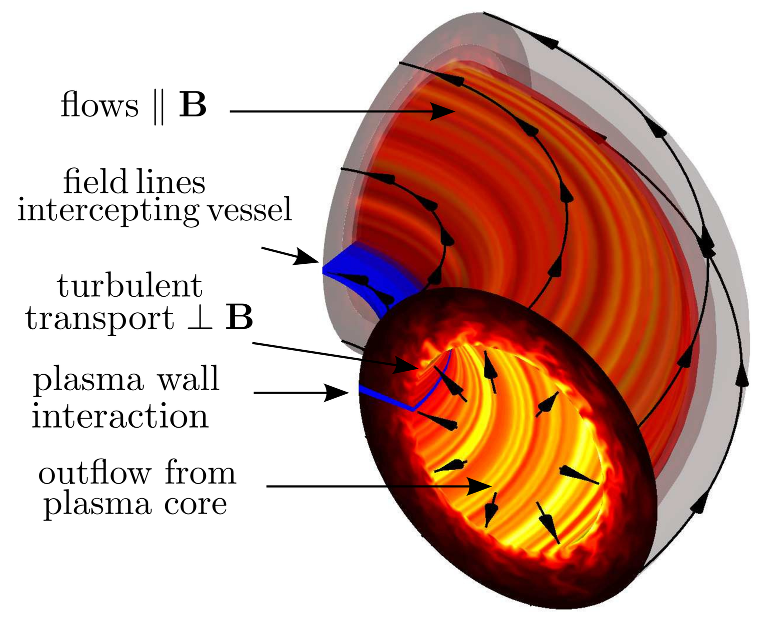
Numerical simulations of plasma fuelling in tokamaks using the GBS code

A. Coroado, P. Ricci, C. Beadle, P. Paruta, F. Riva, C. Wersal

École Polytechnique Fédérale de Lausanne (EPFL), Swiss Plasma Center, CH-1015 Lausanne, Switzerland

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE | SWISS PLASMA CENTER

Introduction



- In tokamaks Scrape-Off Layer (SOL), **magnetic field lines intersect the walls** of the fusion device
- **Heat and particles** flow along magnetic field lines and are **exhausted to the vessel**
- **Turbulence** amplitude and size **comparable** to **steady-state** values
- **Neutral** particles interact with the plasma
- SOL plays a key role on determining the **refuelling** of the plasma

The **Global Braginskii Solver (GBS) code:**
a 3D, flux-driven, global turbulence code in limited geometry
used to study **plasma turbulence in the SOL**

- GBS is a simulation code to evolve plasma turbulence in the edge of fusion devices. [Halpern *et al.*, JCP 2016], [Ricci *et al.*, PPCF 2012]
- GBS solves 3D **fluid equations for electrons and ions**, Poisson's and Ampere's equations, and a **kinetic equation for neutral atoms**.

The Global Braginskii Solver (GBS) code

Two fluid drift-reduced Braginskii equations, $k_\perp^2 \gg k_\parallel^2$, $d/dt \ll \omega_{ci}$

$$\frac{\partial n}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, n] + \frac{2}{B}[C(p_e) - nC(\phi)] - \nabla\cdot(n v_{\parallel e}\mathbf{b}) + \mathcal{D}_n(n) + S_n + n_n\nu_{iz} - n\nu_{rec} \tag{1}$$

$$\frac{\partial \Omega}{\partial t} = -\frac{\rho_*^{-1}}{B}\nabla_\perp\cdot[\phi,\omega] - \nabla_\perp\cdot\left[\nabla_\parallel\left(v_{\parallel i}\omega\right)\right] + B^2\nabla\cdot\left(j_\parallel\mathbf{b}\right) + 2BC(p) + \frac{B}{3}C\left(G_i\right) + \mathcal{D}_\Omega(\Omega) - \frac{n_n}{n}\nu_{cx}\Omega \tag{2}$$

$$\frac{\partial U_{\parallel e}}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, v_{\parallel e}] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e}\left[\frac{\nu j_\parallel}{n} + \nabla_\parallel\phi - \frac{\nabla_\parallel p_e}{n} - 0.71\nabla_\parallel T_e - \frac{2}{3n}\nabla_\parallel G_e\right] + \mathcal{D}_{v_{\parallel e}}(v_{\parallel e}) + \frac{n_n}{n}(\nu_{en} + 2\nu_{iz})(v_{\parallel n} - v_{\parallel e}) \tag{3}$$

$$\frac{\partial v_{\parallel i}}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, v_{\parallel i}] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{\nabla_\parallel p}{n} - \frac{2}{3n}\nabla_\parallel G_i + \mathcal{D}_{v_{\parallel i}}(v_{\parallel i}) + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})(v_{\parallel n} - v_{\parallel i}) \tag{4}$$

$$\frac{\partial T_e}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4T_e}{3B}\left[\frac{C(p_e)}{n} + \frac{5}{2}C(T_e) - C(\phi)\right] + \frac{2T_e}{3n}\left[0.71\nabla\cdot\left(j_\parallel\mathbf{b}\right) - n\nabla\cdot\left(v_{\parallel e}\mathbf{b}\right)\right] + \mathcal{D}_{T_e}(T_e) + \mathcal{D}^\parallel_{T_e}(T_e) + S_{T_e} + \frac{n_n}{n}\nu_{iz}\left[-\frac{2}{3}E_{iz} - T_e + \frac{m_e}{m_i}v_{\parallel e}\left(v_{\parallel e} - \frac{4}{3}v_{\parallel n}\right)\right] - \frac{n_n}{n}\nu_{en}\frac{m_e}{m_i}\frac{2}{3}v_{\parallel e}(v_{\parallel n} - v_{\parallel e}) \tag{5}$$

$$\frac{\partial T_i}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, T_i] - v_{\parallel i}\nabla_\parallel T_i + \frac{4T_i}{3B}\left[\frac{C(p_e)}{n} - \frac{5}{2}\tau C(T_i) - C(\phi)\right] + \frac{2T_i}{3n}\left[\nabla\cdot\left(j_\parallel\mathbf{b}\right) - n\nabla\cdot\left(v_{\parallel i}\mathbf{b}\right)\right] + \mathcal{D}_{T_i}(T_i) + \mathcal{D}^\parallel_{T_i}(T_i) + S_{T_i} + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})\left[\tau^{-1}T_n - T_i + \frac{1}{3\tau}(v_{\parallel n} - v_{\parallel i})^2\right] \tag{6}$$

$$\rho_\star = \rho_s/R_0, \qquad \mathbf{b} = \frac{\mathbf{B}}{B}, \qquad [A, B] = b\cdot(\nabla A\times\nabla B), \qquad C(A) = \frac{B}{2}\left[\nabla\times\frac{b}{B}\right]\cdot\nabla A, \qquad \nabla_\parallel f = \mathbf{b}_0\cdot\nabla f + \frac{\beta_{e0}}{2}\frac{\rho_*^{-1}}{B}[\psi, f]$$

$$p = n(T_e + \tau T_i), \qquad U_{\parallel e} = v_{\parallel e} + \frac{\beta_{e0}}{2}\frac{m_i}{m_e}\psi, \qquad \Omega = \nabla\cdot\omega = \nabla\cdot(n\nabla_\perp\phi + \tau\nabla_\perp p_i)$$

- Equations implemented in GBS, a **flux-driven** plasma turbulence code with limited geometry to study SOL heat and particle transport
- System completed with **first-principles boundary conditions** applicable at the magnetic pre-sheath entrance where the magnetic field lines intersect the limiter [Loizu *et al.*, PoP 2012]
- Parallelized using domain decomposition, **excellent parallel scalability** up to $\sim$ 10000 cores
- Gradients and curvature discretized using **finite differences**, Poisson Brackets using Arakawa scheme, integration in time using **Runge Kutta method**
- Code **fully verified** using method of manufactured solutions [Riva *et al.*, PoP 2014]
- Note: $L_\perp \to \rho_s$, $L_\parallel \to R_0$, $t \to R_0/c_s$, $\nu = ne^2R_0/(m_i\sigma_\parallel c_s)$ normalization

The Poisson and Ampere equations

- **Generalized Poisson equation**, $\nabla\cdot(n\nabla_\perp\phi) = \Omega - \tau\nabla_\perp^2 p_i$
- **Ampere's equation** from Ohm's law, $\left(\nabla_\perp^2 - \frac{\beta_{e0}}{2}\frac{m_i}{m_e}n\right)v_{\parallel e} = \nabla_\perp^2 U_{\parallel e} - \frac{\beta_{e0}}{2}\frac{m_i}{m_e}nv_{\parallel i}$
- Stencil based **parallel multigrid** implemented in GBS
- Elliptic equations separable in parallel direction allow for **independent 2D solutions** for x-y plane

The kinetic neutral atoms equation

$$\frac{\partial f_n}{\partial t} + \vec{v}\cdot\frac{\partial f_n}{\partial \vec{x}} = -\nu_{iz}f_n - \nu_{cx}n_n\left(\frac{f_n}{n_n} - \frac{f_i}{n_i}\right) + \nu_{rec}f_i \tag{7}$$

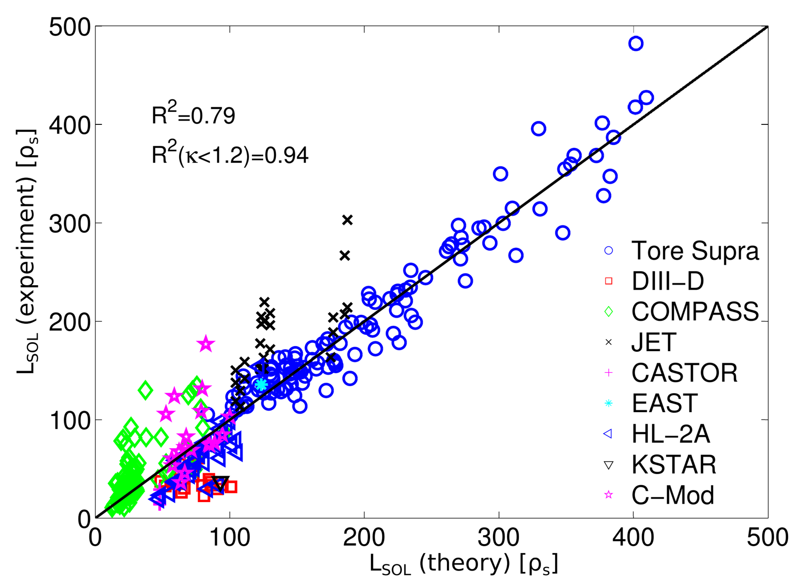
- **Method of characteristics** to obtain the formal solution of f_n [Wersal *et al.*, NF 2015]
- **Two assumptions**, $\tau_{\text{neutral losses}} < \tau_{\text{turbulence}}$ and $\lambda_{\text{mfp, neutrals}} \ll L_{\parallel,\text{plasma}}$, leading to a 2D steady state system for each x-y plane
- **Linear integral equation** for neutral density obtained by integrating f_n over $\vec{v}$
- **Spatial discretization** leading to a linear system of equations

$$\begin{bmatrix} n_n \\ \Gamma_{out} \end{bmatrix} = \begin{bmatrix} K_{p\to p} & K_{b\to p} \\ K_{p\to b} & K_{b\to b} \end{bmatrix}\cdot\begin{bmatrix} n_n \\ \Gamma_{out} \end{bmatrix} + \begin{bmatrix} n_{n,rec} \\ \Gamma_{out,rec} + \Gamma_{out,i} \end{bmatrix} \tag{8}$$

- This system is solved for neutral density, n_n, and neutral particle flux at the boundaries, Γ_{out}, with the threaded LAPACK or MUMPS (serial or parallel) solvers.

Past achievements of GBS

- Characterization of **non-linear turbulent regimes** in the SOL
- **SOL width scaling** as a function of dimensionless / engineering plasma parameters
- Origin and nature of **intrinsic toroidal plasma rotation** in the SOL
- Mechanisms regulating the SOL **equilibrium electrostatic potential**



Open questions on plasma fueling in tokamaks

- Where is the plasma created and how is it transported?
- How do neutral flows influence the plasma density profile?
- How is fueling affected by the n, T profiles and the poloidal location of the gas puffs?

This requires:

- Quantitative assessment of **plasma and neutral flows**
- **Mass-conserving model** (total ions + neutrals kept constant within the simulation)

These will also allow to address:

- Influence of neutrals in the formation of **different SOL regions**
- High density effects (formation of the **density shoulder**, **Greenwald density limit**...)

Moving towards a mass-conserving model

GBS was modified to ensure **mass conservation** (ions + neutrals):

- 2 changes were implemented to make the **continuity equation exactly satisfied**
  - **Radially variable inverse aspect ratio** $\epsilon = \frac{r}{R_0}$ to take into account **curvilinear geometry**
  - **Parallel gradient terms** included in Poisson brackets and curvature operators

$$[\phi, A] = P_{yx}[\phi, A]_{yx} + \mathbf{P}_{\mathbf{x}\parallel}[\phi, \mathbf{A}]_{\mathbf{x}\parallel} + \mathbf{P}_{\parallel\mathbf{y}}[\phi, \mathbf{A}]_{\parallel\mathbf{y}}\,, \quad C(A) = C^x\frac{dA}{dx} + C^y\frac{dA}{dy} + \mathbf{C}^\parallel\nabla_\parallel\mathbf{A}$$

$$[\phi, A]_{uv} = \frac{d\phi}{du}\frac{dA}{dv} - \frac{d\phi}{dv}\frac{dA}{du}\,, \quad P_{yx} = \frac{a_0}{Jb^\varphi}\,, \quad \mathbf{P}_{\mathbf{x}\parallel} = \frac{\mathbf{b}_{\theta^*}}{\mathbf{J}\mathbf{b}^\varphi}\,, \quad \mathbf{P}_{\parallel\mathbf{y}} = \frac{\mathbf{a_0 b_r}}{\mathbf{J}\mathbf{b}^\varphi}$$

$$C^x = -\frac{2B}{J}\frac{dc_\varphi}{d\theta^*}\,, \quad C^y = \frac{a_0B}{2J}\left[\frac{dc_\varphi}{dr} + \frac{1}{q}\left(\frac{dc_{\theta^*}}{dr} - \frac{dc_r}{d\theta^*}\right)\right]\,, \quad \mathbf{C}^\parallel = \frac{\mathbf{B}}{\mathbf{2Jb}^\varphi}\left(\frac{\mathbf{dc_r}}{\mathbf{d}\theta^*} - \frac{\mathbf{dc}_{\theta^*}}{\mathbf{dr}}\right)$$

Straight-field-line right-handed coordinates set: $(y, x, z) = (a_0\theta^*, r, R_0\varphi)$

θ^* defined by $b^\varphi = qb^{\theta^*}$ (with q the safety factor) $\quad c_i = \frac{b_i}{B} \quad J = rR_0\frac{\left(1-\epsilon^2\right)^{3/2}}{(1-\epsilon\cos(\theta^*))^2}$

- **Neutrals generated by boundary recycling** were made to match the **plasma outflow**
  - **Boundary conditions** were changed by adding the diamagnetic and $E\times B$ contributions

$$(n\vec{v})^{\theta^*} = nv_{\parallel e}b^{\theta^*} + (\mathbf{n}\vec{\mathbf{v}}_{de})^{\theta^*} + (\mathbf{n}\vec{\mathbf{v}}_{E\times B})^{\theta^*}, \qquad (n\vec{v})^r = (\mathbf{n}\vec{\mathbf{v}}_{de})^r + (\mathbf{n}\vec{\mathbf{v}}_{E\times B})^r$$

$$\vec{v}_{de} = \frac{1}{B^2}\vec{\nabla}p_e\times\vec{B}, \qquad (n\vec{v}_{de})^{\theta^*} = -\frac{1}{JB^2}\left[\frac{\partial p_i}{\partial x}B_\phi - \frac{\partial p_i}{\partial z}R_0B_r\right], \qquad (n\vec{v}_{de})^r = -\frac{1}{JB^2}\left[-\frac{\partial p_i}{\partial y}a_0B_\phi + \frac{\partial p_i}{\partial z}R_0B_{\theta^*}\right]$$

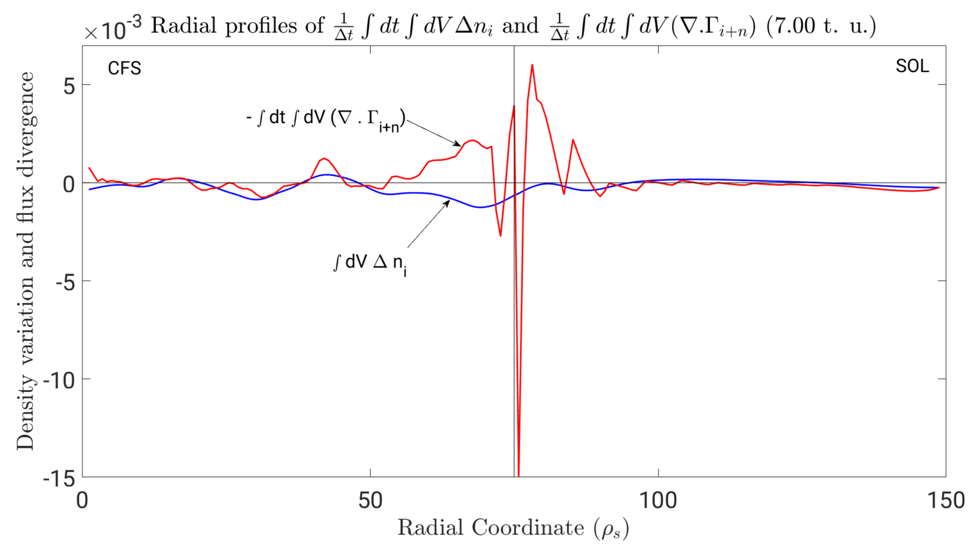
$$v_{\vec{E\times B}} = -\frac{n}{B^2}\vec{\nabla}\phi\times\vec{B}, \qquad (n\vec{v}_{E\times B})^{\theta^*} = -\frac{n}{JB^2}\left[\frac{\partial\phi}{\partial x}B_\phi - \frac{\partial\phi}{\partial z}R_0B_r\right], \qquad (n\vec{v}_{E\times B})^r = -\frac{n}{JB^2}\left[-\frac{\partial\phi}{\partial y}a_0B_\phi + \frac{\partial\phi}{\partial z}R_0B_{\theta^*}\right]$$

Mass conservation is evaluated by checking the balance of the number of particles:

- Continuity equation is integrated over volume and time
- **Neutral density is conserved** within the model, so $(n_n\nu_{iz}) = -\vec{\nabla}\cdot\vec{\Gamma}_{neutral}$
- Density balance given by $\int dt\int dV\,\frac{dn}{dt} = -\int dt\int dV\,(\vec{\nabla}\cdot\vec{\Gamma}_{ion} + \vec{\nabla}\cdot\vec{\Gamma}_{neutral})$

1D radial model

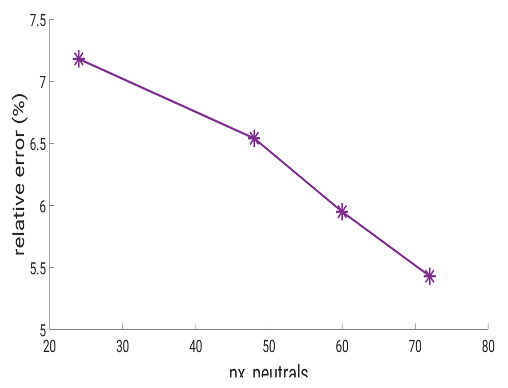
Radial balance of particles by integrating over θ^* and ϕ

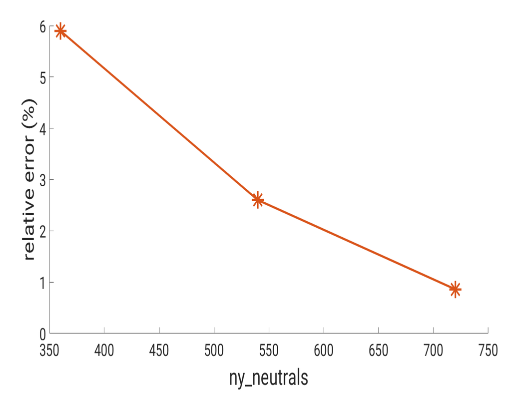


- **Density variation is slightly negative** almost everywhere as a result of **neutral and ion outflow** from the core
- Density variation profile is roughly flat (uniform variation rate)
- Reasonable matching at CFS region and SOL far from LCFS
- Curves strongly mismatch near LCFS due to very large gradients - much greater resolution required.
- Neutrals are conserved during calculation up to an error that **converges with grid resolution** (*nx_neutrals* and *ny_neutrals*)

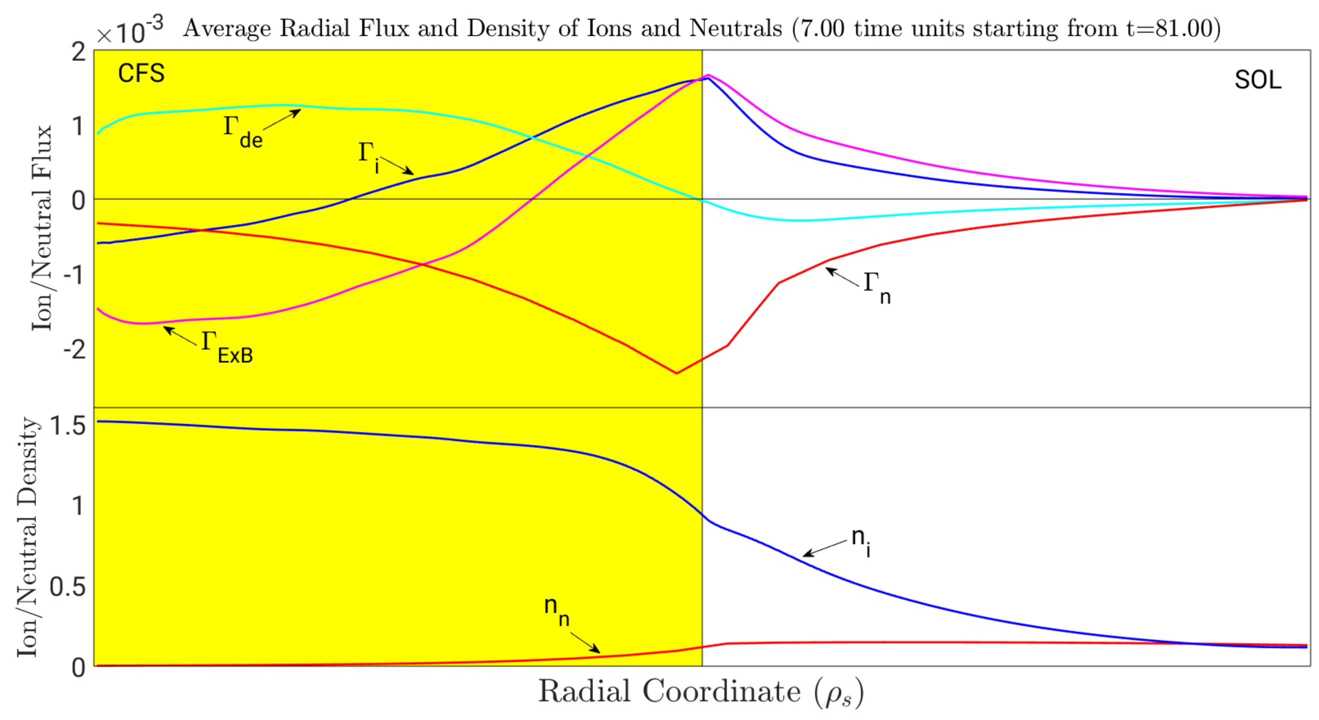
GBS simulation parameters:

- Circular magnetic equilibrium
- $R_0 = 500\rho_s$
- $L_x = 150\rho_s$, $L_y = 800\rho_s$
- Limiter at $x = 75 - 150\rho_s$
- 7 GBS time units
- Time step of $\Delta t = 3.75\times10^{-5}$





Quantitative assessment of ion and neutral fluxes



- Ion and neutral fluxes profiles are similar but not symmetric since system is **not in a steady state**
- Both **ions and neutrals outflow to the core**
- Ion flux in the SOL is dominated by the **E × B flux** (outward pointing)
- Ion flux in CFS region determined by **competition between E × B and diamagnetic contributions**

EUROfusion

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