Vertical Displacement of Bridges using the SOFO\textsuperscript{1} System: 
a Fiber Optic Monitoring Method for Structures

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ABSTRACT

In many bridges, the vertical displacements are the most relevant parameters to be monitored in both the short and long term. Current methods (such as triangulation, water levels or mechanical extensometers…) are often tedious to use and require the intervention of specialised operators. The resulting complexity and costs limit the temporal frequency of these traditional measurements. The spatial resolution obtained is in general low and only the presence of anomalies in the global structural behaviour can be detected and warrant a deeper and more precise evaluation.

To measure bridge vertical displacements at low cost and frequently in time, one solution consists of installing a network of fiber optic sensors during concrete pouring or installing them on the surface of the structure. By subdividing the whole structure into structural elements and those elements into cells that are analysed by the sensors, it is possible to obtain information about the average cell deformation (e.g., mean curvature) that can then be combined to obtain the global structural displacement field.

This measurement methodology was applied to the Lutrive Highway Bridge in Switzerland in order to measure the variation in vertical bridge displacements due to a static load test. The results obtained using the low coherence interferometric sensors of the SOFO system were then compared with the displacements obtained through an optic levelling system. In the case of this cantilever bridge of 60 meters half-span equipped with 30 fiber optic sensors, a discrepancy of 7% was obtained between the two measuring systems.

INTRODUCTION

In industrialised countries, there is a large number of structural works of art and the issue of their maintenance is a real problem. Excessive and non-stabilised displacement are often observed. They rarely affect the global structural security, but they can lead to the serviceability deficiencies. The aesthetic aspect can not be neglected since it often affects the way the public sees the construction.

Calculations of long-term deformation are quite complex. This is why it is necessary to develop an affordable user friendly displacement measurement system that can quickly notice abnormal behaviour as they appear.

The system presented here enables such long-term displacement measurements using optic fibers.

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**THE LUTRIVE HIGHWAY BRIDGE (SWITZERLAND)**

The North and South Lutrive bridges (Switzerland) are two parallel twin bridges. Each bridge supports two lanes of the Swiss national highway RN9 between Lausanne and Vevey. Built in 1972 by the balanced cantilever method with central articulations, the two bridges are gently curved ($r = 1000$ m) and each bridge is approximately 395 m long with four spans. The two bridges have the same cross-section. That consists of a box girder with variable height (from 2.5 m to 8.5 m) and two slightly asymmetric cantilevers, which are meant to reduce the effects of torsion in the curved bridges.

The first half of the third span of the south bridge was used to test SOFO’s system. This half span which is 65.75 meters in length, can be idealised as a cantilever beam with a variable height. 30 relative displacement SOFO sensors [INAUDI] were placed in the interior of the box girder. The half span is divided into 5 cells of 6 meters in length with 6 sensors used in each cell: two sensors for each web and two sensors placed on the upper flag.

**APPLICATION OF THE CURVATURE MEASUREMENT ALGORITHM TO THE LUTRIVE HIGHWAY BRIDGE**

An algorithm developed with the SOFO system enables the calculation of vertical deformations from the expansion of the fibers parallel to the neutral axis. First, we establish geometrically the mean curvature to determine a polynomial curvature. The vertical displacement is the double integration of the polynomial mean curvature [VURPILLOT].

For the Lutrive highway bridge, this algorithm is applied with an adaptation for the calculation of the mean curve: it is determined from the rotation of the plane sections. The six sensors give the normal displacement to the section at six points, which allows finding the equation of the section’s plane after the bridge’s deformation. The plane equation has three unknowns $\alpha$, $\beta$, $\gamma$: 

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\[ \Delta_j = \alpha \cdot y_j + \beta \cdot z_j + \gamma \quad j = 1, 2, \ldots, 6 \]

\[ \Delta_j : \text{half deformation of sensor } j \]
\[ y_j, z_j : \text{position on the transversal section of sensor } j \]

The coefficients of the plane’s equation are the result of a linear system of six equations solved with the least square methods:

\[ \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_6 \end{bmatrix} = A \begin{bmatrix} y_1 \\ \vdots \\ y_6 \\ z_1 \\ \vdots \\ z_6 \end{bmatrix} \quad \vec{\alpha} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \]
\[ \vec{X} = A \cdot \vec{\alpha} \Rightarrow \vec{\alpha} = (A^T \cdot A)^{-1} \cdot A^T \cdot \vec{X} \]

These coefficients allow to find the components of the normal (\( \vec{n} \)) to the deformed transversal section \( \vec{n} = \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix} \), then also the rotation (\( \theta_n \)) of the section \( \theta_n = -\alpha \). The mean curve is geometrically determinate \( \frac{1}{\tau_n} = \frac{2 \cdot \theta_n}{L} \) with \( L \) for the length of the cell. With the mean curve, we use the curvature measurement algorithm for finding the vertical deformation.

The first constant of integration, \( C_1 \), is given with an inclinometer that measures the incline in one precise point of the bridge. The second constant of integration, \( C_2 \), is always base because the rigged mode is not considered.

**VALIDATION OF THE ALGORITHM DURING A STATIC LOAD TEST**

The 20th of November 1997, the Reinforced and Pre-stressed Concrete Institute (IBAP) of the Swiss Federal Institute of Technology of Lausanne (EPFL) performed a static load test on the Lutrive highway bridge. The loads were two trucks of 25 tons. They were placed in four different load cases: A & B for bending and C & D for torsion (see figure 1 & 2), with the possibility of superposition for testing the linear behaviour of bridge: situation A + situation B = situation C + situation D.

Every load case was performed three times with frequent base measures, measures without load that allowed for detecting the presence of permanent deformations.

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figure 2 : Configuration of static load test

IBAP measured the deflections using optical levelling process, as well as inclinometer measurements on five cross sections equally spaced on the half span. The deformation measurement values serve as reference points for the measurement with the deformation fiber optic sensors.
The rotations measured by the inclinometers were used as boundary conditions in the curvature measurement algorithm. The results are good, with maximum 6.5% error (see figure 3). We can make the same calculation with 3 sensors by cell and one inclinometer: in this case, the precision is slightly decreased.

A calculation of vertical displacement was tried without inclinometer values (see figure 4): the necessary boundary condition is a zero incline between the double pillar (see figure 1). The advantage of this second type of calculation is the possibility to make an estimation of the vertical deformations on long term without the presence of an inclinometer, which constitutes a complication of the measuring procedure.

**CONCLUSION**

This static load test shows the possibility to reconstruct the displacements of a bridge from curvature measurement with error maximum of 6.5% for a complicated structure (variable inertia, curvatures beam, …).

Moreover, the curvature measurement allows not only for the determination of the displacements, but also for the determination of the source and importance of a detected problem. Until now, the measurement of vertical displacement has been the most important factor used to determine the health of a bridge. With new monitoring systems measuring the internal displacement (like fiber optic sensor) it will be possible to take the curvature measurement into consideration too.

**REFERENCES**

- [VURPILLOT], D. Inaudi, A. Scanno, “Mathematical model for the determination of the vertical displacement from internal horizontal measurement of a bridge” smart structures and materials, San Diego February 1996, SPIE Volume 2719-05.