On Existence of Equilibria, Voltage Balancing, and Current Sharing in Consensus-Based DC Microgrids*

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Abstract— **This work presents new secondary regulators for current sharing and voltage balancing in DC microgrids, composed of distributed generation units, dynamic RLC lines, and nonlinear ZIP (constant impedance, constant current, and constant power) loads. The proposed controllers sit atop a primary voltage control layer, and utilize information exchanged over a communication network to take necessary control actions. We deduce sufficient conditions for the existence and uniqueness of an equilibrium point, and show that the desired objectives are attained in steady state. Our control design only requires the knowledge of local parameters of the generation units, facilitating plug-and-play operations. We provide a voltage stability analysis, and illustrate the performance and robustness of our designs via simulations. All results hold for arbitrary, albeit connected, microgrid and communication network topologies.**

I. INTRODUCTION

Microgrids (mGs) are electric networks comprising distributed generation units (DGUs), storage devices, and loads. Apart from their manifold advantages like integration of renewables, enhanced power quality, reduced transmission losses, capability to operate in grid-connected and islanded modes, they are compatible with both AC and DC operating standards [1]. In particular, DC microgrids (DCmGs), have gained traction in recent times. Their rising popularity can be attributed to development of efficient converters, natural interface with renewable energy sources (for instance PV modules) and batteries, and availability of electronic loads (various appliances, LEDs, electric vehicles, computers etc) inherently DC in nature [2], [3].

In islanded DCmGs, voltage stability is crucial, for without it voltages may breach a critical level and damage connected loads [3]. Thus, a primary voltage control layer is often employed to track desired voltage references at points of coupling (PCs), whereby DGUs are connected to the DCmG. To this aim, several approaches, for example, based on droop control [2], [4] and plug-and-play control [5], [6], have been proposed in the literature. Besides voltage stability, another desirable objective is to ensure current sharing, that is, DGUs must share mG loads in accordance with their current ratings. Indeed, in its absence, unregulated currents may overload generators and eventually lead to an mG failure. An additional goal of voltage balancing, requiring boundedness of weighted sum of PC voltages, is often sought to complement current sharing [7]. Primary controllers, however, are blind voltage reference emulators and are unable to attain the aforementioned objectives by themselves. Higher-level secondary control architectures are therefore necessary to coordinate the voltage references provided to the primary layer.

Consensus-based secondary regulators guaranteeing current sharing and voltage balancing have been the subject of many recent contributions. Centralized design approaches are proposed in [8], [9], but are prohibitive for large-scale mGs as they require particulars of mG topology, lines, loads, and DGUs. With the aim of overcoming this issue, scalable design procedures [5], [6] have been proposed, which enable the synthesis of decentralized controllers and plug -in/-out of DGUs on the fly without spoiling the overall stability of the network. Distributed consensus-based controllers for DCmGs with generic topologies, discussed in [7], [10], remedy the limitations of centralized design schemes, but presume static lines and abstract DGUs as ideal voltage generators or first-order systems. Efforts to take into account DGU dynamics and RL lines have been made in [11], [12]. In [12], a robust distributed control algorithm is proposed considering both objectives; however, a suitable initialization of the controller is needed. The resistance of the DGU filter is neglected in [11] and hence, voltage balancing cannot be guaranteed in steady state. In addition, the above-mentioned contributions [7], [10], [11], [12] are limited to linear loads. A power consensus algorithm with ZIP loads is studied in [13] under simplified DCmG dynamics and assumptions on existence of a suitable steady state.

A. Paper Contributions

In this paper, we build upon our previous theoretical contributions on primary voltage control [5], and introduce a distributed secondary control layer for proportional current sharing and weighted voltage balancing in DCmGs consisting of DGUs, loads, and interconnecting power lines.

The main technical novelties of this paper are fourfold. First, this work does away with the modeling limitations of several existing contributions. In addition to dynamic RLC lines and nonlinear ZIP loads, we consider DGUs interfaced with DC-DC Buck converters and incorporate complete converter dynamics along with filter resistances. Second, we propose a new consensus-based secondary control scheme relying on the exchange of variables (obtained from DGU filter currents) with nearest communication neighbors. These secondary regulators appropriately modify primary voltage

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references to attain the desired goals. In spite of their distributed structure, the control design is completely decentralized allowing for plug-and-play operations. Third, we throughly investigate the steady-state behavior of the DCmG under secondary control, and deduce sufficient conditions on the existence and uniqueness of an equilibrium point meeting secondary goals. Such an analysis is not trivial due to the nonlinearities introduced by the ZIP loads and entails finding solutions of the DC power-flow equations under consensus constraints on a hyperplane. To the best of our knowledge, this has not been addressed in the literature [14], [15] before. Finally, we present conditions on the controller gains and power consumption of P loads guaranteeing voltage stability of the closed-loop DCmG, and show that stability is independent of DCmG and communication topologies.

The remainder of Section I introduces relevant preliminaries and notation. Section II recaps the DCmG model and primary voltage control. Section III houses our main contributions. It presents consensus-based secondary controllers, and details the steady-state behavior and stability of the closed-loop DCmG in the presence of ZIP loads. Simulations validating theoretical results are provided in Section IV. Finally, conclusions are drawn in Section V.

B. Preliminaries and notation

Sets, vectors, and functions: We let $\mathbb{R}$ (resp. $\mathbb{R}_{>0}$) denote the set of real (resp. strictly positive real) numbers. For a finite set $\mathcal{V}$, let $|\mathcal{V}|$ denote its cardinality. Given $x \in \mathbb{R}^n$, $[x] \in \mathbb{R}^{n\times n}$ is the associated diagonal matrix with x on the diagonal. The inequality $x \leq y$ is component-wise, that is, $x_i \leq y_i,\ \forall i \in 1, ..., n$. Throughout, $\mathbf{1}_n$ and $\mathbf{0}_n$ are the n-dimensional vectors of unit and zero entries, and $\mathbf{0}$ is a matrix of all zeros of appropriate dimensions. The average of a vector $v \in \mathbb{R}^n$ is $\langle v\rangle = \frac{1}{n}\sum_{i=1}^{n} v_i$. We denote with H^1 the subspace composed by all vectors with zero average i.e. $H^1 = \{v \in \mathbb{R}^n : \langle v\rangle = 0\}$. For the matrix $A \in \mathbb{R}^{m\times n}$, $A^\dagger \in \mathbb{R}^{n\times m}$ denotes its pseudo inverse whereas its range and null spaces are indicated by $\mathcal{R}(A)$ and $\mathcal{N}(A)$, respectively.

Algebraic graph theory: We denote by $\mathcal{G}(\mathcal{V}, \mathcal{E}, W)$ an undirected graph, where $\mathcal{V}$ is the node set and $\mathcal{E} = (\mathcal{V} \times \mathcal{V})$ is the edge set. If a number $l \in \{1, ..., |\mathcal{E}|\}$ and an arbitrary direction are assigned to each edge, the incidence matrix $B \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{E}|}$ has non-zero components: $B_{il} = 1$ if node i is the sink node of edge l, and $B_{il} = -1$ if node j is the source node of edge l. The *Kirchoff's Current Law* (KCL) can be represented as $x = B\xi$, where $x \in \mathbb{R}^{|\mathcal{V}|}$ and $\xi \in \mathbb{R}^{|\mathcal{E}|}$ respectively represent the nodal injections and edge flows. Assume that the edge $l \in \{1, ..., |\mathcal{E}|\}$ is oriented from i to j, then for any vector $V \in \mathbb{R}^{|\mathcal{V}|}$, $(B^TV)_l = V_i - V_j$. The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is $\mathcal{L} = BWB^T$. If the graph is connected, then $\mathcal{R}(\mathcal{L}) = H^1$.

II. DCmG MODEL AND PRIMARY VOLTAGE CONTROL

In this section, we start by reviewing our DCmG model [5], [6] comprising multiple DGUs interconnected with each other via power lines, and recall the concepts of primary voltage control.

DCmG Model: The DCmG is modeled as an undirected connected graph $\mathcal{G}_e = (\mathcal{D}, \mathcal{E})$, where $\mathcal{D} = \{1, \dots, N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{D} \times \mathcal{D}$ the edge set. Each node of the DCmG is connected to a DGU and a load, and forms the i^{th} PC. The interconnecting power lines are represented by the edges of $\mathcal{G}_e$. On assigning a number to each line, one can equivalently express $\mathcal{E} = \{1, \dots, M\}$ with M denoting the total number of lines. Note that edge directions are assigned arbitrarily, and provide a reference system for positive currents. We refer the reader to Figure 1 for a representative diagram of the DCmG.

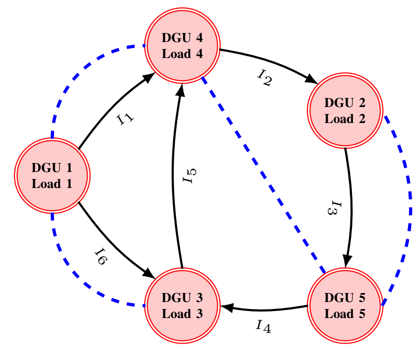

Fig. 1: A representative diagram of the DCmG with the communication network appearing in dashed blue.

Dynamic model of a power line: The power lines are modeled after the π-equivalent model of transmission lines [16]. It is assumed that the line capacitances are lumped with the DGU filter capacitance (capacitor C_{ti} in Figure 2). Therefore, as shown in Figure 2, the power line l is modeled as a RL circuit with resistance $R_l > 0$ and inductance $L_l > 0$. By applying Kirchoff's voltage law (KVL) on the l^{th} line, one obtains

$$\Sigma_{[l]}^{Line} : \left\{ \frac{dI_l}{dt} = -\frac{R_l}{L_l} I_l + \frac{1}{L_l} \sum_{i\in\mathcal{N}_l} B_{il} V_i \right. , \qquad (1)$$

where the variables V_i and I_l represent the voltage at PC_i and the current flowing through the l^{th} line, respectively.

Dynamic model of a DGU: The DGU comprises a DC voltage source (usually generated by a renewable resource), a Buck converter, and a series RLC filter. The i^{th} DGU feeds a local load at PCC_i and is connected to other DGUs through power lines. A schematic electric diagram of the i^{th} DGU along with load, connecting line(s), loads, and local PnP voltage controller is represented in Figure 2. On applying KCL and KVL on the DGU side at PC_i, we obtain

$$\Sigma_{[i]}^{DGU} : \begin{cases} C_{ti}\dfrac{dV_i}{dt} = I_{ti} - I_{Li}(V_i) - \displaystyle\sum_{l\in\mathcal{N}_i} B_{il} I_l \\ L_{ti}\dfrac{dI_{ti}}{dt} = -V_i - R_{ti}I_{ti} + V_{ti} \end{cases} , \qquad (2)$$

where V_{ti} is the command to the Buck converter and I_{ti} is the filter (generator) current. The terms $R_{ti} \in \mathbb{R}_{>0}$, $L_{ti} \in \mathbb{R}_{>0}$, and $C_{ti} \in \mathbb{R}_{>0}$ are the internal resistance, capacitance (lumped with the line capacitances), and inductance of the DGU converter. Each of these DGUs is equipped with local voltage regulators, which forms the *primary control layer*.

The main objective these controllers is to ensure that the voltage at each DGU's PC tracks a reference voltage $V_{ref,i}$ (modified by the *secondary controller*; see Section III for more details). For this purpose, as in [5], [6], we augment each DGU with a multivariable PI regulator

$$\dot{v}_i = e_{[i]} = V_{ref,i} - V_i, \tag{3a}$$

$$\mathcal{C}_{[i]}: \; V_{ti} = K_{[i]}\hat{x}_{[i]},, \tag{3b}$$

where $\hat{x}_{[i]} = [V_i \; I_{ti} \; v_i]^T \in \mathbb{R}^3$ is the state of augmented DGU and $K_{[i]} = [k_{1,i} \; k_{2,i} \; k_{3,i}] \in \mathbb{R}^{1\times 3}$ is the feedback gain. From (2)-(3b), the closed-loop DGU model is obtained as

$$\hat{\Sigma}^{DGU}_{[i]}: \begin{cases} \dfrac{dV_i}{dt} = \dfrac{1}{C_{ti}}I_{ti} - \dfrac{1}{C_{ti}}I_{Li}(V_i) - \dfrac{1}{C_{ti}}\sum_{l\in\mathcal{N}_i} B_{il}I_l \\ \dfrac{dI_{ti}}{dt} = \alpha_i V_i + \beta_i I_{ti} + \gamma_i v_i \\ \dfrac{dv_i}{dt} = -V_i + V_{ref,i} \end{cases}, \tag{4}$$

where

$$\alpha_i = \frac{(k_{1,i}-1)}{L_{ti}}, \; \beta_i = \frac{(k_{2,i}-R_{ti})}{L_{ti}}, \; \gamma_i = \frac{k_{3,i}}{L_{ti}}. \tag{5}$$

We note that the control architecture is decentralized since the computation of V_{ti} uniquely requires the state of $\hat{\Sigma}^{DGU}_{[i]}$.

Load model: In this work, we consider the standard ZIP model [5]. The parallel combination of these three loads is given as

$$I_{Li}(V_i) = \underbrace{Y_{Li}V_i}_{Z} + \underbrace{\bar{I}_{Li}}_{I} + \underbrace{V_i^{-1}P^*_{Li}}_{P}. \tag{6}$$

Assumption 1: The reference signals $V_{ref,i}$ and PC voltages V_i are strictly positive for all $t \geq 0$.

We remark that Assumption 1 is not a limitation, and rather reflects a common constraint in microgrid operation. Indeed, negative PCC voltages reverse the role of loads and make them power generators.

III. Secondary Control in DCmGs

A. Problem formulation

The primary control layer is designed to track a suitable reference voltage $V_{ref,i}$ at the PC_i. As such, they do not ensure current sharing and voltage balancing, defined as follows.

Definition 1: **(Current sharing [7], [10]).** The load is said to be shared proportionally among DGUs if

$$\frac{I_{ti}}{I^s_{ti}} = \frac{I_{tj}}{I^s_{tj}} \quad \text{for all } i,j \in \mathcal{V}, \tag{7}$$

where $I^s_{ti} > 0$ is the rated current of DGU_i.

Currents sharing ensures proportional sharing of loads amongst multiple DGUs. This avoids situations of DGU overloading and prevents harm to the converter modules. As shown in the subsequent sections, in order to attain current sharing, the steady state voltages are not equal to $V_{ref,i}$. It is, however, desirable that PC voltages remain close to the nominal reference voltages for normal operation of the DCmG. To this aim, we state the objective of weighted voltage balancing in the following definition.

Definition 2: **(Weighted voltage balancing [12]).** The voltages are said to be balanced in the steady state if

$$\langle [I^s_t]V \rangle = \langle [I^s_t]V_{ref} \rangle. \tag{8}$$

Voltage balancing implies that the weighted sum of PC voltages is equal to the the weighted sum of voltage references, ensuring boundedness of DCmG voltages. As noticed in [10], in its absence, the PC voltages may experience drifts and increase monotonically despite the filter currents' being shared proportionally.

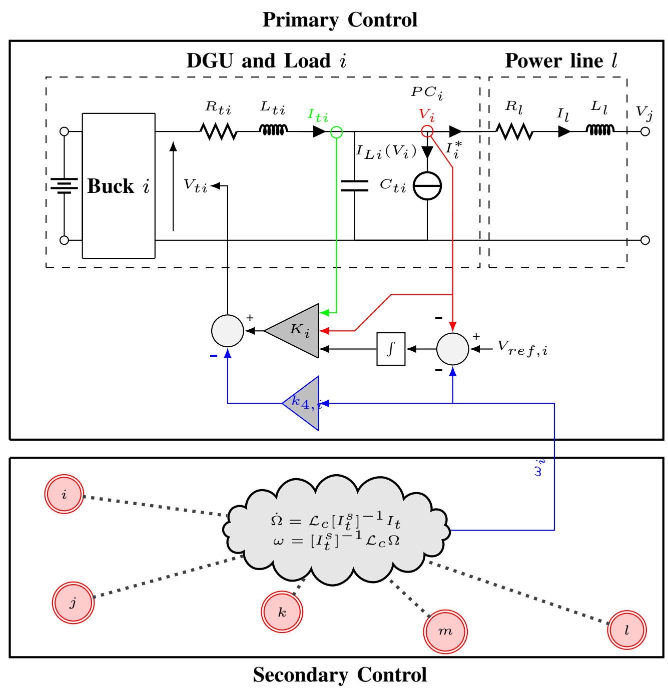

Fig. 2: Schematic diagram showing primary and secondary control layers of the DCmG. Note that the topology of the communication network is not shown.

B. Consensus-based secondary control

In order to achieve the previously stated objectives, we use a consensus-based secondary control layer. Consensus filters are commonly employed for achieving global information sharing or coordination through distributed computations [17]. In our case, we propose the following consensus scheme

$$\dot{\Omega}_i = \sum_{j=1, j\neq i}^{N} a_{ij}\left(\frac{I_{ti}}{I^s_{ti}} - \frac{I_{tj}}{I^s_{tj}}\right), \tag{9}$$

where $a_{ij} > 0$ if DGUs i and j are connected by a communication link ($a_{ij} = 0$, otherwise). The corresponding *communication graph* (see Figure 1), assumed to be undirected and connected, is $\mathcal{G}_c = (\mathcal{D}, \mathcal{E}_c, W_c)$ where $(i,j) \in \mathcal{E}_c \iff a_{ij} > 0$ and $W_c = \text{diag}\{a_{ij}\}$. Note that the topologies of $\mathcal{G}_c$ and $\mathcal{G}_e$ can be completely different. As shown in Figure 2, the consensus variable

$$\omega_i = \frac{1}{I_{ti}^s} \sum_{j=1, j\neq i}^{N} a_{ij} (\Omega_i - \Omega_j) \quad (10)$$

modifies the primary voltage controllers (3a) and (3b) as follows

$$\dot{v}_i = V_{ref,i} - V_i - \omega_i \quad (11a)$$
$$V_{ti}(t) = K_{[i]}\hat{x}_{[i]} - k_{4,i}\omega_i, \quad (11b)$$

where $k_{4,i} \in \mathbb{R}$. Consequently, using equations (11a) and (11b), one obtains the modified DGU dynamics as

$$\hat{\Sigma}_{[i]}^{DGU} : \begin{cases} \dfrac{dV_i}{dt} = \dfrac{1}{C_{ti}} I_{ti} - \dfrac{1}{C_{ti}} I_{Li}(V_i) - \dfrac{1}{C_{ti}} \sum_{l\in\mathcal{N}_i} B_{il} I_l \\ \dfrac{dI_{ti}}{dt} = \alpha_i V_i + \beta_i I_{ti} + \gamma_i v_i + \delta_i \omega_i \\ \dfrac{dv_i}{dt} = -V_i + V_{ref,i} - \omega_i \end{cases}, \quad (12)$$

with

$$\delta_i = \frac{k_{4,i}}{L_{ti}}. \quad (13)$$

The complete dynamics of the DCmG under primary and secondary control are given by (1) along with (9)-(13). These equations can compactly be rewritten as

$$\dot{X} = \mathcal{A}X + \mathcal{B}(V), \quad (14)$$

where $X = \begin{bmatrix} V^T & I_t^T & v^T & I^T & \Omega^T \end{bmatrix}^T \in \mathbb{R}^{4N+M}$,

$$\mathcal{A} = \underbrace{\begin{bmatrix} -C_t^{-1}Y_L & C_t^{-1} & \mathbf{0} & -C_t^{-1}B & \mathbf{0} \\ [\alpha] & [\beta] & [\gamma] & \mathbf{0} & [\delta][I_t^s]^{-1}\mathcal{L}_c \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -[I_t^s]^{-1}\mathcal{L}_c \\ L^{-1}B^T & \mathbf{0} & \mathbf{0} & -L^{-1}R & \mathbf{0} \\ \mathbf{0} & \mathcal{L}_c[I_t^s]^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\mathcal{A}\in\mathbb{R}^{(4N+M)\times(4N+M)}},$$

and

$$\mathcal{B}(V) = \underbrace{\begin{bmatrix} -C_t^{-1}(\bar{I}_L + [V^{-1}]P_L^*) \\ \mathbf{0}_N \\ V_{ref} \\ \mathbf{0}_M \\ \mathbf{0}_N \end{bmatrix}}_{\mathcal{B}(V)\in\mathbb{R}^{(4N+M)}}.$$

Note that $V \in \mathbb{R}^N$, $V_{ref} \in \mathbb{R}^N$, $I_t \in \mathbb{R}^N$, $v \in \mathbb{R}^N$, $I \in \mathbb{R}^M$, $P_L^* \in \mathbb{R}^N$, $\bar{I}_L \in \mathbb{R}^N$, $\alpha \in \mathbb{R}^{\mathrm{N}}$, $\beta \in \mathbb{R}^{\mathrm{N}}$, $\gamma \in \mathbb{R}^{\mathrm{N}}$ are vectors of PC voltages, reference voltages, filter currents, integrator states, line currents, load powers, load currents, and parameters α_i, β_i, γ_i, respectively. The matrices $R \in \mathbb{R}_{>0}^{M\times M}$, $L \in \mathbb{R}_{>0}^{M\times M}$, $Y_L \in \mathbb{R}_{>0}^{N\times N}$, and $C_t \in \mathbb{R}_{>0}^{N\times N}$ are diagonal matrices collecting electrical parameters R_l, L_l, Y_{Li}, and C_{ti}, respectively. The matrix $B \in \mathbb{R}^{N\times M}$ is the incidence matrix of the electrical network and $\mathcal{L}_c \in \mathbb{R}^{N\times N}$ is the Laplacian matrix of the communication network.

C. Analysis of Equilibria

Prior to analyzing the stability of the closed-loop system (14), it is pertinent to establish that an equilibrium exists such that both the objectives (7) and (8) are jointly attained. We emphasize that, in a primary-controlled DCmG, a reference voltage $V_{ref,i}$ is directly enforced at the i^{th} PC. Thus, a unique equilibrium point always exists; see [5]. On the contrary, once the secondary layer is activated, the voltage references are tweaked by ω_i (see (11a)), governed by equations (9) and (10). Since the presence of ZIP loads renders the DCmG dynamics nonlinear, it may occur that an equilibrium point fails to exist (see Section IV for a simulation example). Hence, in this section, we pursue whether the closed-loop system (14) possesses an equilibrium point, and if so, under what conditions on loads, topology of electrical and communication networks, and controller gains.

Lemma 1: **(Steady-state behavior of the DCmG).** Consider the DCmG dynamics (14). The following statements hold:

1) In steady state, the objectives (7) and (8) are attained;
2) A steady-state solution $\bar{X} = [\bar{V}^T, \bar{I}_t^T, \bar{v}^T, \bar{I}^T, \bar{\Omega}^T]^T$ exists only if $[\gamma_i]$ is invertible and there exists a $\bar{V}$ concurrently satisfying the following equations

$$\mathcal{L}_e\bar{V} + \mathcal{L}_t[I_t^s]^{-1}([\bar{V}^{-1}]P_L^* + \bar{I}_L + Y_L\bar{V}) = 0, \quad (15a)$$
$$\mathbf{1}_N^T[I_t^s]\bar{V} = \mathbf{1}_N^T[I_t^s]V_{ref}, \quad (15b)$$

where $\mathcal{L}_t = [I_t^s] - (\mathbf{1}_N^T[I_t^s]\mathbf{1}_N)^{-1}[I_t^s]\mathbf{1}_N\mathbf{1}_N^T[I_t^s]$, and $\mathcal{L}_e = BR^{-1}B^T$ is the Laplacian of the electric network.

Proof: Any steady state solution of (14) satisfies

$$-Y_L\bar{V} - \bar{I}_L - [\bar{V}^{-1}]P_L^* + \bar{I}_t - B\bar{I} = 0, \quad (16a)$$
$$[\alpha]\bar{V} + [\beta]\bar{I}_t + [\gamma]\bar{v} + [\delta][I_t^s]^{-1}\mathcal{L}_c\bar{\Omega} = 0, \quad (16b)$$
$$V_{ref} - \bar{V} - [I_t^s]^{-1}\mathcal{L}_c\bar{\Omega} = 0, \quad (16c)$$
$$B^T\bar{V} - R\bar{I} = 0, \quad (16d)$$
$$\mathcal{L}_c[I_t^s]^{-1}\bar{I}_t = 0. \quad (16e)$$

One has from (16e) that $\bar{I}_t = \epsilon[I_t^s]\mathbf{1}_N$ for some $\epsilon \in \mathbb{R}$, warranting the attainment of (7). Since $\mathbf{1}_N^T B = \mathbf{0}_M$, (16a) implies that $\mathbf{1}_N^T\bar{I}_t = \mathbf{1}_N^T(Y_L\bar{V} + \bar{I}_L + [\bar{V}^{-1}]P_L^*)$, then $\epsilon = (\mathbf{1}_N^T[I_t^s]\mathbf{1}_N)^{-1}\mathbf{1}_N^T(Y_L\bar{V} + \bar{I}_L + [\bar{V}^{-1}]P_L^*)$. We can equivalently represent

$$\bar{I}_t = (\mathbf{1}_N^T[I_t^s]\mathbf{1}_N)^{-1}[I_t^s]\mathbf{1}_N\mathbf{1}_N^T(Y_L\bar{V} + \bar{I}_L + [\bar{V}^{-1}]P_L^*). \quad (17)$$

Using (16d),

$$\bar{I} = R^{-1}B^T\bar{V}. \quad (18)$$

On substituting (17) and (18) into (16a), one obtains (15a). Moreover, for an $\bar{\Omega}$ to exist such that (16c) holds, $[I_t^s](V_{ref} - \bar{V}) \in H^1$, which yields (15b) and guarantees (8) in steady state. If there exists a $\bar{V}$ solving (15), $\bar{I}_t$ and $\bar{I}$ exist due to (17) and (18), respectively. As (15b) holds, from (16c), an equilibrium vector $\bar{\Omega} = \mathcal{L}_c^\dagger[I_t^s](V_{ref} - \bar{V}) + \eta\mathbf{1}_N, \eta \in \mathbb{R}$ exists. Finally, on substituting $\bar{V}, \bar{I}_t, \bar{I}$, and $\bar{\Omega}$ into (16b), one has $\bar{v} = [\gamma]^{-1}\left(([\alpha] + [\delta])\bar{V} - [\delta]V_{ref} + [\beta]\bar{I}_t\right)$. ■

Remark 1: **(Solvability of** (15)**).** We remark that (15a) represents the DC power-flow equations when DGU currents are shared proportionally. The existence and uniqueness of solutions of the power-flow equations have been tackled in [15], [18]. As shown in what follows, the tools therein cannot be applied directly to ascertain the solvability of (15) as (15b) restricts the voltage solutions on a hyperplane.

To analyze the existence of a voltage solution to (15), we rewrite it as

$$\tilde{\mathcal{L}}V = \tilde{I} - \tilde{\mathcal{L}}_t[V^{-1}]P_L^*, \tag{19}$$

where $\tilde{\mathcal{L}} = \begin{bmatrix} \mathcal{L}_p \\ \mathbf{1}_N^T[I_t^s] \end{bmatrix}$, $\mathcal{L}_p = \mathcal{L}_e + \mathcal{L}_t[I_t^s]^{-1}Y_L$, $\tilde{I} = \begin{bmatrix} -\mathcal{L}_t[I_t^s]^{-1}\bar{I}_L \\ \mathbf{1}_N^T[I_t^s]V_{ref} \end{bmatrix}$, and $\tilde{\mathcal{L}}_t = \begin{bmatrix} \mathcal{L}_t[I_t^s]^{-1} \\ \mathbf{0} \end{bmatrix}$.

Theorem 1: **(Existence and uniqueness of a voltage solution).** Consider (19) along with the vector $V^* = \tilde{\mathcal{L}}^\dagger \tilde{I}$. Assume that $[V^*]$ is invertible and define $P_{cri} = 4[V^*]^{-1}\tilde{\mathcal{L}}^\dagger\tilde{\mathcal{L}}_t[V^*]^{-1}$. Assume that the network parameters and loads satisfy

$$\Delta = ||P_{cri}P_L^*||_\infty < 1, \tag{20}$$

and define the percentage deviations $\delta_- \in [0, \frac{1}{2})$ and $\delta_+ \in (\frac{1}{2}, 1]$ as the unique solutions of $\Delta = 4\delta_\pm(1-\delta_\pm)$. The following statements hold:

1) There exists a unique voltage solution $V \in \mathcal{H}(\delta_-)$ of (19), where

$$\mathcal{H}(\delta_-) := \{V \in \mathbb{R}^N | (1-\delta_-)V^* \leq V \leq (1+\delta_-)V^*\}. \tag{21}$$

Moreover, there exist no solutions of (19) in the open set

$$\mathcal{I} := \{V \in \mathbb{R}^N | (V > (1-\delta_+)V^* \text{ and } V \notin \mathcal{H}(\delta_-)\}; \tag{22}$$

2) For $P_L^* = 0$, V^* is the unique solution of (19);
3) If $(1-\delta_+)V^* < V_{ref}$, then, there exist no solutions of (19) in the closed set

$$\mathcal{J} := \{V \in \mathbb{R}^N | (V \leq (1-\delta_+)V^*\}. \tag{23}$$

Proof: For the sake of brevity, the proof is omitted, and can be found in [19]. ■

Remark 2: Under the sufficient conditions provided in Theorem 1, the existence of an equilibrium point depends upon the critical power matrix P_{cri} and the power absorption P_L^*. Clearly, from (19), the communication network topology $\mathcal{G}_c$ has no impact on P_{cri}.

Hereafter, in order to be in line with Assumption 1, we assume that V^* is positive for $V_{ref} > 0$.

D. Stability of the DCmG network

In this section, we aim to study the stability of the closed-loop system (14), necessary for the DCmG to exhibit the desired steady-state behavior described in Section III-C.

Theorem 2: **(Stability of the closed-loop DCmG).** Consider the closed-loop system (14) resulting from equations (9)-(12), along with Assumption 1. For $i \in \mathcal{D}$, if the feedback gains $k_{1,i}$, $k_{2,i}$, and $k_{3,i}$ belong to the set

$$\mathcal{Z}_{[i]} = \left\{ \begin{array}{l} k_{1,i} < 1, \\ k_{2,i} < R_{ti}, \\ 0 < k_{3,i} < \frac{1}{L_{ti}}(k_{1,i}-1)(k_{2,i}-R_{ti}) \end{array} \right\}, \tag{24}$$

$k_{4,i} = k_{1,i}$ and the Z and P components of (6) verify

$$P_{Li}^* < Y_{Li}\bar{V}_i^2, \tag{25}$$

then the equilibrium point $\bar{X}$ is locally asymptotically stable, and is globally asymptotically stable when $\bar{P}_L^* = 0$.

Proof: The proof is provided in [19] and is skipped due to space constraints. ■

Remark 3: **(Compatibility with primary control and behavior under communication collapse).** Equations (1) and (4) represent the DCmG under primary control when the secondary layer is inactive. As shown in [5], (24) and (25) also make it possible to design stabilizing primary controllers. This enables us to reach the following conclusions: (i) the design of the proposed secondary controllers is fully compatible with the primary layer, and solely requires setting an additional control gain $k_{4,i} = k_{1,i} \in \mathcal{Z}_{[i]}$ once activated; (ii) if the DCmG undergoes a communication collapse, the primary controllers maintain voltage stability without any human intervention and force each PC to track $V_{ref,i}$ in steady state.

IV. Simulation results

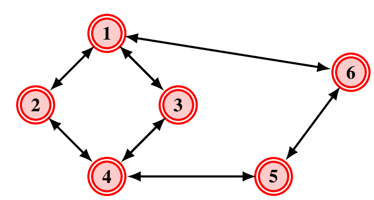

Fig. 3: Simplified DCmG composed of 6 DGUs.

In this section, we aim to demonstrate the capability of the proposed control scheme to guarantee current sharing and voltage balancing. We consider a meshed DCmG composed of 6 DGUs (see Figure 3) with non-identical electrical parameters, adopted from [6]. In our simulations, we assume ZIP loads with powers P_{Li}^*, $i = 1, \ldots, 6$ always fulfilling (25). The PC voltage references $V_{ref,i}$, $i = 1, \ldots, 6$ are set close to 50V and to slightly different values. We highlight that, the control gains $k_{1,i}$, $k_{2,i}$, and $k_{3,i}$ belong to the set $\mathcal{Z}_{[i]}$ defined in (24). For the considered network, the variable Δ is much smaller than 1, guaranteeing the existence of a voltage solution to (19). In the following discussion, we evaluate voltage balancing and current sharing in the DCmG when DGUs are plugged-in and loads are arbitrarily changed.

Plug-in of all DGUs: At time $t < 0$, all the DGUs are isolated and only the primary voltage regulators, designed as in [5], are active tracking $V_{ref,i}$ at their respective PCCs. At time $t = 0$, all the DGUs are connected to from the DCmG shown in Figure 3 and the secondary control layer is activated. The control gain $k_{4,i}$ is set equal to $k_{1,i}$ for all DGUs, and no other control gain is modified. As shown in Figure 4b, the DGU currents are shared proportional to their capacity. This is achieved by automatically adjusting the reference voltages at PC (see Fig 4a). Note that the voltages V^{max} and V^{min} represent the maximum and minimum elements of $(1-\delta_-)V^*$ and $(1+\delta_-)V^*$, respectively.

Although not shown, each PC voltage is bounded as in (21). Moreover, Figure 4c shows that weighted voltage balancing is achieved in steady state.

Robustness to changes in load: At $t = 2$s, an increase takes place in load consumption. Indeed, as in Figure 4, the filter currents attain a new steady state and share the total load proportionally. In addition, the PC voltages are bounded and converge to a new steady state. We highlight that, since the voltage references are left unchanged, the weighted voltage sum remains the same. With the intention of demonstrating that a steady-state voltage may not exist under secondary control due to the presence of P loads, we increase the line resistances multi-fold to violate (20). In such a scenario, the PC voltages do not reach a steady state; see Figure 5. In particular, the voltage V_6 approaches 0, leading to an increasing power absorption by the P load at PC 6, and eventually to a simulation failure.

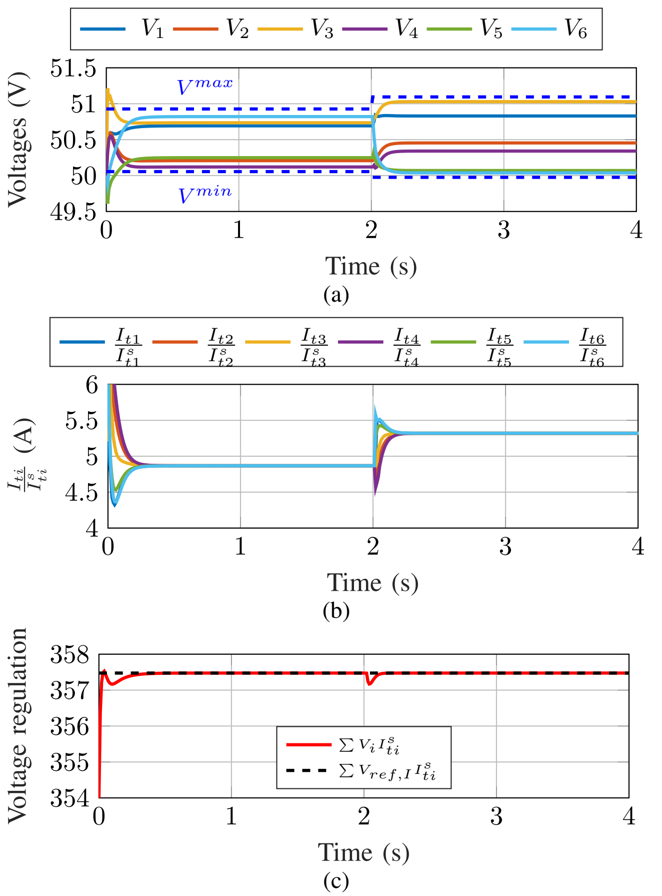

Fig. 4: PCC voltages, weighted filter currents, and weighted voltage sum under secondary control.

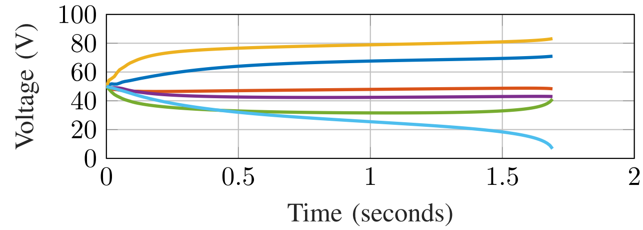

Fig. 5: Voltage collapse in the DCmG network due to nonexistence of a steady state.

V. CONCLUSIONS

In this paper, a novel secondary consensus-based control layer for current sharing and voltage balancing in DCmGs was presented. We considered a DCmG composed of realistic DGUs, RLC lines, and ZIP loads. A rigorous steady-state analysis was conducted, and appropriate conditions were derived to ensure the attainment of both objectives. In addition, a voltage stability analysis was provided showing that the controllers can be synthesized in a decentralized fashion. Future developments will study the impact of non-idealities (such as transmission delays, data quantization and packet drops) on the performance of closed-loop mGs. Further developments can also consider the inclusion of Boost and other DC-DC converters.

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