



Emergence of electric mobility: a nested approach to vehicle choice modeling

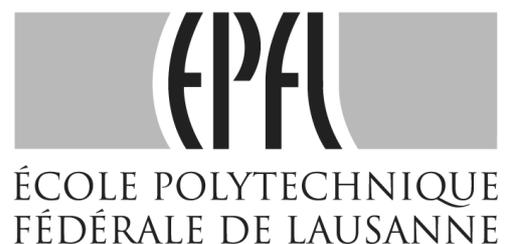
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Abstract

This research aims at modeling the future demand for electric vehicles on the Swiss market. To do so, a nested logit model is calibrated on stated preference data from a survey where each respondent had to face a choice situation context with classical gasoline vehicles, including his own current vehicle, and an electric vehicle. A nested logit model enables us to take into account the existence of common characteristics between alternatives which do not belong to the respondent in comparison to latter's own car, when we analyze the impact on choice of attributes of each alternative as well as socio-economic information of the respondent.

Keywords

Discrete choice models, nested structures, transportation, vehicle choice, stated preferences

1 Introduction

As the large-scale release of electric vehicles on the Swiss market is approaching, the market shares within the automotive sector for the different types of cars (namely gasoline cars, hybrid cars and electric cars) are likely to be significantly affected. This particular context motivated a sound demand analysis for electric cars in Switzerland, in order to identify the characteristics which would influence individuals' purchase decision and the population segments which are the most likely to be interested in such cars.

This demand study required a stated preferences survey that was conducted in two phases in collaboration with Renault Suisse and EPFL's Transportation Center. In the first phase, information was collected about respondents' current vehicle(s) and in the second phase, this information was used to build choice situation contexts involving the respondents' own car(s), a gasoline car in the same segment but from the Renault brand and finally a similar electric car. In the survey, respondents were also asked to report their opinion on statements related to topics such as ecology, new technologies or reliability of the electric vehicle.

The sample consisted of four target groups of respondents, i.e. individuals who bought a new car in the last three years, people who intend to buy a new car in the next six months, Renault customers or future customers who pre-ordered an electric vehicle, and subscribers to the newsletter on electric vehicles. The whole survey involved respondents from three speaking parts of Switzerland, that is the German, French and Italian parts.

This research project leads to the estimation of discrete choice models in order to understand vehicle preferences with respect to their characteristics, as well as socio-economic information of each respondent and be able to predict vehicle purchase behavior. The structure of the choice situation contexts allows for the calibration of models with complex structures, such as nested constructions. The alternatives indeed consist of a owned vehicle, an alternative to the latter from another brand and an electric car, and can hence form two different types of nesting structures, i.e. owned versus Renault vehicles or gasoline versus electric cars.

This paper first presents the data collection. Second, the modeling framework is explained. Third, the specification of the discrete choice model in the case of our study on vehicle preferences is detailed and the estimation results are provided. We conclude by showing the next steps in our research.

2 Data collection

In order to collect information on individuals' preferences towards different types of vehicles, a survey was set up. At the moment, electric vehicles are not widely released on the market and hence their real demand cannot be evaluated yet. Therefore, only the latter can only be evaluated via *hypothetical* choice situations contexts. Surveys involving such hypothetical choice situation contexts are called *stated preference surveys*. In our case, three different types of vehicles are presented to the respondents:

- A vehicle that the respondent's household currently owns;
- An analogous model from the Renault brand, if the respondent's vehicle is from a different brand;
- A similar electric car from the Renault brand.

Despite the impossibility of gathering data about the individuals' real purchase decisions, the survey shows realistic and personalized choice situations by including vehicles currently owned by the respondents. This feature of the survey implied performing two questionnaires: the first one was designed in order to collect information about the respondents' current vehicles and the second one to show choice situations created on the basis of the data reported by the respondents in the first phase. Both questionnaires were performed online in collaboration with the market research institute GfK Switzerland.

This section describes the targeted respondents, the sampling protocol and the structure of the survey.

2.1 Target groups

The survey respondents were selected in order to be representative of the population who was the most likely to face a purchase choice between gasoline and electric cars. They belong to one of the following four target groups:

Recent buyers: Individuals who bought a new car in the last three years.

Prospective buyers: Individuals who plan to buy a new car in the next six months.

Current and future Renault customers: Individuals who already own a Renault car or who pre-ordered an electric vehicle from the Renault brand.

EV-fans: Individuals who joined the Renault newsletter on electric vehicles.

The respondents in the two first groups, i.e. the recent and prospective buyers, were selected via screening questions at the beginning of the questionnaire. For the third and fourth groups,

Group name	Sent	Phase I Number	Rate	Phase II Number	Rate	Phase I vs phase II Rate
1 Recent buyers	3006	150	10.0%	141	9.4%	94.0%
2 Prospective buyers		151		141		93.4%
3 Renault customers	1042	168	16.1%	139	13.3%	82.7%
4 EV-fans	656	197	30.0%	172	26.2%	87.3%
Total	4704	666	14.2%	593	12.6%	89.0%

Table 1: Number of online questionnaires sent, numbers and rates of responses after phase I and phase II, and between both phases, for each sample group.

i.e. the current and future Renault customers, and the EV-fans, the questionnaire was sent to a list of addresses provided by Renault. The number of completed questionnaires, as well as the response rates, after phase I and phase II are reported in Table 1, for each sample group.

For the sample group of EV-fans, a higher response rate (26.2%) was observed after the two phases of the questionnaire. This is not surprising as EV-fans already showed a non-negligible interest in electric vehicles, by being members of a newsletter on electric vehicles. Let us moreover note that the response rate between the two phases is very high for all four target groups ($> 82.7\%$).

2.2 Sampling protocol

The respondents to the survey were sampled in order to match the Swiss proportions of three socio-demographic characteristics: *gender* and *age*, which are classical socio-economic variables for which we wish the survey sample to match the real population proportions, and *language regions*, for each of which different travel behaviors have been observed in previous studies (Bierlaire *et al.* (2006), Hurtubia *et al.* (2010) and Atasoy *et al.* (2010)).

Variable	Level	Targeted rate	Rate phase I	Rate phase II
Language	German	72.5%	67.3%	67.8%
	French	23.0%	27.2%	26.6%
	Italian	4.5%	5.56%	5.56%
Gender	Male	49.4%	74.0%	74.2%
	Female	50.6%	26.0%	25.8%
Age category	18-35 years	33.6%	23.0%	21.8%
	36-55 years	41.6%	51.8%	52.6%
	56-74 years	24.8%	25.2%	25.6%

Table 2: Targeted and real response rates in each socio-economic group (language, gender and age) after phase I and phase II. The targeted rates are the Swiss proportions of each group.

The targeted proportions for each socio-demographic group, as well as the obtained ones are shown in Table 2. For the language group, no obvious difference occurs between the targeted proportions and the ones that were obtained at the end of the survey. Regarding the two other socio-demographic groups, i.e. the language category and the gender, some differences in the response rates are noticeable. Individuals aged between 36 and 55 years were indeed slightly oversampled and more men answered to the survey than women. Between phase I and phase II the proportions did not change much, which could be expected, as the total response rate between the two phases was very high (89.0%).

2.3 Structure of the stated preference survey

The stated preference survey was built in two phases due to the complexity of creating personalized choice situations. In this section, the structure of both phases is described.

Phase I consisted of the three following sections:

Characteristics of the respondent's car(s): The respondent is required to report the characteristics of all cars in his household, that is, their makes, models, types of fuel, motorisations and versions, as far as he knows them. The information reported in this section is then used to create the personalized choice situations shown to the respondent in phase II.

Socio-economic information: The respondent is asked to answer some standard socio-economic questions, such as gender, age, education, etc. This information enables us to uncover the population segments which express different vehicle preferences when it is integrated in the nested logit model.

Mobility habits: The last section of the first phase of the survey consists of questions on the respondent's mobility habits. For example, the respondent has to report the length of his daily trips or the transport mode(s) he uses for some particular types of trips. Our assumption is that different mobility habits, such as the occasional or frequent use of public transports, can induce preferences for different vehicles types.

Phase II was launched two weeks after phase I. It included the two important sections:

Opinions on five topics related to electric vehicles: In order to model complex underlying attitudes that might affect the decision maker's choice to purchase an electric vehicle, sentences related to five different topics were shown to him. For each of the statements, the respondent was asked to rate his agreement on a five-point *Likert scale*, ranging from 'Total disagreement' to 'Total agreement'. Five sentences were shown per theme. Such

theme could include for example the ecological perception of an electric vehicle, the attitude towards new technologies or the perception of the reliability of an electric car.

Examples of sentences related to the perception of the electric vehicle as an environmentally friendly solution are reported below:

- Renewable energies should be promoted, so that the energy used to charge the battery is also clean.
- Finding a solution for the second life of batteries is not a major problem.
- I prefer driving a car with a powerful engine than a car that emits little carbon dioxide.

Choice situations: This section builds the core of the entire survey. We wish indeed to be able to explain each individual's preference towards a particular type of vehicle. Five choice situations contexts are shown to each respondent. The aim of each choice situation is to show three different cars to the respondent: his own car, the analogous model of the Renault brand (also with combustion engine) and finally a similar model in the Renault product line of electric cars. Such a choice situation experiment enables us to define nests of alternatives, which could be 'gasoline' vehicles for the respondent's own vehicle and the analogous gasoline vehicle by Renault, or 'Renault alternatives' for the Renault car with combustion engine which is analogous to the respondent's current vehicle and the electric model. It allows for the application of nested logit models, which will be presented in section 4.

Nevertheless it is not always possible to present these three exact alternatives to a respondent as he may own a Renault model. In that particular case, the respondent will not be shown the gasoline vehicle from the Renault brand and that alternative will be declared unavailable in the model specification part.

Table 3 is an example of personalized choice situation for an individual owning a particular car model, an Audi A4. The information on the make, model and fuel type of the respondent's car are obtained from the vehicle description filled in during phase I, and the corresponding purchase price and the fuel cost of driving 100 km are inferred from a data base containing information on the vehicles currently released on the market. The information on the analogous gasoline vehicle from the Renault brand are also obtained using the vehicle data base.

For the electric vehicle, levels were defined for its purchase price, a possibly attributed governmental incentive, the cost of driving 100 km and its battery lease (see Table 4). The levels of each variable were combined according to a *fractional factorial design* of resolution V . Let us note that *blocking* was also performed relatively to the four sample groups presented in section 2.1, i.e. the recent buyers, the prospective buyers, the current and future Renault customers, and the EV-fans, in order to avoid undesirable variability

Characteristics	Your vehicle	Renault vehicle with combustion engine	Renault electric vehicle
Make	Audi	Renault	Renault
Model	A4	Laguna	Fluence
Fuel	Petrol	Petrol	Electricity
Purchase price (in CHF)	42'400	37'200	56'880
Incentive (in CHF)	0	0	-1'000
Total purchase price (in CHF)	42'400	37'200	55'880
OR: Monthly leasing price (in CHF)	477	399	693
Maintenance costs (in CHF for 30'000 km)	850	850	425
Cost in fuel/electricity for 100 km (in CHF)	11.70	13.55	3.55
Battery lease (in CHF per month)	0	0	125
	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Table 3: An example of choice situation presented to respondents with a standard non-Renault car in their household. The respondent had to tick the box below the column corresponding to the vehicle he would choose if he had to change his car at present.

in the answers of the respondents of each group. EV-fans could have a higher tendency to select the electric car in a choice situation context than the recent buyers, for instance. More information fractional factorial designs and blocking procedures can be found in Montgomery (2001).

Level	Purchase price P	Incentive I	Cost C of 100 km	Battery lease L
1	$(P_{\text{own}} + 5'000) \cdot 0.8$	-0 CHF	1.70 CHF	85 CHF
2	$(P_{\text{own}} + 5'000) \cdot 1$	-500 CHF	3.55 CHF	105 CHF
3	$(P_{\text{own}} + 5'000) \cdot 1.2$	-1'000 CHF	5.40 CHF	125 CHF
4	-	-5'000 CHF	-	-

Table 4: Levels of the variables related to the electric vehicles which are subject to an experimental design, that is, the purchase price P , based on the price P_{own} of the respondent's car, a possible governmental incentive I , the cost C of driving 100 km and the battery lease L .

3 Methodology

The methodology used to analyze the potential demand for electric vehicles is *discrete choice modeling*. It enables us to analyse the effect on vehicle choice of attributes of each alternative and socio-economic information of the respondents. When some alternatives share common characteristics, a particularly appropriate model can be applied, that is, a *nested logit model*, where such alternatives are said to belong to the same nest.

One of the outcomes of a discrete choice model is the possibility to compute the probability that an individual n chooses an alternative i . In a nested logit model, such probability depends on the nest m the alternative i belongs to, i.e. it is given by the following formula:

$$P_n(i|C_n) = P_n(i|m, C_n)P_n(m|C_n), \quad (1)$$

where C_n is the choice set of individual n . For some individuals, all alternatives might indeed not always be available. Hence, a choice set C_n is defined for each individual n , containing the alternatives n has access to. Equation (1) is the product of two factors: $P_n(i|m, C_n)$, which is the probability that individual n chooses alternative i given that alternative i belongs to nest m , and $P_n(m|C_n)$, which is the probability that individual n selects an alternative in nest m .

A derivation of formula (1) leads to the following expression of probability $P_n(i|C_n)$:

$$P_n(i|C_n) = \frac{e^{\mu_m V_{in}}}{\sum_{j \in C_{mn}} \sum_p \left(\sum_{l \in C_{pn}} e^{\mu_p V_{ln}} \right)^{\frac{\mu}{\mu_p}}}} \left(\sum_{l \in C_{mn}} e^{\mu_m V_{ln}} \right)^{\frac{\mu}{\mu_m}}, \quad (2)$$

where μ_m is a coefficient associated to nest m and which needs to be estimated, μ is a scale parameter, C_{mn} is the choice set of alternatives in nest m for an individual n and V_{in} is the deterministic utility associated to alternative i for individual n .

The deterministic utility V_{in} is a function V of characteristics X_{in} of alternative i and respondent n , and of a vector of parameter β , i.e.

$$V_{in} = V(X_{in}; \beta). \quad (3)$$

Function V must be specified by the modeler. In order to identify which variables have an effect on individuals choices, we need to estimate the vector β of parameters on the collected data. This is performed by maximum likelihood estimation, where the following likelihood function \mathcal{L} is maximized:

$$\mathcal{L} = \prod_{n=1}^N \prod_{i=1}^{J_{C_n}} P_n(i|C_n)^{y_{in}}, \quad (4)$$

where C_n is the choice set for respondent n , J_{C_n} is the number of available alternatives in C_n and y_{in} is an indicator that respondent n chose alternative i . Precisely, variable y_{in} is defined as follows:

$$y_{in} = \begin{cases} 1 & \text{if } U_{in} = \max_j U_{jn} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

The vector of parameters β is estimated by maximizing \mathcal{L} . For that purpose, we use the extended version of software BIOGEME (Bierlaire (2003)), which is described in Bierlaire and Fethiarison (2009).

4 Model specification and estimation

In this section we present the specification of the nested logit model used to analyze the demand for the three vehicle types, i.e. the respondent's own vehicle, the analogous gasoline car from the Renault brand and the electric car. As explained in section 3, deterministic utility functions must be specified for each alternative. We denote them as V_{own} , V_{Renault} and V_{elec} for each alternative mentioned above, respectively. Table 5 shows the specification of all three utilities. Each utility V_i is given by the inner product between the left-hand column 'Utilities' and the column corresponding to alternative i . For example, utility V_{own} is given by the inner product between column 'Utility' and column 'Own car'.

Utilities	Own car	Renault car	Electric car
ASC_{own}	1	-	-
ASC_{Renault}	-	1	-
$\beta_{\text{price}_{\text{own}}}$	price _{own}	-	-
$\beta_{\text{price}_{\text{Renault}}}$	-	price _{Renault}	-
$\beta_{\text{price}_{\text{elec}}}$	-	-	price _{elec}
β_{UseCost}	UseCost _{own}	UseCost _{Renault}	-
$\beta_{\text{UseCost}_{\text{elec}}}$	-	-	UseCost _{elec}
$\beta_{\text{BatteryHigh}}$	-	-	BatteryHigh
$\beta_{\text{IncentiveHigh}}$	-	-	IncentiveHigh
β_{PTWork}	PTWork	PTWork	-
$\beta_{\text{IncomeHigh}}$	IncomeHigh	IncomeHigh	-
β_{NbCars}	NbCars	NbCars	-
β_{FamChild}	-	FamChild	FamChild
β_{Age}	-	Age	Age

Table 5: Specification table of the utilities

Several types of variables were assumed to have an influence of individuals' vehicle choices:

Common characteristics of all cars: We specified alternative specific price coefficients,

since we assume that the impact on the purchase decision of the prices $price_{own}$ of a vehicle currently owned, $price_{Renault}$ of a similar vehicle or $price_{elec}$ of a vehicle with a totally new type of engine can be perceived differently. We made the hypothesis that individuals perceived fuel costs $UseCost_{own}$ and $UseCost_{Renault}$ for their own car and the analogous model from Renault, respectively, the same way. Hence, a generic coefficient is specified for both. Since the charging cost $UseCost_{elec}$ of the electric alternative is a discrete value indicating a high cost of electricity, a coefficient specific to that alternative was specified.

Characteristics of the electric car: Besides vehicle price and usage costs, two other variables were part of the experimental design related to the electric alternative, that is, a possible governmental incentive and the battery monthly lease. Precisely, the highest levels of these variables, denoted as *IncentiveHigh* and *BatteryHigh* were introduced in the utility of the electric alternative.

Socio-economic characteristics: Variables related to the usage of public transportation for work-related trips, the income or the total number of cars in the household were included in the utilities of the respondent's car and the analogous car by Renault, in order to observe their effect on the choice of gasoline cars in comparison with the electric one. Other characteristics, such as the family composition or the respondent's age, were introduced in the utilities of the gasoline car by Renault and the electric car. Their impact on choice was assumed to differ depending on whether the car belongs to the respondent or not.

Let us remark that due to the fact that some respondents may already own a Renault vehicle, their choice sets C_n might be restricted to their car and the electric one, i.e. for such an individual n , we have:

$$C_n^{Renault} = \{\text{own, electric}\}.$$

For all other individuals, the choice set C_n is made of their own non-Renault car, the analogous gasoline model by Renault and the electric vehicle. It is given by the following expression:

$$C_n^{\text{non-Renault}} = \{\text{own, Renault, electric}\}.$$

Our assumption to calibrate a nested logit model is that alternatives which are not owned by a respondent are perceived differently than the alternative consisting of his own car. Hence the two Renault models, driven by gasoline or electricity, belong to a same nest, denoted as the 'Renault' nest. A scale parameter $\mu_{Renault}$ relative to this nest must consequently be specified. Let us remark that scale parameter μ of equation (2) is set to 1, for normalization purposes.

The parameters of Table 5 are estimated by maximum likelihood. Table 6 shows the estimates of the parameters of the nested logit model whose specification is described above and of a logit model with the same deterministic utilities, for comparison purposes.

Variable	Nested logit model		Base model	
	Estimate	<i>t</i> -test	Estimate	<i>t</i> -test
ASC _{own}	0.05	0.19	0.37	1.51
ASC _{Renault}	-0.35	-1.47	-0.72	-2.14
Price _{own}	-0.03	-2.32	-0.03	-2.11
Price _{Renault}	-0.30	-5.70	-0.26	-3.66
Price _{elec}	-0.40	-9.84	-0.45	-10.73
UseCost	-0.05	-2.33	-0.08	-3.59
UseCost _{elec}	-0.18	-2.50	-0.21	-2.44
BatteryHigh	-0.12	-1.63	-0.18	-2.07
IncentiveHigh	0.57	6.69	0.65	7.18
PTWork	-0.39	-4.40	-0.49	-5.28
IncomeHigh	-0.22	-2.98	-0.29	-3.53
NbCars	-0.15	-3.25	-0.20	-3.96
FamChild	0.25	3.10	0.25	3.07
Age	-0.23	-2.47	-0.22	-2.37
$\mu_{Renault}$	1.69	6.20	-	-
Log-likelihood	-2237.64		-2242.49	

Table 6: Estimation Results

All estimates of the nested logit model are significant at a 95% level of confidence, except for the coefficient for the battery lease, which is significant at a 90%, and for the alternative specific constants. From the signs of the estimates, the following conclusions can be made:

- The negative sign of the price coefficients show that the higher the purchase price of a vehicle is, the lower its utility becomes. The effect of the purchase price is the most important for the electric alternative, the second most important for the gasoline car from the Renault car and the least important for the vehicle owned by the respondent.
- Refueling and recharging costs have a negative effect on the utility of all vehicles. For the electric vehicle, only the highest level of price of electricity affects the choice significantly.
- The highest levels of a potential governmental incentive and of the battery lease significantly decrease the utility of the electric vehicle.
- The signs of the estimates of the parameters relative to the socio-economic variables have meaningful interpretations and characterize the potential customers.

The nest parameter $\mu_{Renault}$ is moreover significantly different from 1, as we have:

$$\frac{\mu_{Renault} - 1}{\sigma_{\mu_{Renault}}} = \frac{1.69 - 1}{0.272} \cong 2.54,$$

where $\sigma_{\mu_{Renault}}$ denotes the standard deviation for parameter $\mu_{Renault}$.

This results shows evidence for the existence of a nested structure. A likelihood ratio test between the nested logit model and the logit model demonstrates that the nested logit model is

more appropriate.

A nested logit model with a nest including all gasoline vehicles was also estimated, but the results did not show any improvement over a logit model with the same specification. The same holds for the cross-nested structure including nests with Renault cars and with gasoline cars. These reasons moreover confirm that the nested logit model with a nest consisting of the vehicle owned by the respondent and another nest gathering the two Renault vehicles is the most adequate nested structure.

5 Conclusion

In this paper we presented the framework of a survey designed to analyze the future demand for electric vehicles, in a context of their near large-scale release on the market, as well as results from the estimation of a nested logit model on the obtained vehicle preference data.

The calibration of such model enabled us to assess the effect on choice of general characteristics of the cars, such as their purchase price, or of particular attributes of electric vehicles, such as the battery lease. Moreover, by including socio-economic characteristics in the model, we could uncover segments of the population which have greater preference for electric vehicles and which might hence be potential customers.

The impact on choice of common characteristics of vehicles which are not owned by the respondent in comparison with the vehicle he currently owns could be captured by the calibration of a nested logit model. Such model happened to be more adapted than a simple logit model.

One of the particularity of the survey was the inclusion of statements related to the opinions of the respondents on certain topics such as their perception of an electric vehicle as an ecological alternative to gasoline cars. A next step in this research would be to include these attitudes or perceptions into the discrete choice model via an integrated framework with latent constructs (see Walker (2001) and Walker and Ben-Akiva (2002)).

Further work also include a forecasting analysis, in order to evaluate the potential market share for electric vehicles in Switzerland.

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References

- Atasoy, B., A. Glerum, R. Hurtubia and M. Bierlaire (2010) Demand for public transport services: Integrating qualitative and quantitative methods, paper presented at *Swiss Transport Research Conference (STRC)*, September 2010.
- Bierlaire, M. (2003) Biogeme: a free package for the estimation of discrete choice models, paper presented at *Swiss Transport Research Conference (STRC)*, March 2003.
- Bierlaire, M. and M. Fetiariison (2009) Estimation of discrete choice models: extending biogeme., paper presented at *Swiss Transport Research Conference (STRC)*, September 2009.
- Bierlaire, M., M. Thémans and K. Axhausen (2006) Analysis of driver's response to real-time information in Switzerland, *European Transport*, **34**, 21–41.
- Hurtubia, R., B. Atasoy, A. Glerum, A. Curchod and M. Bierlaire (2010) Considering latent attitudes in mode choice: The case of switzerland, paper presented at *World Conference on Transport Research (WCTR)*, July 2010.
- Montgomery, D. C. (2001) *Design and Analysis of Experiments*, fifth edition edn., John Wiley and Sons, New York.
- Walker, J. and M. Ben-Akiva (2002) Generalized random utility model, *Mathematical Social Sciences*, **43** (3) 303–343.
- Walker, J. L. (2001) Extended discrete choice models: Integrated framework, flexible error structures, and latent variables, Ph.D. Thesis, Massachusetts Institute of Technology.