by including deteriorating items in the analysis. Chung [7] refines Goyal [16]'s study and provides a simpler optimal ordering condition. Building on this work, Jaggi et al. [22] study the effect of trade credit from the retailer to the customer when the customer's demand depends on the length of the credit. Their results suggest that “offering such credit has a positive effect on unrealized demand.” For an extensive review of the trade credit literature applied to operations research refer to Seifert et al. [34].

Likewise, the newsvendor model has been extended to study the interaction between operations and finance. With a focus on capital structure, Xu and Birge [37] analyze the value of a firm when bond debt is included. They compare the unlevered and levered value of the company and observe the changes in the optimal ordering quantities. Concerning debt pricing, Dada and Hu [11] study a Stackelberg game in which the banker (leader) provides finance to the newsvendor (follower). They find that to achieve channel coordination, a nonlinear loan schedule is needed. In a similar setting, Kouvelis and Zhao [24] compare short-term debt and supplier-financed trade credit. They determine the retailer's optimal inventory level and the supplier's wholesale discount rate. They find that the retailer always prefers trade credit financing, which improves supply chain efficiency but does not coordinate the chain completely.

The closest works to ours are financial supply chain management papers for a multi-period setting with budget constraints. Buzacott and Zhang [4] discuss budget constraints in two settings: multi-period with deterministic demand and single period with stochastic demand. Both settings focus on the analysis of asset-based financing, which means that the maximum loan depends on the amount of working capital. The authors conclude that the retailer benefits from access to this type of financing. In a similar approach, Chao et al. [5] develop a model that has a dynamic budget constraint. They deal with three instances: one that permits no loans, one that allows loans with no boundaries and one that allows loans bounded by the working capital at a given period. Using dynamic programming, they have to make restrictive assumptions to solve their model such as lost sales without penalty costs, no backorders and no holding costs. In a similar vein, Protopappa-Sieke and Seifert [31] have contributed to the understanding of working capital management for a single-product environment. Within the framework of a finite horizon with stochastic demand, they explicitly incorporate upstream/downstream payment delays and allow for lead time. They obtain results for special cases and use simulation to develop their analysis for the general case. In their study, short-term debt is not allowed, although some of the above studies included it. These studies are stepping stones for our study of the effects of short-term debt; however, they demonstrate the difficulty of finding an analytical solution when short-term debt is included.

Based on this previous work, we can now position our paper and explicitly establish its contribution. Our study goes beyond the EOQ and newsvendor models which demonstrate the importance of sources of financing but are limited for the analysis of working capital constraints. The multi-period papers are closer in nature to our study, but in order to find analytical solutions, they do not include payment delays, multiple sources of financing or working capital constraints in a unified model. Our objective is to study how different sources of financing and how long- and short-term debt affect the optimal ordering policy when working capital constraints, payment delays and lead time are taken into account. Therefore, we contribute to the existing literature with results from a more holistic model that takes into account these different factors while highlighting working capital policy.

3. Mathematical model

To study the relationship between working capital policy, short-term debt and payment delays, we develop a mathematical model that describes the operational and financial flows for a simple supply chain that consists of a supplier and a retailer. We focus on the operational and financial decisions of a retailer whose goal is to maximize the expected present value of cash flows. Next, we present the relevant parameters and discuss in detail our working capital policy assumption which affects both the financial and operational sides. Then, we include a description of the retailer's decision variables and express our objective function and the transition equations for the most important endogenous variables. We end this section with an analysis of the cash flow margins with and without loans and the implications for the level of working capital target.

On the operational side, we consider a standard single product, finite horizon, periodic review inventory setting. At the beginning of the period, the retailer orders at the unit ordering cost c from a supplier that has infinite capacity. There is no set-up cost for the orders. The supplier, in turn, delivers the goods with a lead time of L periods. The retailer sells them to the customer at a price p, where $p>c$. At every period, the customer's demand is independent and identically distributed with mean μ and standard deviation σ. The variable ξ_n represents the demand at period n. If demand cannot be satisfied, there is a backlogging cost b. Otherwise, if there is excess inventory, there is a holding cost h. We assume that if there are unsatisfied orders at the end of the time horizon, these are satisfied with a last order. Otherwise, if there is inventory left, it is salvaged at a price s, where $s \leq c$. We note that the holding cost parameter h should represent only cash expenditures such as material holding costs, storage costs, damage costs and taxes but not the opportunity costs of capital. The opportunity cost of capital is already included by using a net present value function (see Eq. (1)). With respect to the backorder cost parameter b, our model can handle only cash expenditures related to extra administrative costs, price discounts, material handling and transportation. However, the model could be adjusted to include loss of goodwill by adding a service level constraint[1] or by performing the optimization of Eq. (1) in two steps: first for the inventory level with loss of goodwill, and then for the working capital without the loss of goodwill.[2] This is a standard setting, and although restrictive in terms of assumptions, it enables us to include the financial assumptions and parameters for payment delays, sources of financing and working capital policy.

On the financial side, we model trade credit payment terms, multiple sources of financing and a working capital restriction. The retailer receives payments from the customer with a delay of d periods and pays the supplier with a delay of u periods. The retailer has two sources of financing: long-term and short-term debt. The long-term financing is a capital endowment E that the retailer uses to finance his operations throughout all periods. The interest rate for the long-term financing is r_E and is accrued at every period. Then, E and its accumulated interest rate are paid back in the last period. In addition to the endowment, the retailer has access to short-term debt, similar to a traditional credit line, to cover short-term cash deficiencies. The interest rate for the short-term loan is r_s. We assume that $r_s > r_E$ as there is a surcharge for this credit line. We assume that the retailer sets a working capital allowance level A. At the end of each period, the retailer computes the total working capital, as the inventory value plus cash. If the total working capital is higher than the working capital allowance A, all excess in the form of cash is sent to an external depository and will not be used in the future to finance the retailer's day-to-day operations. This assumption includes in the model a specific working capital policy to be decided by the retailer. At its optimal level, it will balance the high costs of a short-term loan with the operational costs from inventory and the opportunity costs of cash holdings. Any cash left within this permissible limit at the end of a period will earn interest at rate r_c, where $r_c \leq r_E < r_s$. Cash flows are discounted with a factor $\alpha = 1/(1+r_d)$, where the rate of discount is equal to the weighted average cost of capital of the retailer. In our model we do not include taxes or bankruptcy costs.

The importance of the working capital allowance in the model can be summarized as follows: (1) The working capital allowance, as opposed to other stochastic capacity constraints, involves two components: cash and inventory, (2) its effect on the available working capital at the beginning of every period is stochastic rather than constant and it is a result of its operations and

[1] Axsäter [2, p. 26] argues that "because backorder costs are so difficult to estimate, it is very common to replace them by a suitable service constraint."

[2] In the first step, it is important to consider the loss of goodwill as a cash penalty since it represents the importance the retailer places on backordered demand when ordering inventory. Therefore, to find the optimal ordering level for a given level of working capital allowance A, the parameter of the backorder cost should include the loss-of-goodwill part. Once the optimal ordering level is found, we can calculate the pure cash flow value of the objective function by setting the backorder cost to the part that excludes the loss of goodwill. In the second step, we can repeat this procedure for multiple values of working capital allowance A until the optimal working capital level is found.

finances, (3) since it determines a level after which cash is sent to a depository to finance other firm operations, the working capital cannot accumulate indefinitely over time. In fact, it would be unreasonable for the retailer to accumulate cash for long periods of time and (4) the working capital allowance assumption distances our model from the setting developed by Modigliani and Miller [29]. Next, we discuss this last point in more detail.

In their famous study, Modigliani and Miller [29] (hereafter MM) present their irrelevance principle which states that under restrictive conditions (given investment opportunities, perfect competition in the capital markets, and the existence of equivalent return classes for firms), the cost of capital does not depend on the capital structure of the firm. In our field, this proposition has been understood as a statement of the independence between a firm's operational and financial decisions. Several revisions to MM's proof and their assumptions have been developed. In particular, some of the work that has refined their meaning can be found in Stiglitz [35], Miller [27], Modigliani [28], and Ross [32]. A review of this evolution has been presented by Rubinstein [33]. In this review, Rubinstein states that one of the main assumptions in order for the MM propositions to hold is that operating income (from assets) and the present value function are not affected by capital structure. Therefore, if we think of profit generated by operations as a random variable, the assumption states that this random variable is not changed by the choice of financing. However, by tying together the financial and operational decisions with our working capital assumption, profit can be affected by the choice of working capital allowance. We illustrate this in the following scenario. Suppose inventory and cash are bounded by a working capital restriction in period n. In period $n+1$, if short-term debt is cheap enough, the firm may be able to achieve its full optimal ordering policy, but it will incur financial costs. Conversely, if loans are expensive, the capital-restricted firm will order less than what is ideal, and will consequently suffer from a stronger negative effect on profit. Therefore, because the choice of capital structure, embodied in the level of endowment and working capital allowance, can influence operations and the present value function, we depart from the MM setting and examine how strongly their results are impacted.

Having presented the model's operational and financial assumptions, we now list the sequence of events at the beginning of each period. (1) The retailer's total short-term loan, endowment and retained cash are compounded with their respective interest rates, (2) the retailer reviews his initial inventory, working capital position and loan position, (3) the retailer places a new order with the supplier, (4) the retailer's order placed with the supplier L periods ago arrives, (5) the retailer satisfies backorders as far as possible, (6) the customer places an order, (7) the retailer satisfies the customer's order as much as possible. The unsatisfied backorders and unsatisfied customer demand are counted as the next backorders. Otherwise, the excess of products is counted as the next inventory, (8) revenue arrives from satisfied backorders and the customer's orders of d periods ago, (9) the retailer pays the procurement costs from u periods ago and the current period's holding and shortage costs from his working capital. If no working capital is available, these costs are paid with a new short-term loan and (10) working capital restrictions are applied, which ensures that any remaining cash is used initially to repay debt before it is either used to further finance the retailer's operations or sent to the external depository.

Protopappa-Sieke and Seifert [31] study a case in which working capital restrictions constrain the optimal ordering decision, but the authors do not consider access to short-term debt financing. We extend their model by studying an *expected cash flow maximization problem* in which *short-term debt levels*, working capital policy and payment delays jointly influence profitability. The planning horizon in our model extends from 1 to N. Our decision variables are the working capital allowance level A and the ordering quantity at period n denoted by q_n. We define vector $\mathbf{q}=[q_1,\ldots,q_N]$. To simplify the analysis, we assume that the initial endowment equates to the working capital allowance, i.e. $E=A$. The objective function value OFV is defined in Eq. (1) as the accumulated expected present value (PV) of deposits in the depository DV_n for $n=1\ldots N$, the last period's cash level after the last transactions $\mathbf{C}_{N+1}$, the last period's compounded long-term debt level $\mathbf{EI_{N+1}}$, and the last period's short-term debt level after the last transactions $\mathbf{TL_{N+1}}$.

$$OFV=\max_{\mathbf{q},A=E}\mathrm{E}\Big[\mathrm{PV}[\mathbf{C_{N+1}}(\mathbf{q},A)-\mathbf{TL_{N+1}}(\mathbf{q},A)-\mathbf{EI_{N+1}}(\mathbf{A})] + \sum_{n=1}^{N}\mathrm{PV}[DV_n(\mathbf{q},A)]\Big] \tag{1}$$

This objective function is subject to operational and financial flow constraints. Let $(w)^+=\max(w,0)$. Then, we express the inventory flows as $I_{n+1}(\mathbf{q})=(q_{n-L}+I_n(\mathbf{q})-\xi_n-B_n(\mathbf{q}))^+$ and the backorder flows as $B_{n+1}(\mathbf{q})=(\xi_n+B_n(\mathbf{q})-q_{n-L}-I_n(\mathbf{q}))^+$, where I_{n+1} and B_{n+1} represent the inventory and backorder levels respectively at the end of period n, beginning of period $n+1$. In Eq. (2), cash after operations CO_n is defined as the cash at the beginning of the period C_n, plus customer payment for satisfied demand d periods ago, adjusted by payments to the supplier for orders made $L+u$ periods ago and the period's holding and backorder costs.

$$CO_n(\mathbf{q},A)=C_n(\mathbf{q},A)+p\min[\xi_{n-d}+B_{n-d}(\mathbf{q}),q_{n-d-L}+I_{n-d}(\mathbf{q})] -cq_{n-L-u}-hI_{n+1}(\mathbf{q})-bB_{n+1}(\mathbf{q}) \tag{2}$$

Cash after operations CO_n can be either negative or positive. If $CO_n<0$, the retailer takes a short-term loan to cover expenses. If $CO_n>0$, the retailer can use the cash to repay short-term loans. In Eq. (3), the accumulated short-term debt TL_{n+1} is defined as the accrued remaining short-term debt after repayments from the previous period. The cash after loans COL_n is presented in Eq. (4).

$$TL_{n+1}(\mathbf{q},A)=(1+r_s)[TL_n(\mathbf{q},A)-CO_n(\mathbf{q},A)]^+ \tag{3}$$

$$COL_n(\mathbf{q},A)=[CO_n(\mathbf{q},A)-TL_n(\mathbf{q},A)]^+ \tag{4}$$

Cash after loans might be sent to the depository once the working capital policy is applied. In Eq. (5), DV_n represents the amount sent to the depository, and in Eq. (6), C_{n+1} stands for the amount of money left for the next period.

$$DV_n(\mathbf{q},A)=[COL_n(\mathbf{q},A)-[A-cI_{n+1}(\mathbf{q})+cB_{n+1}(\mathbf{q})]^+]^+ \tag{5}$$

$$C_{n+1}(\mathbf{q},A)=(1+r_c)\min[COL_n(\mathbf{q},A),[A-cI_{n+1}(\mathbf{q})+cB_{n+1}(\mathbf{q})]^+] \tag{6}$$

In Eq. (7), we accumulate the endowment interest payments:

$$EI_{n+1}(A=E)=EI_n(1+r_E)+Er_E \tag{7}$$

To complete the model, we detail the end-of-horizon accounting (end of period N): If inventory is left over, it is salvaged at price s and the payment is received d periods later. If there are backorders, they are satisfied with a last product order which arrives L periods later. The supplier is paid for this order $L+u$ periods later, and the payment from the customer is received $L+d$ periods later. The backorder cost for the L periods that the customer had to wait is taken into account. Therefore, the cash level after these transactions is

$$\mathbf{C_{N+1}}=\left(C_{N+1}+\alpha^d sI_{n+1}+\left(\alpha^{L+d}p-\alpha^{L+u}c-\sum_{i=1}^{L}\alpha^i b\right)B_{n+1}\right)^+.$$

If the cash holdings do not suffice to cover the last expenses, a short-term debt is used. Therefore, the total short-term debt level after these transactions is $\mathbf{TL_{N+1}}=TL_{N+1}+(-\mathbf{C_{N+1}})^+$. Finally, the long-term debt and its accumulated interest rate is

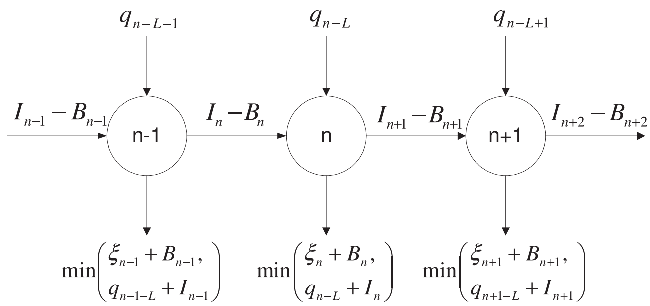

Fig. 1. Inflows and outflows of products at periods $n-1$, n and $n+1$.

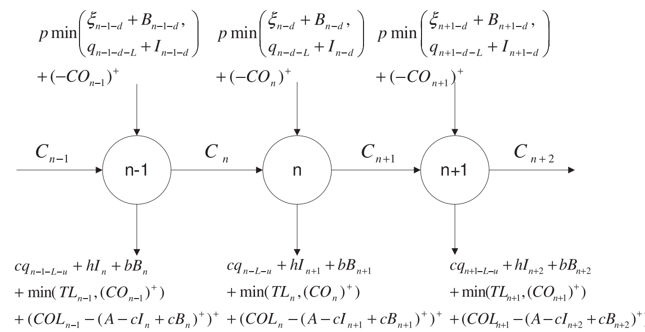

Fig. 2. Inflows and outflows of cash at periods $n-1$, n and $n+1$.

$\mathbf{EI_{N+1}} = EI_N(1+r_E)+Er_E+E$. This completes the intermediate and last period operational and financial flow equations.

Figs. 1 and 2 show how product flows and cash flows are interrelated. Fig. 1 shows product inflows and outflows and their evolution from period $n-1$ to period $n+1$. At period n, for example, inventory starts at level I_n-B_n. There is an inflow of products from the order made L periods ago due to lead time, q_{n-L}. The outflow of products is the quantity of satisfied demand and backorders with the current inventory level, $\min(\xi_n+B_n, q_{n-L}+I_n)$. The end inventory level that will be kept for the next period is $I_{n+1}-B_{n+1}$. Fig. 2 illustrates the inflows and outflows of cash and their evolution from period $n-1$ to period $n+1$. At period n, cash starts at level C_n. There are two inflow quantities. The first is the income from satisfied demand and backorders from d periods ago represented by $p\min(\xi_{n-d}+B_{n-d}, q_{n-L-d}+I_{n-d})$. The second is the amount of short-term debt acquired, which equals a potential cash deficit after operations and is represented by $(-CO_n)^+$. The outflow quantities can be divided in two: cash outflows due to operations and those due to finances. Operational cash outflows are as follows. Payment for the order made $L+u$ periods ago due to lead time and upstream payment delay is cq_{n-L-u}. The holding cost and backorder cost are hI_{n+1} and bB_{n+1} respectively. If cash after operations is positive, it is used to pay as much as possible of any outstanding short-term debt, $\min(TL_n, (-CO_n)^+)$. Then, with the cash after operations and finances and leftover inventory, we determine the amount that goes to the depository with the working capital condition, $(CO_n-(A-cIn+1+cBn+1)^+)^+$. Finally, the end cash level that will be kept for the next period is C_{n+1}.

The optimal solution to our single product, finite horizon, periodic review inventory model has to coordinate the inventory ordering decisions with the level of working capital and the level of short-term debt needed. A complete analysis of these decisions is fairly complex and is addressed in Section 4. To gain insights into the effect of short-term debt and working capital, we perform sensitivity analysis on the marginal revenue after operations. We define the *marginal revenue* after operations as the change in cash position by a unit increase in the inventory position for different scenarios. Proposition 1 specifies how the marginal revenue behaves with respect to the loan position.

Proposition 1. *The marginal revenue after operations without loans is higher than that with loans.*

Proposition 1 states that it is beneficial to be in the region where no short-term loans are outstanding. This implies that the retailer should set higher working capital allowances, since this reduces the probability of taking short-term loans, resulting in higher marginal revenue after operations. However, as the endowment increases, the associated financial costs also increase. Eventually, the endowment level is too high and the associated financial charges outweigh the benefits of higher marginal revenues. Therefore, this analysis hints at the existence of an optimal working capital and endowment level that balances the endowment and the short-term financial costs. However, we do not consider the effect of the working capital allowance on the depository amounts. We can investigate this effect only numerically as we will see in Section 4.1.

4. Numerical analysis

In this section, we simulate and numerically analyze the dynamics of the multi-period financial inventory control model. As required by this approach, we detail how we initialize the simulation and how we perform the end-of-horizon accounting. We describe the numerical algorithm to approximate the optimal decision variables and the list of parameters explored. For the rest of this section, the word “optimal” refers to the approximated values obtained with the numerical algorithm. Therefore, it should be read as “numerical optimal.” In Section 4.1, we discuss the shape of the objective function with respect to the working capital allowance. In Section 4.2, we build on the previous analysis and explain how the most relevant parameters—payment delays, lead time and short-term debt levels—affect the objective function and optimal working capital decisions. In Section 4.3, we compare the sensitivity of optimal working capital levels and optimal objective function values for varying payment delays and lead times. To contextualize our results, in Section 4.4, we perform a working capital analysis on products that fall in the classic product quadrants of low-high profit margin and low-high demand variability.

For proper bookkeeping, the simulation requires well-defined initial and end-of-horizon assumptions. To avoid transient effects on cash due to the lead time, we initialize the simulation with incoming orders equal to the average demand for every period $n<L$. To simplify the analysis, we assume that the initial endowment is equal to the working capital allowance, i.e. $E=A$. The retailer acquires this endowment at the beginning of the first period, i.e. $C_1=E$. Lastly, there are no outstanding short-term loans, i.e. $TL_1=0$. Here we summarize the end-of-horizon accounting that was presented in Section 3. When the simulation reaches the terminal period N, delayed payments to the supplier and from the customer are accounted for. Unused inventory is sold at salvage value s, where $s<c$. Outstanding backorders are satisfied with a last order that arrives L periods later and revenues are discounted accordingly. Finally, the endowment E is returned in the last period and the accrued interest is in turn deducted.

With the initial and end-of-horizon assumptions in place, we describe the inventory ordering policy and the algorithm to approximate the optimal values for the decision variables and objective function. Consistent with the results and assumptions for multi-period inventory models with stochastic constraints by Ciarallo et al. [8] and Güllü [17], we assume that the retailer uses

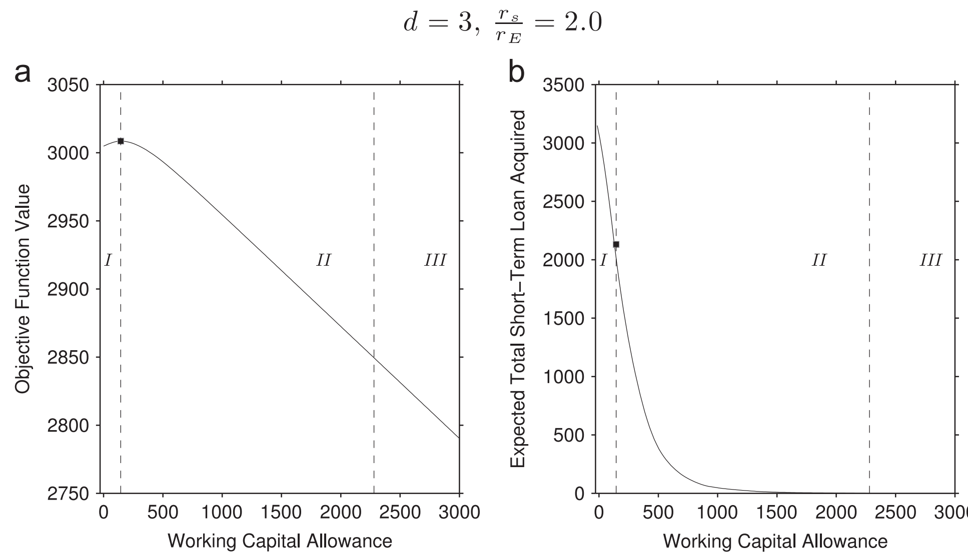

Fig. 3. (a) Objective function value vs. working capital allowance and (b) expected total short-term loan acquired vs. allowance. The marked squares show the optimal working capital policy and the optimal short-term debt level for this setting. Regions I and II use short-term debt. Region III does not.

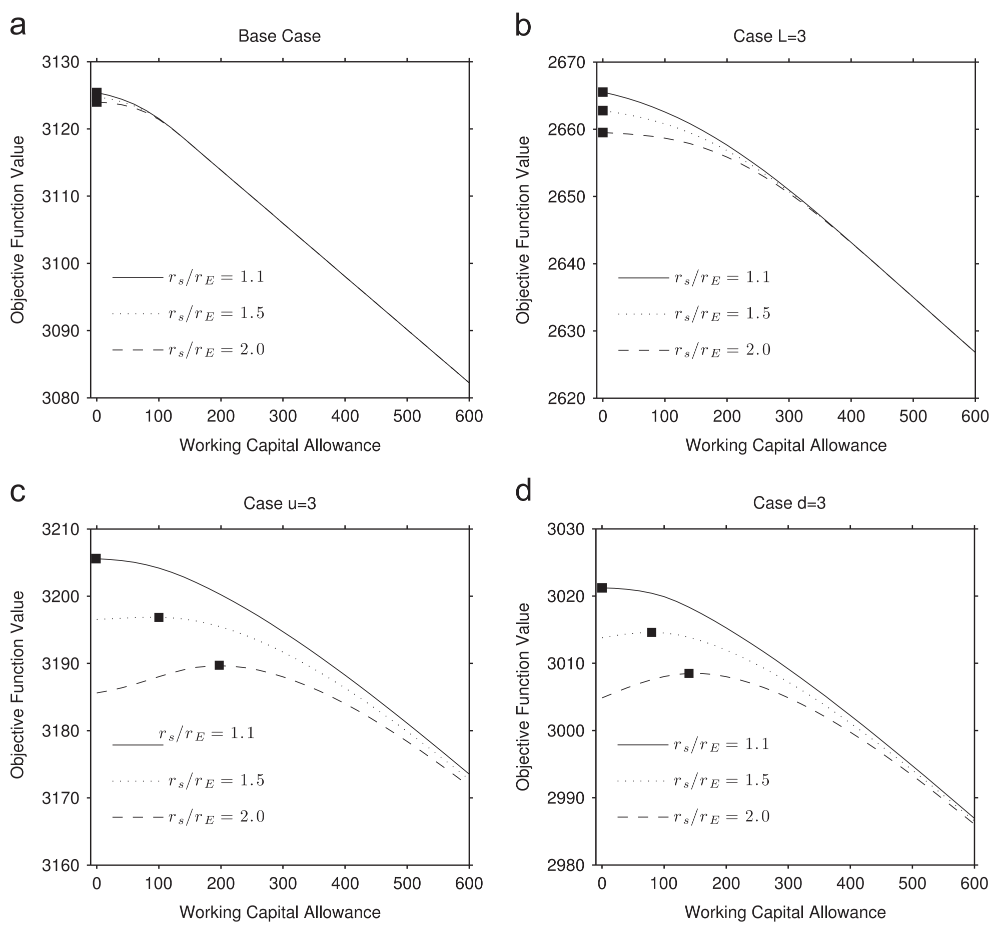

Fig. 4. Objective function value vs. allowance for (a) base case, (b) lead time case, (c) upstream payment delay case and (d) downstream payment delay case. All delays are equal to 3 periods. Each case is examined for three levels of relative short-term debt interest rate premiums, $m_s = r_s/r_E = 1.1, 1.5, 2.0$.

an order-up-to level policy S. Even though the structure of the proposed financial inventory control model corresponds to a stochastic dynamic inventory model, this is not sufficient to guarantee the optimality of a base stock policy as explained by Protopappa-Sieke and Seifert [31]. The principal reason is that the optimal order decision for a period depends on the future working capital position when the payment for an order is due. Therefore, we use the order-up-to level policy since it is consistent with previous work and since it has been proven to be optimal in settings that are the closest to ours.

The algorithm starts with an initial guess for the decision variables. It sets a feasible solution for the allowance, $\overline{A_0} = 0$, and it sets the order-up-to policy equal to the mean demand, $\overline{S_0} = \mu$. For these values, the exogenous and endogenous variables are

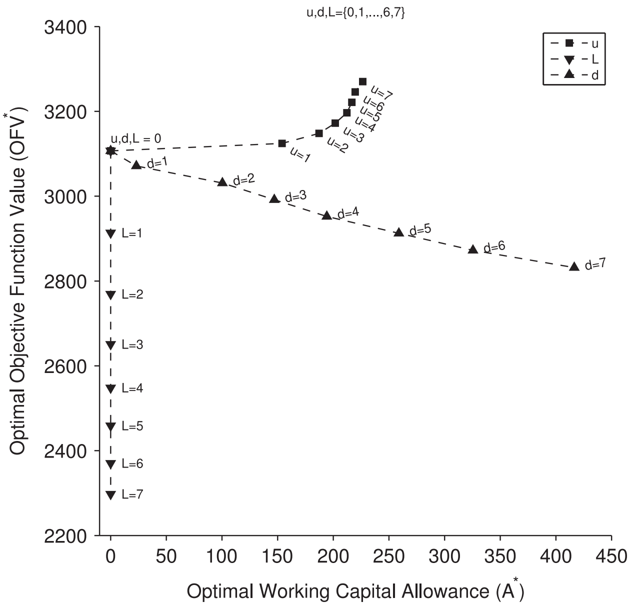

Fig. 5. Scattered plot of optimal objective function values vs. optimal working capital allowance levels for payment delays and lead time ranging from 0 to 7.

initialized and for each period $n = 1 \dots N$, the following steps are followed: (1) cash available, loans and endowment are compounded with their respective interest rates; (2) the order quantity is calculated with the order-up-to policy, $\overline{S_0}$; and (3) the operational and financial variables are updated. Once steps (1)–(3) are completed for every period, the objective function value for $\overline{A_0}$ and $\overline{S_0}$ is computed. Keeping $\overline{A_0}$ constant, a new value $\overline{S_1}$ is calculated with the Nelder–Mead Simplex Method, a nonlinear multivariate optimizer developed by Nelder and Mead [30] and Lagarias et al. [25]. This optimizer was chosen since it can be used for multi-variable nonlinear objective functions, which serves our model and allows the implementation to handle higher complexity. The order-up-to policy is updated until the optimizer converges to a numerical optimal value, $S^*(\overline{A_0})$, or a maximum number of iterations is reached, which is set to 200. Once $S^*(\overline{A_0})$ is known, a new value $\overline{A_1}$ is calculated with the Nelder–Mead Simplex Method. Similarly, the allowance is updated until it converges to a numerical optimal value A^*. Afterwards, A^* and $S^*(A^*)$ are used as the optimal decision variables. The algorithm was implemented in Matlab 7.9. The convergence of the Nelder–Mead Simplex Method has been proved by Lagarias et al. [25] for a minimization of convex functions in low dimensions while using its standard coefficients. The multiple numerical experiments performed for this manuscript show a concave shape for the objective function (Figs. 3(a) and 4) that is to be maximized. Even though, the concave shape is supported by the characteristics of the problem: too low or too high order-up-to levels and working capital allowances will decrease the objective function value, however, we can not prove such findings mathematically. The concavity results needed for convergence are based on observations in the numerical experiments. The inverse used in the minimization has a convex shape, which is required for the proof by Lagarias et al. [25]. In addition our problem is only two dimensional since we only search for the optimal values of A and S. Lastly, we use the standard values for the parameters of the numerical search algorithm. These values are 1, 2, 0.5 and 0.5 for the coefficients of reflection, expansion, contraction and shrinkage, respectively.

Before we proceed with our analysis, we detail the parameter values used in our simulations. The model will track the decisions for $N = 80$ periods, which define the life cycle of a product. In order to calculate the number of replications needed to obtain a specific absolute error of β we used the methodology described by Law and Kelton [26]. The authors claim that an approximate expression for the total number of replications, $n^*_\alpha(\beta)$, required to obtain an absolute error of β is given by $n^*_\alpha(\beta) = (i \ge n : t_{i-1;1-\alpha/2}\sqrt{S^2(n)/i} \le \beta)$, where t is the student t-distribution and $S^2(n)$ is an estimate of the population of the variance. For our data we chose an absolute error β of 0.01 and a confidence interval of 90% which corresponds to $\alpha = 0.1$ and approximately $CI = 50$ replications. The per period operational parameters are $p = 2.08$, $c = 1.60$, $h = 0.08$, $b = 0.14$, $\mu = 100$ and $CV = \sigma/\mu = 0.70$. The price and procurement cost correspond to a profit margin of 30%. This level of profit margin in conjunction with a coefficient of variation of 70% represent a product that has a potentially high revenue but at the same time high variability of demand. The use of short-term debt for these types of products should be more relevant since they may experience periods of very low demand followed by periods of high demand with high profitability. Moreover, we have chosen the holding and backorder costs in such a way that the critical ratio, $(p + b - c)/(p + b - c + h + c - s)$, is higher than 0.5 (0.88) which means that the order-up-to level would be higher than the mean demand. The firm would therefore more often experience pressure to cover the procurement and holding costs. In Section 4.4 we generalize this analysis by changing the values for profitability and variability of demand. The customer's demand distribution is lognormal. The yearly financial base parameters are $r_E = 12\%$ and $\alpha = 0.89$. The discount rate, short-term debt rate, and cash investment rate are set with respect to the endowment rate. The short-term interest rate is $r_s = m_s r_E$, where $m_s > 1$. The cash investment rate is $r_c = m_c r_E$, where $0 < m_c < 1$. Finally, lead time and upstream and downstream payment delays are varied in our numerical experiments.

4.1. Impact of working capital policies

In this section, we analyze how profitability and short-term debt levels vary with different working capital allowances. We illustrate this relationship in Fig. 3, in which two plots, (a) and (b), show the objective function value and the expected total short-term debt acquired for the same ranges of working capital allowance. To illustrate this sensitivity analysis, we use the case of downstream payment delay and highlight the relevant regions of interest. However, parallel results apply for the cases of no payment delay, upstream payment delay and lead time. Both plots (a) and (b) are split into three regions. Regions I and II are separated by the point with the optimal objective function value and optimal working capital allowance. Regions II and III are separated by the required minimum level of working capital allowance for which short-term loans are no longer used.

In Region I, as we increase working capital allowance, there is an increase in the objective function value. Although an increase in working capital allowance and endowment results in higher endowment capital financial costs, it is still beneficial due to higher outflows to the depository and lower short-term debt financial costs. In Regions II and III, with increasing working capital allowance, there is a reduction in the objective function values. This is because the reduction in short-term debt financial costs is outweighed by an increase in endowment financing costs. We note that at the optimal allowance level, the use of some short-term debt—even if it is relatively more expensive—is most favorable. Lastly, if short-term debt is inaccessible, the feasible region would be limited to Region III which is far off the optimal level. This reinforces the importance of the availability of short-term debt and warns of the negative effects of a lack of short-term financial credit, since even small credit lines can significantly lower average requirements.

Table 1
Optimal objective function value.

Profit margin	r_s/r_E	Coefficient of variation = 0.3				Coefficient of variation = 0.7			
		Base	$L=3$	$u=3$	$d=3$	Base	$L=3$	$u=3$	$d=3$
Profit margin 10%	1.1	989.5	803.6	1069.5	900.8	776.2	318.2	855.2	687.2
	1.5	989.0	801.9	1065.2	890.7	775.0	312.5	843.3	678.0
	2.0	988.4	799.9	1059.9	878.1	773.8	308.4	835.1	671.8
Profit margin 30%	1.1	3334.1	3147.4	3414.3	3230.1	3125.4	2665.5	3205.6	3021.2
	1.5	3333.9	3146.8	3412.5	3226.5	3124.8	2662.8	3196.9	3014.6
	2.0	3333.8	3146.2	3410.3	3222.7	3124.0	2659.5	3189.7	3008.5

Furthermore, Fig. 3 relates back to MM's assumptions that the financial structure does not affect profit levels, therefore implying that there is no preferred capital structure. In this figure, we observe that the relative weights in the combined use of short- and long-term debt indeed affect the objective function. We will explore the sensitivity further in the next sections.

4.2. Sensitivity to interest rate premiums

Building on the working capital allowance analysis of Section 4.1, we extend our numerical studies to understand how sensitive the optimal working capital allowance is to changes in short-term interest rate premiums for lead time and payment delays. In Fig. 4, for the same working capital range, we individually plot the objective function values for the base case, lead-time case (L), upstream delay case (u) and downstream delay case (d). As expected, upstream payment delay affects the retailer's objective function positively; downstream payment delay has the opposite effect; and lead time lowers it.

Observation 1. *The optimal working capital allowance is more sensitive to changes in short-term interest rate in cases of changed payment terms (both upstream and downstream) compared with changes in lead time.*

In the base case and the lead time case, cash after operations almost always suffices to pay all expenses; therefore, short-term debt is rarely used, which means that the objective function is insensitive to short-term interest rate changes. In both the upstream and downstream payment delay cases, short-term debt is used more extensively; consequently, the change in the objective function is clearly sensitive. The extent of this change in the objective function is much lower than one might expect, especially when compared to Section 4.1 in the case when short-term debt is unavailable.

Therefore, from our numerical analysis in the previous sections, we observe that the MM results are robust to varying interest rate premiums. Nonetheless, we observe that companies will experience higher working capital requirements and lower objective function values when credit dries up. The Quarterly Report of the Bank of England clearly expresses the effects of the lack of credit. In this report, Benito et al. [3] explain that “businesses typically use working capital to fund their day-to-day business activities. But if credit lines dry up and businesses are unable to access working capital, they may be constrained in the amount they can ‘effectively’ supply.” Similarly, the Confederation of British Industry [10] explains that the lack of credit caused by the financial crisis of 2008 has affected British companies in terms of their levels of working capital.

4.3. Impact of payment delays and lead time

In this section, we discuss how different levels of payment delays and lead time affect the optimal objective function value and working capital allowance. Fig. 5 shows how optimal objective function values and the corresponding optimal levels of working capital allowance vary for upstream and downstream payment delays and lead times that range from 0 to 7 periods. The center point reflects the base case where there are no payment delays or lead time.

Observation 2. *Increasing lead time strongly decreases optimal objective function value levels, but it does not increase the optimal working capital allowance.*

The increase in lead time increases operational costs; therefore, it has a negative effect on the objective function value. However, since payments arrive as soon as orders are received, a small level of working capital allowance proves optimal.

Observation 3. *As upstream and downstream payment delays increase, the optimal working capital allowance increases.*

The magnitude of the optimal working capital allowance increase differs for upstream and downstream payment delays. In the case of upstream payment delays, the optimal working capital allowance tends to increase slowly. This is because cash needs to be kept for the last payments to the supplier instead of being sent to the depository. However, since revenue is received without delay, there is rarely a need to leverage short-term loans. Conversely, the case of downstream payment delays requires higher working capital allowance levels, since increasing downstream delays imply longer periods without revenue to pay for operations, which implies that they either have to be financed by expensive short-term debt or an increased capital endowment.

Observation 4. *The increase in the objective function value from greater upstream payment delays is less than the losses from greater downstream payment delays.*

Seifert et al. [34] observe that suppliers typically face more downstream payment delays compared with retailers. However, our result warns against overly increasing suppliers' downstream payment delays since this can cause strong working capital pressures and lower objective function values. Moreover, suppliers could be further affected when credit lines dry up.

4.4. Impact of product profitability and demand variability

Finally, in this section we extend our numerical analysis to discern how sensitive our results are to varying profit margins and demand variability. Table 1 summarizes the optimal objective values for low and high levels of both profit margin and coefficient of variation. Each quadrant can be thought of as a different type of product characterized by its profitability and demand variability. For each quadrant, we find the optimal values for the base case, lead time case and payment delay cases, as well as for different rates of short-term debt. As expected, the optimal objective function value increases with an increase in profit margin, while

Table 2
Second quadrant: profit margin = 10%, coefficient of variation = 0.7.

r_s/r_E	Base	$L=3$	$u=3$	$d=3$	Base	$L=3$	$u=3$	$d=3$
	Objective function value				Order-up-to level			
1.1	776.2	318.2	855.2	687.2	103.8	128.7	103.9	103.8
1.5	775.0	312.5	843.3	678.0	103.4	128.2	103.9	103.9
2.0	773.8	308.4	835.1	671.8	103.3	128.1	103.8	104.0
	Optimal allowance				Average short-term loan			
1.1	0.0	0.0	38.2	79.4	15.2	29.2	50.2	48.4
1.5	0.0	66.9	263.8	227.0	15.0	24.0	32.0	34.2
2.0	38.1	146.8	374.7	320.6	9.8	16.2	22.0	25.0
	Exp total long-term interest				Exp total short-term interest			
1.1	0.0	0.0	7.3	15.1	4.2	20.1	55.5	39.6
1.5	0.0	12.7	50.2	43.2	5.5	20.0	37.9	28.2
2.0	7.3	28.0	71.4	61.1	4.3	15.0	32.2	22.7

an increase in the coefficient of variation decreases the optimal objective function value. Note that this result applies for all levels of short-term debt rate and for all four cases.

Observation 5. *The optimal objective function value is sensitive to lead time and payment delays, but it is relatively robust to changes in the short-term debt factor*, $m_s = r_s/r_E$.

To understand Observation 5, we focus on the quadrant with low profit margin and high coefficient of variation. In Table 2, we show the values for the order-up-to level, the optimal working capital allowance, the total average short-term loan acquired, the total long-term interest paid and the total short-term interest paid.

Observation 6. *The optimal order-up-to level is fairly insensitive to increases in the short-term debt factor.*

Although the working capital restriction may affect the operational side, it should be noted that due to the access to short-term debt, it is always feasible for the retailer to order up to any level. Observation 6 highlights that even though keeping the same order-up-to level may increase the financial costs of short-term debt, it is still rational to do it. In addition, this means that when analyzing the changes in the objective function due to changes in short-term debt interest rates, we should focus on the financial aspects of the model.

Observation 7. *The optimal objective function value is fairly insensitive to changes in the short-term debt factor due to the possibility of shifting short-term and long-term financial costs by changing the optimal working capital allowance level.*

Therefore, although the total average short-term debt level reduces due to a higher short-term interest rate, the optimal working capital allowance increases. The result of this increase is that there is enough cash in the system so the total financial costs due to short-term debt and long-term debt are fairly similar. This result is consistent with Proposition 1.

5. Conclusion

Recognizing the seminal work by Modigliani and Miller [29], operational aspects and financial considerations have typically been dealt with independently in operations research. Recent events, however, have demonstrated that a lack of short-term credit could halt the whole supply chain; therefore, the robustness of the MM model should be examined in more complex settings. In this paper, we have developed a mathematical model that includes key financial aspects such as working capital requirements, short-term debt usage, and upstream and downstream payment delays in a standard operational setting. We consciously deviate from the assumptions underlying the work of MM and numerically solve for the optimal working capital allowance and order-up-to level using a Nelder and Mead [30] optimization embedded in a simulation. We performed extensive sensitivity analyses by varying the most relevant model parameters such as working capital allowance, short-term debt premium, payment delays, profit margin and the coefficient of demand variation. The three main findings from our numerical analysis are as follows:

- When working capital restrictions and short-term debt are considered, the MM results do not strictly speaking apply, but looking at relative sensitivities, they carry over in spirit.
- The lack of access to short-term debt drastically inflates working capital requirements and lowers cash flows.
- Increasing downstream payment delay accentuates working capital requirements and reduces cash flows. This result is particularly relevant for suppliers, since payment delays typically increase further up the supply chain.

Given the multifaceted aspect of our model and to ensure the clarity of our analysis, we made simplifying assumptions, which we acknowledge could limit the scope of our findings. Thus, our results should be seen as exploratory in nature, but they could nonetheless help to better understand the trade-offs between working capital and short-term debt requirements and their relative sensitivities. Building on this work, future research could test the robustness of these results by replacing the order-up-to level with other inventory control policies such as the (R,S) or (R,s,S). In addition, future research could focus on the merits of multi-product working capital pooling strategies. Finally, working capital requirements should be more explicitly analyzed in a dynamic setting to determine how product rollovers impact operational cash flows.

Appendix A

A.1. Proof of Proposition 1

In the shortage region, inventory is not sufficient to fill demand. For each unit bought, the retailer receives the unit revenue from the customer d periods later. Moreover, he regains the shortage cost that otherwise had to be paid. However, he needs to pay the procurement cost u periods later. If the retailer already has loans, either he will pay short-term loans if the profit margin is positive or he will take more short-term loans to cover the negative margin. Consequently, the discount factor is $\alpha_s = 1/(1+r_s)$ and the cash flow margin is $\alpha_s^{L+d}p + \alpha_s^L b - \alpha_s^u c$. If the retailer does not have loans, either he will increase his cash holdings if the cash flow margin is positive or he will pay it with his current cash holdings if the margin is negative. Therefore, the discount factor is $\alpha_s = 1/(1+r_E)$ and the cash flow margin is $\alpha_E^{L+d}p + \alpha_E^L b - \alpha_E^u c$. In the abundance region, inventory is sufficient to cover demand. Therefore, for each unit bought, there are holding costs to be paid at the end of the period and procurement costs to be paid u periods later. Nonetheless, the new inventory excess can be sold at the salvage value if needed. Therefore, if the retailer already has loans, the discount factor is $\alpha_s = 1/(1+r_s)$ and the cash flow margin is $\alpha_s^L s - \alpha_s^L h - \alpha_s^u c$. If the retailer does not have loans, the cash flow margin is $\alpha_E^L s - \alpha_E^L h - \alpha_E^u c$. Given that $r_s > r_E > 0$, then $1+r_s > 1+r_E$, which follows $1/(1+r_E) > 1/(1+r_s)$. Substituting the definitions of α_E and α_s, we get that $\alpha_E > \alpha_s$. Using this inequality, we see that

$\alpha_E^{L+d}p+\alpha_E^L b-\alpha_E^u c>\alpha_s^{L+d}p+\alpha_s^L b-\alpha_s^u c$ in the shortage region and $\alpha_E^L s-\alpha_E^L h-\alpha_E^u c>\alpha_s^L s-\alpha_s^L h-\alpha_s^u c$ in the abundance region.

A.2. Nomenclature table

Symbol	Description
p	unit price
c	unit ordering cost
b	backlog cost
h	holding cost
s	salvage value
n	current period
N	number of periods in the planning horizon
ξ_n	random variable for demand
μ	mean demand
σ	standard deviation of demand
L	lead time in number of periods
d	payment delay from retailer to supplier in periods
u	payment delay from customer to retailer in periods
E	capital endowment amount
A	working capital allowance
r_E	interest rate for long-term financing
r_s	short-term interest rate
r_c	interest rate for a cash deposit
α	discount rate
q_n	ordering quantity at period n
I_n	inventory level at the beginning of period n
B_n	backorder level at the end of period n
C_n	cash level at the beginning of period n
CO_n	cash after operations in period n
COL_n	cash after operations and loans in period n
DV_n	depository value at period n
TL_n	short-term loan level at the beginning of period n
$\mathbf{C_{N+1}}$	last period's cash level after the last transactions
$\mathbf{TL_{N+1}}$	last period's short-term debt level after the last transactions
$\mathbf{EI_{N+1}}$	last period's compounded long-term debt level
OFV	objective function value

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