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NONLINEAR SYSTEMS CONTROL USING MSEV APPROACH

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Abstract

A class of modified state-space self-tuning controllers of the MSEV (minimum state error variance) type is has been considered in this article. A suitable chosen structure for the proposed controller provides for tracking the time-varying reference input, and makes it possible to apply this solution to nonlinear and nonstationary plants. Starting from the changes of innovations sequence statistics, an efficient load disturbance detector is also constructed, and the estimated disturbance amplitude is used to correct the control signal, in order to eliminate the influence of disturbances. The advantage in using the proposed algorithm for nonlinear systems control, in the presence of load disturbances and stochastic disturbances of unknown statistics, is demonstrated through simulation results.

Key Words

Adaptive control, self-tuning control, nonlinear systems, nonstationary plants, load disturbance

1. Introduction

Two groups of optimization-based adaptive controllers have drawn wide attention in recent years, and have been widely studied in the literature. The first makes use of the state-space representation of a system, coupled with the linear quadratic Gaussian (LQG) optimal control theory and a sequential parameter estimation technique, in order to obtain an adaptive filtered state feedback controller [1 – 3]. The optimal adaptive control algorithms so obtained have the advantage of being globally stable, of being applicable to any finite-dimensional controllable and observable system, and of providing effective control of the errors in the state trajectories. These are, however, achieved at the cost of rather large computational burden, which makes it difficult to implement these algorithms in real time for some practical systems. The second group makes use of the input-output representation of a system, coupled with the minimization of a generalized output error variance [4–6]. The main advantage of such controllers, named self-tuning controllers, is the relative simplicity of their derivation and implementation. However, the performance index selected for this approach does not minimize the errors in the state trajectories, as may be required in some applications. Also, the global stability of the controlled system requires the inverse system to be stable, which may exclude some nonminimum-phase systems [5]. A class of state-space self-tuning controllers, named minimum state error variance (MSEV) algorithm, that represents a combination of the mentioned approaches was presented in [7]. The MSEV controller is analogous to the form to the LQG optimal adaptive controller, but it achieved a considerable reduction of the computational requirement, at the cost of some performance loss.

Extensions of this approach to the control of nonlinear and nonstationary plants are proposed in this article. In contrast to the original MSEV approach, these extensions provide for tracking a prespecified nominal state-space trajectory in the presence of load disturbance and stochastic disturbances with unknown statistics.

2. Problem Formulation

Let us consider the system:

$$x(k+1) = \Phi x(k) + \Psi u(k) + \Gamma w(k) \qquad (1a)$$

$$y(k) = Hx(k) + v(k) \qquad (1b)$$

where $x(k) \in R^n$ is the state, $u(k) \in R^m$ is the input, and $y(k) \in R^p$ is the output of the system; $\{w(k)\}$ and $\{v(k)\}$ represent the zero-mean disturbance terms with covariance matrices Q and R, respectively; and Φ, Ψ, Γ, and H are the known system matrices.

Let us introduce the single stage performance index [7]:

$$J(u) = E\{L[x(k+1), u(k)]\} \qquad (2a)$$

$$L[x(k+1), u(k)] = \begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix}^T \begin{bmatrix} W & S \\ S^T & U \end{bmatrix} \begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix} \qquad (2b)$$

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where W and S are constant, bounded, and non-negative definite weighting matrices ($W \geq 0$), ($S \geq 0$), and U is a constant, bounded and positive-definite weighting matrix ($U > 0$). The control obtained by minimizing (2) will be referred to as the minimum state error variance control, and is given by [1, 3, 4]:

$$u(k) = M\hat{x}(k);\ \hat{x}(k) = E\{x(k)|Y(k-1)\}, \quad (3a)$$

$$M = -(\Psi^T W\Psi + U + 2\Psi^T S)^{-1}(\Psi W\Phi + S^T\Phi) \quad (3b)$$

where $Y(k-1) = \{y(0), y(1), \ldots, y(k-1)\}$. Here, $\hat{x}(k)$ represents the minimum variance one-step-ahead prediction of the state $x(k)$, and may be generated by using the well-known Kalman predictor [1, 8]

$$\hat{x}(k+1) = \Phi\hat{x}(k) + \Psi u(k) + \Phi K(k)(y(k) - H\hat{x}(k)) \quad (4)$$

which requires the noise statistics Q and R to be known. The Kalman gain matrix may be computed from the relation:

$$K(k) = P(k)H^T(HP(k)H^T + R)^{-1} \quad (5)$$

Here $R = cov\{v(k)\}$ and the error is $\tilde{x}(k) = x(k) - \hat{x}(k)$, while $P = cov\{\tilde{x}(k)\}$ may be obtained by solving the Riccati equation [1, 8]. Thus, the Riccati equation is required to be solved for calculating the optimal state prediction (4) and the MSEV control signal (3). However, in the next section we will show that in the case of an unknown system, an asymptotic state prediction may be achieved after a direct estimation of the parameters of the steady-state prediction model in observer canonical form, the so-called innovation model [4, 8, 9], without requiring explicit knowledge of the noise covariances Q and R.

3. Adaptive MSEV Controller

Let us consider now the case where all the system parameters of the model (1) are unknown and need to be estimated recursively before the control algorithm may be implemented. To ensure identifiability, we replace the model (1) by the steady state innovations model in observer canonical form [4, 8, 10]:

$$x(k+1) = \Phi x(k) + \Psi u(k) + \overline{\Gamma} e(k);\ y(k) = Hx(k) + e(k) \quad (6)$$

where the matrices are assumed to have the following form:

$$\Phi = \left[\begin{array}{c|c} -a_1 & \\ \vdots & I_{(n-1)\times(n-1)} \\ -a_{n-1} & \\ \hline -a_n & 0 \cdots 0 \end{array}\right];$$

$$H^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix};\ \Psi = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_n \end{bmatrix};\ \overline{\Gamma} = \begin{bmatrix} \overline{\gamma}_1 \\ \vdots \\ \overline{\gamma}_n \end{bmatrix} \quad (7)$$

The problem of parameter estimation of the observer state-space form (6) may be solved by taking advantage of the fact that the model (6) may be equivalently expressed in the form of an n-dimensional autoregressive moving average model with exogenous input, the so-called ARMAX model, given below [4, 8, 10]:

$$A(q^{-1})y(k) = B(q^{-1})u(k) + D(q^{-1})e(k) \quad (8)$$

where q^{-1} is the unit delay operator. The polynomials A, B, and D have the following forms:

$$A(q^{-1}) = 1 + \sum_{i=1}^{n} a_i q^{-1};\ B(q^{-1})$$

$$= \sum_{i=1}^{n} b_i q^{-1};\ D(q^{-1}) = 1 + \sum_{i=1}^{n} d_i q^{-1} \quad (9)$$

The coefficients a_i, b_i, and d_i, $i = 1, \ldots, n$ are easily obtained from the elements of the matrices Ψ and $\overline{\Gamma}$; that is [9]:

$$\psi_i = b_i;\ \overline{\gamma}_i = d_i - a_i;\ i = 1, \ldots, n \quad (10)$$

Moreover, it is possible to express the ARMAX form (9) in the following linear regression form:

$$y(k) = Z^T(k)\Theta(k-1) + e(k) \quad (11)$$

where Θ is the vector of unknown parameters given as:

$$\Theta(k-1) = [a_1, \ldots, a_n, b_1, \ldots, b_n, \overline{\gamma}_1, \ldots, \overline{\gamma}_n]^T \quad (12)$$

and $Z(k)$ is the regression vector containing appropriate set of past outputs, inputs, and innovations:

$$Z(k) = [-y(k-1), \ldots, -y(k-n), u(k-1), \ldots,$$

$$u(k-n), e(k-1), \ldots, e(k-n)]^T \quad (13)$$

Once the regression model (11) is obtained, a recursive estimation of the parameter vector Θ may be achieved using a number of alternative algorithms [4, 10, 11]. Here, we have made use of the pseudo-linear regression algorithm:

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + G(k)\varepsilon(k) \quad (14a)$$

$$\varepsilon(k) = y(k) - Z^T(k)\hat{\Theta}(k-1) \quad (14b)$$

$$G(k) = T(k-1)Z(k)\left[1 + Z^T(k)T(k-1)Z(k)\right]^{-1} \quad (14c)$$

$$T(k) = T(k-1) - T(k-1)Z(k)Z^T(k)T(k-1)$$

$$\left[1 + Z^T(k)T(k-1)Z(k)\right]^{-1} \quad (14d)$$

These equations are initialized with an assumed initial estimate $\hat{\Theta}(0)$ and initial covariance $T(-1)$ and may be expected to converge to the correct parameter values under certain conditions [4, 10]. In this way, the adaptive MSEV controller is obtained easily from (3) using the state prediction from (6), where the unknown system matrices are replaced by their estimates [7]. These estimates may be obtained from (7) and (10), when the unknown parameters in (9) are replaced by their estimates in (14a). It must be pointed out that the innovations sequence $e(k)$ in (13) is usually unmeasurable and consequently must be replaced by its estimate $\varepsilon(k)$ in (14b). Thus, the proposed MSEV controller is analogous in form to the LQG theory-based optimal adaptive controller, but an asymptotic state prediction $\hat{x}(k)$ is achieved after direct estimation of the parameters of the innovations model (6), without requiring explicit knowledge of the noise covariance. On the other hand, the classical LQG approach requires the Riccati equation to be solved for the prediction error covariance $P(k)$, in order to calculate the Kalman gain in (5). This results in a considerable reduction of the computational requirement for the MSEV controller, at the cost of some performance loss, as a single-stage quadratic performance index (2) has been minimized. The major computational requirements of the MSEV controller arise from the parameter estimation algorithm (14), which is central to the implementation of any form of adaptive controller. It is possible to show that the MSEV controller for a known system will be globally asymptotically stable provided the triple (Φ, Ψ, H) is completely controllable and observable, as well as that the weighting matrices W, S, and U are bounded, constant, and satisfy the following conditions: $W \geq 0$, $S \geq 0$, $W - SU^{-1}S^T \geq 0$, and$\Psi^T S \geq 0$ [7].

4. Modified Adaptive MSEV Controller

The MSEV method discussed above may be extended so to design an estimated state feedback controller for nonlinear and nonstationary systems with a time-varying reference signal, in the presence of load disturbances and stochastic disturbances with unknown statistics. The scheme for such implementation of the MSEV controller, named modified MSEV controller, is shown in Fig. 1. Here, $u_{ref}(k)$ and $y_{ref}(k)$ represent a given time-varying deterministic reference, or nominal trajectory; and $u(k)$ and $y(k)$ are deviations, or misalignments, from the nominal signals $u_{ref}(k)$ and $y_{ref}(k)$, respectively. Additionally, $e_m(k)$ is the measurement noise, d is the load disturbance, and $p_e(k)$ is an augmented pseudo-random binary sequence, which has to ensure the convergence of parameter estimation procedure (14). In this way, the signal generated by the controller represents a correction around the nominal, or reference, control signal. So the adopted structure of the control system enables one to track the reference trajectory of a desired form. Furthermore, because the MSEV approach uses an on-line identification of linear system (this system describes a plant in the vicinity of an operating point on the nominal trajectory), it can be also applied to nonlinear systems. This requires the linearization of the nonlinear system model around the properly chosen operating points. The choice of operating points should be done in such a way as to cover characteristic nonlinear regimes of the system concerned. In this way, various linear models will describe behaviour of the system in the vicinity of the operating points. Finally, starting from the fact that linearized model parameter estimation is performed on-line, this approach is also appropriate for control of nonstationary systems, provided that the dynamics of these nonstationarities is negligible compared to the system dynamics.

In practice, reference trajectory is usually obtained either by developing a complex nonlinear model of the system in question or by simulation under some reasonable operating conditions. Here, we shall adopt a numerical approach, based on the strategy of predictive control and optimization under cconstraints (i.e., $u_{min} \leq u_{ref} \leq u_{max}$, where bounds u_{min} and u_{max} have to be determined on the basis of known process properties).

Let y_{ref}^{nom} represent the desired nominal trajectory; then the nominal or reference control signal u_{ref}, at stage k, can be calculated so as to minimize the deflection from the desired nominal trajectory, that is, by solving the minimization problem:

$$\min_{u_{ref}} \left(y_{ref}\{(k+N)T\} - y_{ref}^{nom}\{(k+N)T\} \right)^2 \tag{15}$$

where $y_{ref}\{(k+N)T\}$ denotes the value of y_{ref}–coordinate at the instant $(k+N)T$. The sequence $y_{ref}\{(k+N)T\}$ is obtained by solving the corresponding nonlinear state-space model of the system concerned under the condition that the control signal $u_{ref}(t)$ is constant over N consecutive sampling periods $k, k+1, \ldots, k+N-1$, respectively. Prediction horizon N represents a free parameter, which has to be adopted in advance. The choice of N represents a compromise between two opposite requirements concerning the allowable values and dynamics of the reference control input u_{ref} and the corresponding admissible errors in tracking the prespecified reference trajectory. A smaller value of N will result in the control signal u_{ref} being very close to the prespecified bounds u_{min} and u_{max}, as a consequence of the effort to minimize as fast as possible the deflections from the nominal trajectory (the so-called bang-bang control). On the other hand, higher values of N will result in a control signal u_{ref} with smaller dynamics, which is not influenced too much by the given control bounds. However, this will yield rather large deflections of y_{ref} from the given nominal trajectory y_{ref}^{nom}.

The influence of load disturbance must be also incorporated into the controller design, in order to avoid system performance degradation and bad parameters estimation. The disturbance detection can be based on the changes of innovation sequence statistics. The detector has to be designed so as to prevent its frequent activation under the influence of high-intensity measurement noise realizations e_m. However, most industrial and scientific data contain unavoidable large noise realizations, named outliers, owing to meter and communication errors, incomplete measurements, errors in mathematical models, and the like [9]. Therefore, it is important to design the detector in a robust manner in the sense of its insensitivity to outliers.

This task can be accomplished by extending the adopted ARMAX model in (8) as:

$$A(q^{-1})y(k) = B(q^{-1})u(k) + D(q^{-1})e(k) + d \qquad (16)$$

where d denotes the true unknown value of load disturbance. In order to estimate the value of load disturbance and to suppress its influence, the relations (12) and (13) have to be modified in the following manner:

$$\Theta(k-1) = [a_1, \dots, a_n, b_1, \dots, b_n, \overline{\gamma}_1, \dots, \overline{\gamma}_n, 1]^T \qquad (17)$$

$$Z(k) = [-y(k-1), \dots, -y(k-n),$$

$$u(k-1), \dots, u(k-n), e(k-1), \dots, e(k-n), \hat{d}(k)]^T \qquad (18)$$

and the reference control signal u_{ref} has to be corrected as (see Fig. 1):

$$u^*_{ref}(k) = u_{ref}(k) + \Delta u(k) \qquad (19)$$

The control signal correction $\Delta u(k)$ has to be related to the estimated load disturbance and the model parameters as:

$$\Delta u(k) = -\hat{d}(k-1)/\hat{B}(1) \qquad (20)$$

where the sign ˆ denotes the estimated quantities. Such control signal correction eliminates the influence of disturbance on the steady-state process output (eq. (16)). Additionally, load disturbance estimation can be based on this correction as:

$$\hat{d}(k) = \varphi\left(E\left\{y(k) - H\hat{x}(k) - \Delta u(k)\frac{\hat{B}(1)}{\hat{A}(1)}\right\}\hat{A}(1)\right) \qquad (21)$$

Similarly to (20), the relation (21) is derived from (16) when the average noise e is taken to be zero and the control signal u is replaced by $u + \Delta u$. Here, $\varphi(.)$ is taken to be a dead-zone type nonlinearity with the width B_φ:

$$\varphi(x) = \begin{cases} 0 \; ; \; |x| \leq B_\varphi \\ x \; ; \; |x| > B_\varphi \end{cases} \qquad (22)$$

Finally, as the innovations statistics are not known in practice, the expectation $E\{.\}$ in (21) can be approximated by the arithmetic mean, calculated within the sliding frame of appropriate size I:

$$\hat{d}(k) = \varphi\left(\frac{1}{I}\left\{\sum_{i=0}^{I} y(k-i) - H\hat{x}(k-i) - \Delta u(k)\frac{\hat{B}(1)}{\hat{A}(1)}\right\}\hat{A}(1)\right) \qquad (23)$$

In general, the window size I has to be large enough to reduce the influence of measurement noise, but not so large as to obscure the nonstationarity of the data. Moreover, the detector parameter B_φ has to be chosen in accordance with the a priori knowledge on the load disturbance intensity and outliers variance. The relations (3), (6), (7), (14), and (16)–(23) define the modified MSEV controller. It should be noted that the large signals u_{ref} and y_{ref} cover the nonlinear process properties, and the small signals u and y represent the deviation from the reference trajectory; the latter are obtained by minimizing the criterion (2). Finally, it should be noted that the signal $p_e(k)$ (Fig. 1) has to be chosen according to the process dynamics, in order to provide identifiability condition [4, 10].

5. Experimental Results

The feasibility of the proposed approach has been demonstrated through simulations. The algorithm has been applied to the second-order nonlinear system:

$$\dot{x}_1(t) = -x_1(t)sin(x_1(t)) - 2.5x_2(t)x_1(t) + \tilde{u}(t) \qquad (24)$$

$$\dot{x}_2(t) = x_1(t) - 4x_2(t)cos\{x_1(t)(x_2(t)-1)\} + d \qquad (25)$$

where the system output $\tilde{y}(t)$ is equal to $x_2(t)$. Here d and $\tilde{u}(t)$ represent the load disturbance and the system input, respectively (see Fig. 1). The reference control signal (Fig. 2) is obtained by minimizing the criterion (15) under constraints $u_{min} \leq u_{ref} \leq u_{max}$ ($u_{min} = 0.5$ and $u_{max} = 6.5$), with prediction horizon $N = 3$ and sampling period $T = 0.1s$, where the nominal trajectory is defined as:

$$y^{nom}_{ref}\{kT\} = \begin{cases} 0.50 \; ; \; 0 \leq k \leq 250 \\ 0.25 \; ; \; 250 \leq k \leq 400 \\ 0.75 \; ; \; 400 \leq k \leq 500. \end{cases} \qquad (26)$$

The corresponding reference output signal y_{ref} in (15) (Fig. 3) is computed as the response of the nonlinear system (24) and (25) using Runge-Kutta method of the fifth order, with initial conditions $x_0 = [0.5 \;\; 1]^T$. The prediction horizon N is chosen in accordance to the system dynamics and the nature of nominal trajectory, thus resulting in small deviations between desired y^{nom}_{ref} and obtained reference trajectory y_{ref}. Simulation of the closed-loop system in the presence of additive measurement noise e_m and load disturbance d is performed, where the state weighting matrix W in (2) is adopted as null, and the cross-weighting matrix S in (2) and the input weighting matrix U in (2) has been chosen to be $[0.2 \;\; -0.1]^T$ and 0.5, respectively. The measurement noise is generated as zero-mean, stationary, white Gaussian sequence with variance chosen so as to provide the signal to noise ratio $SNR = 5dB$.

The order n of the linearized system model (16) represents a free parameter that has to be adopted in advance. In general, the choice of n is a nontrivial problem that requires a trade-off between good description of the data and model complexity. The basic approach is to compare the performance of models of different orders and to test whether the higher order model is worthwhile. Figs. 4–12 illustrate the MSEV control system performance for different values of n. It should be noted that the true order of the system (24) and (25) is two. Obviously, there is a penalty for going beyond the true order (see Figs. 4–6). Specifically, in the case of $n = 1$ the unmodelled dynamics may result in an inadequate estimation of the load disturbance in (23) (Fig. 6), which in turn will produce a corresponding control signal correction (20), such that the tracking of nominal trajectory (26) will be unsatisfactory (Fig. 5). It can be seen from Fig. 5 that the load disturbance was detected and eliminated properly during the first period of its duration (time index $k \in (100, 200)$), whereas during the second period of its existence ($k \in (270, 350)$) it was detected but its magnitude was estimated wrongly. One can also conclude from Fig. 5 that the process output will be very close to the desired one when there is no influence of load disturbance, even though the order of the system model used in the algorithm is less than the true one. This can be explained by the fact that the proposed algorithm estimates the steady-state gain efficiently, as well as that the desired output is a piece-wise linear function. When the model order n is chosen properly, the control system performs quite well (Figs. 7–9), and the sensitivity of the algorithm to the load disturbance is decreased significantly: the influence of load disturbance exists only at the begining and the end of the interval of its duration. Furthermore, the time needed for the load disturbance detection can be shortened by tuning the window size I in (23). owever,too small a value of I could make the algorithm rather sensitive to the measurement noise. This will be also the case when the model order underestimates the true one (Figs. 10–12). In the last case, the estimated coefficients (14) of the linearized model (16) will result in the pole-zero cancellations within the corresponding transfer function, and consequently the system dynamics will be approximated adequately. Moreover, we note that the most significant deviations from the reference trajectory are related to the initial phase, that is, during the presence of persistent excitation signal (see Fig. 1).

The control system performance in the presence of outliers contaminating the normal observations is depicted in Figs. 13–15. Such a measurement sequence is generated from the heavy-tailed distribution $0.9N(0, r_1) + 0.1N(0, r_2)$, where $N(0, r_1)$ is the zero-mean normal distribution with the variance r_1 chosen as before, and $r_2 = 10r_1$. With respect to the simulation results, it can be concluded that the detector performs well in the presence of outliers if the detector parameter B_φ is adjusted properly. Simulations have found that the sliding frames $I = 7$ and detector parameter $B_\varphi = 0.004$ give satisfactory results for the problem concerned. The robustness property of the proposed algorithm can be explained by the fact that (22) and (23) define some kind of robust influence function that reduces the influence of spiky noise realizations, that is, outliers [9]. It should be noted that such a type of influence function cannot be used efficiently in the presence of patchy outliers in the measurement data. In the last case, we need to modify the form of the influence function using the robust estimation theory [9].

6. Conclusion

A form of self-tuning controller, the modified minimum state error variance controller, has been proposed. In contrast to the MSEV approach known from the literature, the proposed modifications enable tracking of a time-varying reference trajectory, as well as the load disturbance rejection. A way of generating the reference trajectory by using numerical optimization has also been proposed. Furthermore, a special type of nonlinear detector, which provides for load disturbance detection and estimation, together with the reduction of outliers influence, has been derived. Once the disturbance detection and its magnitude estimation are finished, the control signal correction is performed in order to eliminate the disturbance influence on the control system performances. The possibility of applying the modified MSEV approach for nonlinear and nonstationary plants control is also analyzed. The feasibility of the proposed approach for such applications is demonstrated through simulations. Special emphasis is devoted to the choice of the linearized order model and the disturbance detector parameters, in order to suppress the influence of unmodelled dynamics and outliers contaminating the measurement data. The obtained results have shown that the proposed controller may be an efficient tool for tracking a prespecified reference trajectory in the case of nonlinear and nonstationary system dynamics, as well as in the presence of load disturbance and stochastic disturbances with unknown statistics.

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Figure 1. The modified adaptive MSEV controller for non-zero reference in the presence of load d and stochastic disturbances e_m.

Figure 2. Reference control signal u_{ref} generated by numerical optimization (15).

Figure 3. Reference trajectory y_{ref} corresponding to u_{ref} and the nominal reference trajectory (26).

Figure 4. Control signal u for adopted first-order model (16).

Figure 5. Process output y for adopted first-order model (16).

Figure 6. Load disturbance d and its estimate $\hat{d}$ for adopted first-order model (16).

Figure 7. Control signal u for adopted second-order model (16).

Figure 8. Process output y for adopted second-order model (16).

Figure 9. Load disturbance d and its estimate $\hat{d}$ for adopted second-order model (16).

Figure 10. Control signal u for adopted third-order model (16).

Figure 11. Process output y for adopted third-order model (16).

Figure 12. Load disturbance d and its estimate $\hat{d}$ for adopted third-order model (16).

Figure 13. Control signal u in the presence of outliers for adopted second-order model (16).

Figure 14. Process output y in the presence of outliers for adopted second-order model (16).

Figure 15. Load disturbance d and its estimate $\hat{d}$ in the presence of outliers for adopted second-order model (16).