Automatica ▮ (▮▮▮▮) ▮▮▮–▮▮▮

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Automatica

journal homepage: www.elsevier.com/locate/automatica

Iterative minimization of H_2 control performance criteria☆

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ARTICLE INFO

Article history:
Received 16 April 2007
Received in revised form
3 March 2008
Accepted 10 March 2008
Available online xxxx

Keywords:
Data-based control design
H_2 performance criteria

ABSTRACT

Data-based control design methods most often consist of iterative adjustment of the controller's parameters towards the parameter values which minimize an H_2 performance criterion. Typically, batches of input–output data collected from the system are used to feed directly a gradient descent optimization — no process model is used. A limiting factor in the application of these methods is the lack of useful conditions guaranteeing convergence to the global minimum; several adaptive control algorithms suffer from the same limitation. In this paper the H_2 performance criterion is analyzed in order to characterize and enlarge the set of initial parameter values from which a gradient descent algorithm can converge to its global minimum.

1. Introduction

Control performance criteria are a key element in control systems theory. Not only are they fundamental from a conceptual point of view but this concept also leads to a large variety of control design methods, which are formulated as optimization problems. The solution of these optimization problems usually relies on full knowledge of the process to be controlled and of the characteristics of its disturbances. Often it is also required that the controller's transfer function can be freely chosen. These conditions are often not fulfilled in practice, which motivates the development of methods for the design of fixed structure regulators with partial or even conceivably no *a priori* modelling of the process. The minimization of the performance criterion, in these methods, is performed directly from data collected from the system, which motivates the designation *data-based control design*.

Several data-based control design methods explicitly optimizing performance criteria have appeared in the literature, with different approaches and for different performance criteria. These criteria express either one or a combination of the fundamental control objectives: reference tracking, noise rejection, and economic use of control energy. In Kammer, Bitmead, and Bartlett (2000) an iterative procedure based on spectral analysis, named *Frequency Domain Tuning* (FDT), has been proposed for the minimization of an H_2 performance criterion for a system with zero reference; hence, no tracking objective is pursued. The Virtual Reference Feedback Tuning (VRFT) method (Campi, Lecchini, & Savaresi, 2002; Campi & Savaresi, 2006) is based on a clever manipulation of variables which transforms an H_2 performance criterion into one which is quadratic in the design parameters. The resulting quadratic cost function can be minimized directly, so that no iterations are required. However, only the reference tracking objective is treated (unless a two degree of freedom controller is used, as in Lecchini, Campi, and Savaresi (2002)) and the global minimum of the resulting quadratic function coincides with that of the original criterion only under ideal conditions. Not suffering from this second limitation, but again an iterative procedure, is Correlation-based Tuning (CbT) (Karimi, Mišković, & Bonvin, 2003, 2004), which uses instrumental variable ideas to eliminate the deleterious effect of noise on the achievement of its reference tracking objective. Data-based optimization of a general H_2 performance criterion appears in Hjalmarsson, Gunnarsson, and Gevers (1994). There, a method for obtaining an unbiased estimate of the gradient of the cost function directly from closed-loop data is proposed; this method has been named Iterative Feedback Tuning (IFT). IFT is discussed in depth in Hjalmarsson, Gevers, Gunnarsson, and Lequin (1998), Hjalmarsson (2002) and extended in Procházka, Gevers, Anderson, and Ferrera (2005) to even more general performance criteria, which contain robustness enhancement objectives.

☆ Work supported by the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office, by the Brazilian Ministry of Education through CAPES, and by ARC Discovery-Projects Grants (DP0342683 and DP0664427) and National ICT Australia. National ICT Australia is funded through the Australian Government's *Backing Australia's Ability* initiative, in part through the Australian Research Council. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Er-Wei Bai under the direction of Editor Torsten Söderström.

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0005-1098/$ – see front matter © 2008 Published by Elsevier Ltd
doi:10.1016/j.automatica.2008.03.014

It has been found that a limiting factor in the application of data-based control design is the possible lack of convergence to the global minimum of the criterion. Closely related is the problem of convergence of model reference adaptive control (MRAC) algorithms, which is also formulated as the optimization of an H_2 criterion. The fundamental difference between MRAC and data-based design is that in MRAC, controller updates are made at each time sample, whereas in data-based control design each controller update is done based on a batch of data. Convergence of MRAC is a thoroughly studied subject (Åström & Wittenmark, 1995; Goodwin & Sin, 1984). Convergence properties of IFT have also been studied but not fully established (Hildebrand, Lecchini, Solari, & Gevers, 2005). Typically, convergence to local minima cannot be excluded, even under ideal conditions. The convergence problem is usually approached, in data-based control design as well as in adaptive control, through analysis of the optimization algorithm. Remedies to lack of convergence are accordingly prescribed as modifications to the optimization algorithm, usually ad hoc and requiring additional experiments or complexity of the algorithm.

In this paper, we take a different approach to the problem of securing convergence to the global optimum of H_2 performance criteria via a data-based approach. We do not concern ourselves with analyzing or deriving algorithms to solve the optimization problem; instead, we focus on the cost function itself. If the cost function to be optimized is sufficiently "well-behaved", then the properties of any one specific optimization procedure become less material for the purpose of assuring convergence: any (correct) algorithm will converge properly. We thus aim at requiring simpler algorithms and less data for the optimization — and simplicity is a major credential for a data-based design algorithm. In contrast with some other data-based schemes mentioned above, we deal with general H_2 optimization, which includes tracking, noise rejection and control effort.

Our principal concern is to give intuitively meaningful and appealing conditions for the set of parameter values that are guaranteed to lie within the domain of attraction of the global minimum of a performance index, when minimization is attempted using gradient descent approximations. When these conditions are not satisfied by the desired criterion, we show how it can be changed, possibly with the use of intermediate criteria that do satisfy these conditions and whose minima converge to the minimum of the desired criterion. The idea behind the introduction of intermediate criteria is to enlarge the domain of attraction to the global minimum of the global procedure without compromising the final performance. This procedure has been baptized *cost function shaping*.

The paper is organized as follows. Notation, definitions, preliminary results and a formal statement of the problem are presented in Section 2. The core of our analysis appears in Section 3. There the H_2 performance criterion is broken up into three terms, each one representing one of the fundamental control objectives. It is shown that, under some structural hypothesis, the three terms present a similar analytical structure which allows the derivation of convergence properties. Then the cost function shaping is presented in Section 4. In Section 5 the effect of relaxing the previous structural assumptions is analyzed. A concluding discussion is given in Section 6.

2. Preliminaries

2.1. Definitions

Consider a linear time-invariant discrete-time single-input–single-output process

$$y(t) = G(z)u(t) + v(t). \tag{1}$$

In (1), z is the forward-shift operator, $G(z)$ is the process transfer function, $u(t)$ is the control input and $v(t)$ is the process noise. The noise is a quasi-stationary process which can be written as $v(t) = H(z)e(t)$ where $e(t)$ is white noise with variance σ_e^2. Both transfer functions, $G(z)$ and $H(z)$, are rational and causal (proper). It is assumed that $H(\infty) = 1$, that is the impulse response $h(t)$ of the filter $H(z)$ satisfies $h(0) = 1$.

This process is controlled by a linear time-invariant controller which belongs to a given – user specified – class $\mathcal{C}$ of linear transfer functions. This class is such that $C(z)G(z)$ has positive relative degree for all $C(z) \in \mathcal{C}$; equivalently, the closed loop is not delay-free. The controller is parametrized by a parameter vector $\rho \in \mathfrak{R}^p$, so that the control action $u(t)$ can be written as

$$u(t) = C(z, \rho)(r(t) - y(t)) \tag{2}$$

where $r(t)$ is the reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise, that is

$$E[r(t)e(s)] = 0 \quad \forall t, s$$

where $E[\cdot]$ denotes expectation. The system (1) and (2) in closed loop becomes

$$y(t, \rho) = T(z, \rho)r(t) + S(z, \rho)v(t)$$

$$T(z, \rho) = \frac{C(z, \rho)G(z)}{1 + C(z, \rho)G(z)} = C(z, \rho)G(z)S(z, \rho)$$

where we have now made the dependence on the controller parameter ρ explicit in the output signal $y(t, \rho)$. It is also assumed that the controller parametrization has a certain structure, as specified below.

Assumption A. Linear parametrization:

$$C(z, \rho) = \rho^{\mathrm{T}}\bar{C}(z), \quad \rho \in \mathfrak{R}^p \tag{3}$$

where $\bar{C}(z)$ is a column vector of fixed rational functions.

Some of the most common controller structures are indeed linearly parametrized, PID with fixed derivative pole being the most popular. In addition, one can create a set of stable basis functions, a finite sum of which can approximate any stable controller in L_2 norm on the unit circle as close as desired; similarly for a controller that is totally unstable. If a controller has poles on the unit circle, the best approximation is to duplicate the poles and the residues, but that is strictly not necessary if the controller is part of a stable closed loop. Hence, Assumption A does not represent a significant loss of generality.

For later reference, we provide here some additional definitions. We say that a scalar quasi-stationary signal $x(t)$ is persistently exciting of order k – in short, PEk – if its spectrum $\Phi_x(e^{j\omega})$ has at least k nonzero components. For a vector field $V(e^{j\omega}) = [v_1(e^{j\omega})v_2(e^{j\omega}) \dots v_k(e^{j\omega})]^{\mathrm{T}}$, where each $v_i(e^{j\omega})$ is a function of the frequency variable ω, we say that the vector $V(e^{j\omega})$ has full-rank if the functions $v_i(e^{j\omega})$ form a linearly independent (LI) set over the reals, that is, if $\nexists \eta \in \mathfrak{R}^k, \eta \neq \mathbf{0} : \eta^{\mathrm{T}}V(e^{j\omega}) = 0 \ \forall \omega \in \mathfrak{R}$.

2.2. Problem statement

We want the closed loop to achieve a given performance, which is specified by a "desired" closed-loop transfer function $T_d(z)$, called *reference model* in the literature. We thus search for the controller parameters that make the output of the system the closest to the desired one, that is, we solve the following optimization problem.

$$\min_{\rho} J(\rho)$$

$$J(\rho) \triangleq E[\lambda(y(t, \rho) - y_d(t))^2 + (1 - \lambda)u^2(t, \rho)] \tag{4}$$

where $y_d(t) = T_d(z)r(t)$ is the desired output and $\lambda \in [0, 1]$ is a user-specified constant. This control design formulation is representative of several well-known *model-based* control design

methods, such as LQR/LQG (Anderson & Moore, 1989) and generalized predictive control (GPC) (Bitmead, Gevers, & Wertz, 1990), in which solutions of (4) may be achieved by means of tools such as Riccati equations, Linear Matrix Inequalities (LMIs), Bilinear Matrix Inequalities (BMIs), etc (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994).

Such *model-based* design requires the knowledge of a process model, namely the transfer functions $G(z)$ and $H(z)$, and possibly the noise variance σ_e^2 as well. Obtaining a good model for a real process usually demands, among other tasks, collecting data from real system operation. *Data-based design*, on the other hand, addresses the minimization of the criterion (4) directly from data collected from the system, without the intermediate step of deriving a process model from these data. Such methods as IFT (Hjalmarsson et al., 1998) and FDT (Kammer et al., 2000) rely on iterative optimization procedures, mostly gradient descent algorithms. The quantities required in the optimization procedure are the cost function's gradient and possibly its Hessian, which are estimated pointwise directly from batches of input–output data collected from the closed-loop system. These methods bear no claim on convergence to the global minimum of the criterion; convergence to a local minimum only can be guaranteed (Hildebrand et al., 2005; Hjalmarsson et al., 1994).

In this paper we are concerned with this particular but core aspect of data-based design: the convergence of iterative solutions of (4) to its global minimum. In order to proceed we need some definitions on optimization, which are given in the following subsection.

2.3. Optimization

Let $J(\rho) : \Re^n \to \Re^+$ be a differentiable cost function with an isolated global minimum ρ_*. Successive approximations for the global minimum are obtained by a recursion $\rho_{i+1} = f(\rho_i)$. A set of initial conditions for which the algorithm converges to ρ_* is called a *domain of attraction* (DOA).

Definition 2.1. Let ρ_* be the global minimum of a function $J(\rho) : \Re^n \to \Re^+$. A set $\Omega \subset \Re^n$ is a domain of attraction of an algorithm $\rho_i = f(\rho_{i-1})$ for the function $J(\rho)$ if $\lim_{i\to\infty} \rho_i = \rho_* \forall \rho_0 \in \Omega$. ◇

A gradient descent algorithm is one in which the iteration is given by

$$\rho_{i+1} = \rho_i - \gamma_i \nabla J(\rho_i) \tag{5}$$

where $\nabla J(\rho) = \frac{\partial J(\rho)}{\partial \rho}$ and $\gamma_i > 0\ \forall i$. The rationale behind this algorithm is clear: updates are made in the opposite direction of the gradient, so, at least for sufficiently small γ_i, at each iteration a smaller value for the cost is achieved. Its convergence properties which are relevant for our analysis are established in the following theorem.

Theorem 2.1. *Consider a twice-differentiable function $J(\cdot) : \Re^n \to \Re^+$. Assume that this function has an isolated global minimum ρ_* and define the set $\mathcal{B}_\alpha(\rho_*) = \{\rho : (\rho - \rho_*)^{\mathrm{T}}(\rho - \rho_*) < \alpha\}$. If*

$$(\rho - \rho_*)^{\mathrm{T}} \nabla J(\rho) > 0 \quad \forall \rho \in \mathcal{B}_\alpha(\rho_*),\ \rho \neq \rho_* \tag{6}$$

then there exists a sequence γ_i, $i = 1, \ldots, \infty$ such that $\mathcal{B}_\alpha(\rho_*)$ is a DOA of algorithm (5) for $J(\rho)$.

Proof. Let $V(\rho) = (\rho - \rho_*)^{\mathrm{T}}(\rho - \rho_*)$ be a Lyapunov function for the discrete-time system (5). Then

$$\begin{aligned} V(\rho_{i+1}) - V(\rho_i) &= (\rho_i - \gamma_i \nabla J(\rho_i) - \rho_*)^{\mathrm{T}} \\ &\quad \times (\rho_i - \gamma_i \nabla J(\rho_i) - \rho_*) - (\rho_i - \rho_*)^{\mathrm{T}}(\rho_i - \rho_*) \\ &= -2\gamma_i (\rho_i - \rho_*)^{\mathrm{T}} \nabla J(\rho_i) + \gamma_i^2 \nabla J(\rho_i)^{\mathrm{T}} \nabla J(\rho_i) \end{aligned}$$

which is negative provided that $0 < \gamma_i < \frac{2(\rho_i - \rho_*)^{\mathrm{T}} \nabla J(\rho_i)}{\nabla J(\rho_i)^{\mathrm{T}} \nabla J(\rho_i)}$. For $\rho_i \in \mathcal{B}_\alpha(\rho_*)$ the existence of such γ_i is guaranteed by condition (6), which also implies that $\nabla J(\rho_i) \neq \mathbf{0}\ \forall \rho_i \neq \rho_*, \rho_i \in \mathcal{B}_\alpha(\rho_*)$. The proof is completed by noting that $\mathcal{B}_\alpha(\rho_*)$ is a connected and bounded level set of $V(\rho)$. ◇

Condition (6) implies that:

- the angle between the gradient and the vector $\rho - \rho_*$ is always in the interval $(-\pi/2, \pi/2)$;
- the gradient is never zero — there are no extrema (minima, maxima) or saddle-points in $\mathcal{B}_\alpha(\rho_*)$ other than the global minimum ρ_*.

Actual convergence also involves the proper choice of algorithmic parameters, the sequence γ_i in particular, an issue which we do not address in this paper. When the cost function does obey condition (6) in a given set, then we say that this set is a *candidate* DOA, because then we can find γ_i such that this set will be a DOA for the gradient descent algorithm (5).

Definition 2.2. Let ρ_* be the global minimum of a function $J(\rho) : \Re^n \to \Re^+$. A ball $\mathcal{B}_\alpha(\rho_*)$ is a *candidate* DOA for $J(\rho)$ if $\rho_* \in \Omega$ and (6) is satisfied for all $\rho \in \Omega$. ◇

In an actual data-based design (IFT, CbT, FDT), the gradient in (5) is replaced by an estimate $\widehat{\nabla J(\rho_i)}$. It is a well-known fact (see Hjalmarsson et al. (1998), for instance) that such a gradient descent algorithm converges, in a stochastic sense, to a minimum of the cost function provided that the estimate is unbiased and that appropriate γ_i have been chosen. The reasoning in Theorem 2.1 remains valid, with $\widehat{\nabla J(\rho_i)}$ replacing $\nabla J(\rho_i)$, and some slack in condition (6) becomes necessary to compensate for the estimate errors. Also notice that in this stochastic setting only convergence to a neighborhood of the global optimum can be achieved (stochastic convergence) and hence condition (6) must be verified only outside this neighborhood.

3. Analysis of the cost function

Under the assumption that the reference and the noise are uncorrelated, we can split the cost $J(\rho)$ in (4) into three components:

$$J(\rho) = \lambda[J_y(\rho) + J_e(\rho)] + (1 - \lambda) J_u(\rho) \tag{7}$$

where we have defined

$$J_y(\rho) = E[((T(z, \rho) - T_d(z))r(t))^2]$$
$$J_e(\rho) = E[(S(z, \rho)v(t))^2]$$
$$J_u(\rho) = E[u(t, \rho)^2].$$

The minimization of $J_e(\rho)$ for a free-form controller with known process model is a classical problem known as *minimum variance control*, whose solution is well known (Åström, 1970; Åström & Wittenmark, 1973; Kammer et al., 2000). The minimization of $J_y(\rho)$ has also been widely studied in the literature, particularly in the context of model reference adaptive control (Åström & Wittenmark, 1995; Goodwin & Sin, 1984). In this paper we refer to $J_y(\rho)$ as the *reference tracking* criterion.

Let Γ be the set of all control parameter values that render the closed-loop system BIBO-stable, that is, $\Gamma \triangleq \{\rho : T(z, \rho)$ is *BIBO-stable*$\}$. For $\rho \in \Gamma$ Parseval's theorem yields:

$$J_y(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |T(\mathrm{e}^{\mathrm{J}\omega}, \rho) - T_d(\mathrm{e}^{\mathrm{J}\omega})|^2 \Phi_r(\mathrm{e}^{\mathrm{J}\omega}) \mathrm{d}\omega \tag{8}$$

$$J_e(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |S(\mathrm{e}^{\mathrm{J}\omega}, \rho)|^2 \Phi_v(\mathrm{e}^{\mathrm{J}\omega}) \mathrm{d}\omega \tag{9}$$

$$\begin{aligned} J_u(\rho) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |C(\mathrm{e}^{\mathrm{J}\omega}, \rho)|^2 |S(\mathrm{e}^{\mathrm{J}\omega}, \rho)|^2 \\ &\quad \times [\Phi_r(\mathrm{e}^{\mathrm{J}\omega}) + \Phi_v(\mathrm{e}^{\mathrm{J}\omega})] \mathrm{d}\omega \end{aligned} \tag{10}$$

where $\Phi_x(\mathrm{e}^{\mathrm{J}\omega})$ indicates the spectrum of a signal $x(t)$.

3.1. Reference tracking

Let us start by studying the reference tracking cost, that is, $J_y(\rho)$. To that end, we define the ideal controller $C_d(z)$, which is the controller transfer function that would exactly achieve the desired closed-loop transfer function $T_d(z)$:

$$C_d(z) = \frac{T_d(z)}{G(z)(1 - T_d(z))}. \tag{11}$$

Let us assume that none of the unstable poles and zeros of $G(z)$ (if there are any) are cancelled by $C_d(z)$. Clearly, this assumption is automatically verified for processes that are stable and minimum phase. For unstable and/or nonminimum-phase processes, the satisfaction of this property is determined by the choice of the reference model; this is an inherent feature of model reference design. When a process model is available, the reference model can and must be chosen to satisfy this assumption, otherwise the global minimum of the performance criterion would deliver a destabilizing controller. In data-based design, it is also possible to cope with this limitation by leaving the zeros of the reference model as free parameters, as shown in Lecchini and Gevers (2002).

If and only if the ideal controller $C_d(z)$ lies within the class of controllers considered, then the closed-loop system can be made to behave exactly as the reference model by a proper choice of the parameter ρ. We will assume that this is the case in most results that follow, so let us formalize this assumption.

Assumption $\mathbf{B_y}$ $-$ $C_d(z) \in \mathcal{C}$, or, equivalently,

$$\exists \rho_d : C(z, \rho_d) = C_d(z) = \rho_d^{\mathrm{T}} \bar{C}(z). \tag{12}$$

This assumption is dual to the assumption in identification theory that the process model belongs to the model class considered. Similar assumptions will be made for the two other cost components J_u and J_e. Though assumptions of this nature are standard in our context (Åström & Wittenmark, 1995; Campi et al., 2002), they are not weak ones. We can, however, expect them to be violated only moderately in a well-formulated design problem; it does not make good engineering sense to formulate a problem in which one searches for a performance that is radically different from what can be achieved. The case where Assumption B_y is "almost" satisfied is analyzed in Section 5.

Under Assumptions A and B_y the global minimum of J_y can be found by VRFT (Lecchini & Gevers, 2002), which is most opportune, since it is a direct (noniterative) method. However, VRFT design for controllers with just one degree of freedom can only cope with J_y and not with general H_2 criteria, as we do later in this paper. Moreover, when assumption B_y is not satisfied VRFT yields a controller that is not the minimum of J_y. In these cases, VRFT can still be used to provide an initial condition for an iterative optimization procedure. Note also that without Assumption A and B_y we cannot guarantee that the global minimum will be reached, but they are not necessary for the application of data-based controller tuning.

From (3) and (12), and adopting the convention that gradients are column vectors, we get:

$$C(z, \rho) - C_d(z) = (\rho - \rho_d)^{\mathrm{T}} \bar{C}(z)$$
$$\nabla C(z, \rho) = \bar{C}(z) \qquad \nabla C^*(z, \rho) = \bar{C}^{*T}(z)$$

where $\bar{C}^*(\mathrm{e}^{j\omega}) = \bar{C}^T(\mathrm{e}^{-j\omega})$. Calculating the gradient of $J_y(\rho)$ in (8) and using the above yield (the calculations are given in Appendix A)[1]:

$$\nabla J_y(\rho) = M(\rho)(\rho - \rho_d) \tag{13}$$
$$M(\rho) \triangleq \frac{1}{\pi} \left(\int_{-\pi}^{\pi} \Phi_r |G|^2 |S(\rho)|^2 \Re\{S_d^* S(\rho) \bar{C} \bar{C}^*\} \mathrm{d}\omega \right).$$

Here $\Re\{\cdot\}$ denotes the real part of a complex number and the desired sensitivity function is defined as $S_d(z) = 1 - T_d(z)$. It is clear that $J_y(\rho_d) = 0$ and $\nabla J_y(\rho_d) = \mathbf{0}$ so that ρ_d is the global minimum of $J_y(\rho)$. Writing condition (6) for this function we get

$$(\rho - \rho_d)^{\mathrm{T}} \nabla J_y(\rho) = (\rho - \rho_d)^{\mathrm{T}} M(\rho)(\rho - \rho_d) > 0$$
$$\forall \rho \in \mathcal{B}_\alpha(\rho_d),\ \rho \neq \rho_d. \tag{14}$$

Thus, whether this condition is satisfied in a given set depends on the properties of the matrix $M(\rho)$ for all ρ contained in this set; let us examine these properties. From (13):

$$M(\rho) = \frac{1}{\pi} \int_{-\pi}^{\pi} \Phi_r |G|^2 |S(\rho)|^2 \Re\{S_d^* S(\rho)\} \Re\{\bar{C} \bar{C}^*\} \mathrm{d}\omega$$
$$- \frac{1}{\pi} \int_{-\pi}^{\pi} \Phi_r |G|^2 |S(\rho)|^2 \Im\{S_d^* S(\rho)\} \Im\{\bar{C} \bar{C}^*\} \mathrm{d}\omega$$
$$\triangleq M_s(\rho) + M_a(\rho). \tag{15}$$

It is straightforward to verify that $M_a(\rho)$ is anti-symmetric, whereas $M_s(\rho)$ is symmetric $-$ hence their subscripts. Since $x^{\mathrm{T}} Q x = 0$ for any anti-symmetric matrix Q and any $x \in \Re^n$, (14) can be written as

$$(\rho - \rho_d)^{\mathrm{T}} M_s(\rho)(\rho - \rho_d) > 0 \quad \forall \rho \in \mathcal{B}_\alpha(\rho_d), \rho \neq \rho_d \tag{16}$$

which is satisfied if $M_s(\rho)$ is positive definite.

Now, $\bar{C}(\mathrm{e}^{j\omega}) \bar{C}^*(\mathrm{e}^{j\omega})$ is positive semi-definite by construction and its sum over a sufficiently large number of frequencies is positive definite provided that the vector field $\bar{C}(\mathrm{e}^{j\omega})$ is full-rank (the functions in $\bar{C}(\mathrm{e}^{j\omega})$ form an LI set over the reals). The same is true for its real part. With this we have proven the following result.

Theorem 3.1. *Let $\bar{C}(\mathrm{e}^{j\omega})$ be full-rank and $r(t)$ be PEp.*[2] *Let $\Upsilon \subseteq \Gamma$ be a connected set such that $\rho_d \in \Upsilon$ and, for all $\rho \in \Upsilon$:*

$$\Re\{S_d^*(\mathrm{e}^{j\omega}) S(\mathrm{e}^{j\omega}, \rho)\} > 0 \quad \forall \omega. \tag{17}$$

Then

$$(\rho - \rho_d)^{\mathrm{T}} \nabla J_y(\rho) > 0 \quad \forall \rho \in \Upsilon, \rho \neq \rho_d. \quad \diamond$$

Some comments are in order:

(1) Whereas the first two conditions of the theorem (reference is PEp and $\bar{C}$ is full-rank) are necessary for the integral in (15) to be bounded away from zero, condition (17) is not. If (17) is violated only at those frequencies where the input power is low, then the integral is likely to be positive. This is a powerful idea in adaptive control which will be further explored in what follows.

(2) Condition (17) can also be expressed as

$$\max_{\omega} |\angle S(\mathrm{e}^{j\omega}, \rho) - \angle S_d(\mathrm{e}^{j\omega})| < \pi/2 \quad \forall \rho \in \Upsilon. \tag{18}$$

This *maximum-phase difference* between two transfer functions is a metric that can be used to measure the distance between a given ρ and the global optimum ρ_d.

(3) Condition (17) is satisfied if and only if the following transfer function is strictly positive real (SPR).

$$\frac{S_d(\mathrm{e}^{j\omega})}{S(\mathrm{e}^{j\omega}, \rho)}. \tag{19}$$

(4) To check the SPR condition (18) we need to know roughly the phase of the sensitivity $S(z, \rho)$, and a rough estimate can be obtained from the same data used for the design. So, checking condition (18) does not require a process model.

(5) A similar SPR condition involving sensitivity functions has been obtained for correlation-based tuning (CbT) in Karimi et al. (2004). This SPR condition is established for the class of controllers in the form of rational functions.

[1] In order to make the expressions shorter we will henceforth omit the dependence on $\mathrm{e}^{j\omega}$ when the signals appear inside integrals.

[2] Recall that p is the dimension of the parameter vector ρ.

The conditions of Theorem 3.1 have a clear interpretation:

- the input must be persistently exciting of order at least equal to the dimension of the parameter vector;
- the parametrization of the controller cannot be redundant, that is, it must represent the class of controllers considered with a minimum number of parameters;
- the parameter values in Υ must be "close enough" to the ideal parameter value ρ_d, so that the respective sensitivity functions are not too different, in the sense that their phases are close, but actually with no specific restriction on their magnitudes.

Example 3.1. Let $G(z) = \frac{1}{z-a}$, $|a| < 1$, $C(z,\rho) = \rho\frac{z-a}{z-1}$. The ideal controller $C_d(z)$ belongs to the controller class considered if and only if the reference model is of the form $T_d(z) = \frac{1-b}{z-b}$; then $\rho_d = 1-b$. The sensitivity function is given by

$$S(z,\rho) = \frac{1}{1+C(z,\rho)G(z)} = \frac{z-1}{z-(1-\rho)}.$$

For two arbitrary sensitivity functions generated by $\rho_1, \rho_2 \in \Gamma$, we have

$$\frac{S(z,\rho_1)}{S(z,\rho_2)} = \frac{z-(1-\rho_2)}{z-(1-\rho_1)}.$$

The set of stability parameters ρ is $\Gamma = \{\rho : |1-\rho| < 1\}$. Let us define $\alpha_i = 1-\rho_i$ for convenience of notation. From the positive real lemma (Boyd et al., 1994), this function is SPR if and only if there exists $q \in \Re^+$ satisfying the inequalities:

$$q(1-\alpha_2^2) > 0$$
$$(\alpha_2-\alpha_1)^2q^2 + 2(\alpha_1\alpha_2-1)q + 1 \triangleq \eta(q) < 0.$$

The first inequality requires $\alpha_2 < 1$, which is the case for $\rho_2 \in \Gamma$. The second one will be satisfied for some real positive q if and only if the roots of the polynomial $\eta(q)$ are real and at least one of them is positive. The roots of $\eta(q)$ are given by

$$q = \frac{-(\alpha_1\alpha_2-1) \pm \sqrt{(\alpha_1^2-1)(\alpha_2^2-1)}}{(\alpha_2-\alpha_1)^2}.$$

These roots are real if and only if $(\alpha_1^2-1)(\alpha_2^2-1) \geq 0$, which is satisfied for all $\rho_1, \rho_2 \in \Gamma$. Moreover, for $\rho_1, \rho_2 \in \Gamma$, $-(\alpha_1\alpha_2-1) > 0$, so one of the roots is positive. Hence, whatever reference model $T_d(z)$ we choose such that it is BIBO-stable and can be achieved exactly with the controller class considered, Γ will be a candidate DOA for the global minimum $\rho_d = 1-b$. ◇

It is typically the case that some information on $G(z)$ is necessary in order to *verify* the satisfaction of Assumption B_y, as well as to check condition (17). In the example, the knowledge of the model structure alone is enough to verify that Assumption B_y can be satisfied with a PI controller class $C(z,\rho) = \rho\frac{z-a}{z-1}$. In order to actually choose a controller class that satisfies Assumptions B_y and A we need to know also the pole value a. This knowledge, on the other hand, allows also characterizing the whole set of reference models for which Assumption B_y is satisfied, and verifying that condition (17) is satisfied for all these reference models. So, although some information on the process must be available in order to use the results of this paper, this required information is far less than what is required for model-based design — namely, knowledge of $G(z)$ to a reasonable degree of accuracy.

Example 3.2. Let $G(z) = \frac{1}{z-0.5}$, $C(z,\rho) = \rho\frac{z}{z-0.9}$ and $T_d(z) = \frac{2.4z}{z^2+z+0.45}$. The ideal controller, which minimizes $J_y(\rho)$, is achieved for $\rho_d = 2.4$. The stability set is $\Gamma = (-0.05, 2.85)$. It is straightforward to verify that $\frac{S_d(e^{j\omega})}{S(e^{j\omega},\rho)}$ is SPR for all $\rho \in \Upsilon = (1.35, 2.85)$. So, from Theorem 3.1, the cost $J_y(\rho)$ has no other extrema than ρ_d within the set Υ and this set is a candidate DOA for $J_y(\rho)$. For $\rho \notin \Upsilon$ the SPR condition is not satisfied, so there may exist local minima or maxima in this set. Whether such extrema exist depends on the particular reference applied to the system, as will be seen later. ◇

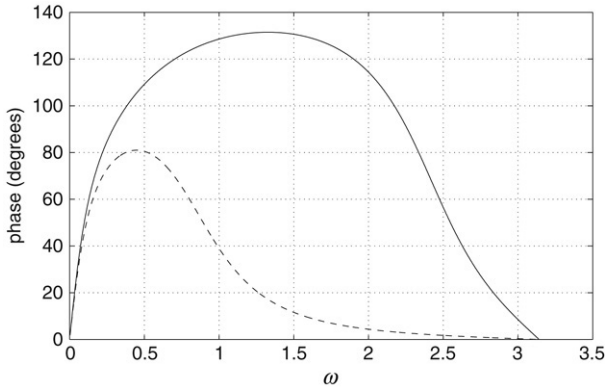

Fig. 1. Example 3.2 — frequency response of $S_d(e^{j\omega})$ (full line) and $S(e^{j\omega},\rho)$ (dashed line) for $\rho = 0.5$ ($\rho_d = 2.4$).

Let us re-examine (13) and (16) in the light of the final remark in the example above, noting that $M(\rho)$ is a scalar here. Theorem 3.1 tells us, based on these equations, that if all the factors inside the integral that forms $M_s(\rho)$ are positive, then the integral cannot be zero and (16) is satisfied. There is only one term inside that integral that can be nonpositive: $\Re\{S_d^* S(z,\rho)\}$. If this term is negative in a range of frequencies and positive in another range, then there exists a Φ_r such that it "weighs" equally these two frequency ranges, thus causing the integral to vanish. The following corollary results immediately from this argument.

Corollary 3.1. *Let $\bar{C}(e^{j\omega})$ be full-rank and consider a given set $\Upsilon \subseteq \Gamma$, with $\rho_d \in \Upsilon$. If for some ρ_1, $\exists\omega : \Re\{\frac{S_d(e^{j\omega})}{S(e^{j\omega},\rho_1)}\} < 0$ then there exist PEp reference signals $r(t)$ such that $\nabla J_y(\rho_1) = \mathbf{0}$. This, in turn, implies that for such reference signals: condition (6) is not satisfied at ρ_1; ρ_1 is an extremum of $J_y(\rho)$; and any set $\Upsilon \ni \rho_1$ is **not** a candidate* DOA *for $J_y(\rho)$.* ◇

Example 3.3. Consider again the system of Example 3.2. The SPR condition is not satisfied for $\rho < 1.35$. For $\rho = 0.5$, for instance, we have the situation presented in Fig. 1, where it is clear that the SPR condition is indeed not satisfied, since there are frequencies where the phases of $S_d(z)$ and $S(z, 0.5)$ differ by more than $\frac{\pi}{2}$ rad.

It can be seen in this figure that $\Re\{\frac{S_d(e^{j1})}{S(e^{j1},0.5)}\} = 0$, so if only this frequency ($\omega = 1$ rad) is excited, the gradient will be zero at this particular value of ρ. Indeed, by applying $r(t) = \sin(1\cdot t)$ we have the cost presented in Fig. 2, which has a local maximum at $\rho = 0.5$. ◇

Hence, convergence can be guaranteed by properly restricting the reference spectrum to those frequencies where the phase difference is small. This idea has been explored in adaptive control (Riedle, Praly, & Kokotovic, 1986) and is one of the bases of the cost function shaping presented in what follows.

3.2. Control effort

From (10), the cost function associated with the control energy is given by

$$J_u(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{\rho^T\bar{C}(e^{j\omega})}{1+\rho^T\bar{C}(e^{j\omega})G(e^{j\omega})}\right|^2 \times [\Phi_r(e^{j\omega}) + \Phi_v(e^{j\omega})]d\omega. \quad (20)$$

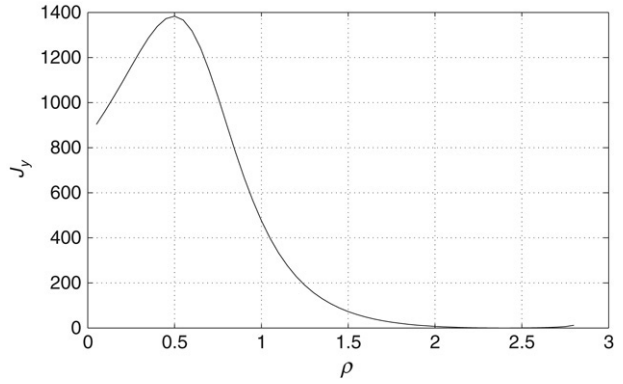

Fig. 2. Cost $J_y(\rho)$: global optimum at $\rho_d = 2.4$ and a maximum at $\rho = 0.5$.

It is evident from (20) that $\rho_u \triangleq \mathbf{0}$ is the global minimum of $J_u(\rho)$, which results in $J_u(\mathbf{0}) = 0$. The gradient is given by (see Appendix A.2)

$$\nabla J_u(\rho) = M_u(\rho)\rho \tag{21}$$

$$M_u(\rho) = \frac{1}{\pi}\left[\int_{-\pi}^{\pi}(\Phi_r + \Phi_v)|S(\rho)|^2\Re\{S(\rho)\bar{C}\bar{C}^*\}\mathrm{d}\omega\right].$$

From (21) it is also clear that $\nabla J_u(\mathbf{0}) = \mathbf{0}$. Note that this optimal controller belongs to any linearly parametrized class of controllers; it is achievable with $\rho \in \Gamma$ if and only if the process is open-loop BIBO-stable. We can apply for $J_u(\rho)$ the same reasoning as for $J_y(\rho)$, *mutatis mutandis*, which gives the following result.

Theorem 3.2. *Let $\bar{C}(e^{j\omega})$ be full-rank, let $\Upsilon \subseteq \Gamma$ be a connected set such that $\mathbf{0} \in \Upsilon$, and let*

$$\mathrm{Re}\{S(e^{j\omega}, \rho)\} > 0 \quad \forall \rho \in \Upsilon. \tag{22}$$

Then

$$\rho^{\mathrm{T}}\nabla J_u(\rho) > 0 \quad \forall \rho \in \Upsilon,\ \rho \neq \mathbf{0}. \quad \diamond$$

The SPR condition (22) has the same interpretation as (17): it represents the distance of the achieved sensitivity to the desired one. The difference between the two cases is the value of the desired sensitivity, which now is given by $S_u(e^{j\omega}) = S(e^{j\omega}, \mathbf{0}) = 1$.

3.3. Noise rejection

The cost $J_e(\rho)$ represents the variance of the output due to the noise. The minimization of this variance for the case where the controller is not constrained is a classical topic in control theory known as minimum variance control. The solution of this problem, when $G(z)$ is minimum phase, is given by the following controller (Åström, 1970):

$$C_e(z) = \frac{H(z) - 1}{G(z)}.$$

Applying this controller, the obtained sensitivity is

$$S_e(z) = \frac{1}{1 + C_e(z)G(z)} = \frac{1}{H(z)}.$$

The sensitivity $S_e(z)$, being the one that is obtained with the best possible controller, is the desired sensitivity for the performance criterion $J_e(\rho)$. We can use this fact to rewrite the cost (9) in a more insightful form (see Appendix A.3):

$$J_e(\rho) = \sigma_e^2 + \frac{1}{2\pi}\sigma_e^2\int_{-\pi}^{\pi}|H|^2|S(\rho) - S_e|^2\mathrm{d}\omega. \tag{23}$$

Clearly, if the minimum variance controller were applied, then the sensitivity would equal the desired sensitivity $S_e(z)$ and the cost would evaluate to σ_e^2. We can give J_e the same treatment as before and obtain similar results. We start with the assumption that the minimum variance controller can be implemented in our class $\mathcal{C}$.

Assumption $\mathbf{B_e}$ — There exists ρ_e: $C(\rho_e, z) = C_e(z)$.

The remarks previously made about Assumption B_y apply *ipsis literis* here. Under Assumption B_e the gradient is given by

$$\nabla J_e(\rho) = M_e(\rho)(\rho - \rho_e) \tag{24}$$

$$M_e(\rho) = \frac{\sigma_e^2}{\pi}\int_{-\pi}^{\pi}|H|^2|G|^2|S(\rho)|^2\Re\{S_e^*S(\rho)\bar{C}\bar{C}^*\}\mathrm{d}\omega$$

which is similar to (13) and (21). So, once again we have a similar result.

Theorem 3.3. *Let $\bar{C}(e^{j\omega})$ be full-rank. Let $\Upsilon \subseteq \Gamma$ be a connected set such that $\rho_e \in \Upsilon$ and that, for all $\rho \in \Upsilon$,*

$$\Re\left\{\frac{S_e(e^{j\omega})}{S(e^{j\omega}, \rho)}\right\} > 0 \quad \forall \rho \in \Upsilon. \tag{25}$$

Then

$$(\rho - \rho_e)^{\mathrm{T}}\nabla J_e(\rho) > 0 \quad \forall \rho \in \Upsilon, \rho \neq \rho_e. \quad \diamond$$

3.4. Combining the three objectives

Each one of the three cost components represents a different control objective whose minimum is achieved at different values of the parameter vector. When the three components are put together to form the whole cost, we can also write the gradient of the combined criterion in the nice form of a symmetric matrix multiplying the difference to the minimum, as done previously for the gradient of each individual cost.

$$\begin{aligned}\nabla J(\rho) &= \lambda M_y(\rho)(\rho - \rho_d) + \lambda M_e(\rho)(\rho - \rho_e) \\ &\quad + (1 - \lambda)M_u(\rho)\rho \\ &= M(\rho)(\rho - \rho_*)\end{aligned} \tag{26}$$

where

$$M(\rho) = [\lambda M_y(\rho) + \lambda M_e(\rho) + (1 - \lambda)M_u(\rho)]$$

$$\rho_* = M^{-1}(\rho)[\lambda M_y(\rho)\rho_d + \lambda M_e(\rho)\rho_e].$$

A sufficient condition for $M(\rho)$ to be nonsingular is that each one of the individual matrices $M_y(\rho)$, $M_e(\rho)$ and $M_u(\rho)$ has a positive definite symmetric part. On the other hand, if the symmetric parts of each one of these matrices – $M_y(\rho)$, $M_e(\rho)$ and $M_u(\rho)$ – are positive definite, then so is the symmetric part of $M(\rho)$. Under these conditions the total cost $J(\rho)$ has similar properties to each one of its components and we can state a result which is similar to the ones presented previously for each term separately.

Theorem 3.4. *Consider the cost (26), let $\bar{C}(e^{j\omega})$ be full-rank and let $r(t)$ be PEp. Let $\Upsilon \subseteq \Gamma$ be a connected set containing ρ_e, ρ_d and $\mathbf{0}$ such that, for all $\rho \in \Upsilon$, $\frac{S_d(e^{j\omega})}{S(e^{j\omega},\rho)}$, $\frac{S_e(e^{j\omega})}{S(e^{j\omega},\rho)}$ and $S(e^{j\omega}, \rho)$ are* SPR. *Assume further that $\rho_* \in \Upsilon$. Then*

$$(\rho - \rho_*)^{\mathrm{T}}\nabla J(\rho) > 0 \quad \forall \rho \in \Upsilon, \rho \neq \rho_* \quad \diamond$$

4. Cost function shaping

4.1. The problem data

For simplicity of presentation we shall focus on the reference tracking criterion J_y, but the results to be discussed are also valid for the other two criteria, since they have the same analytical structure. Among the variables present in (15) and (17), which determine the convergence properties, some can be manipulated by the designer and others cannot. Let us take a closer look at each one of these variables to see which ones are under the designer's control. Start with the process characteristics — $G(z)$, $H(z)$ and σ_e^2; these are given and unknown, and we certainly cannot change them.

The controller class $\mathcal{C}$ is given and known; it is a designer's choice that is usually made in a previous stage of system conception, so we cannot change it here. For example, the available hardware often imposes the controller class. On the other hand, the particular parametrization $\bar{C}(z)$ used to represent this class can often be manipulated. It has been proven in this paper, and also noted elsewhere (Kammer et al., 2000), that this parametrization must represent the class $\mathcal{C}$ with the minimal number of parameters but it can otherwise be freely chosen. Though this choice does play a role in the properties of the cost function, and thus might be useful for cost function shaping, we do not explore it in this paper.

The initial controller parameter ρ_0 could, in theory, be selected at will. However, finding an initial controller that is guaranteed to provide a stable closed loop is not without danger when the process is unknown, even though a method like VRFT can be very helpful. Also, the main application field envisaged for model-free tuning methods is the performance improvement of controllers that have been operating in a stable, but not optimally performing way. In this situation, which is probably the most common in practice, the initial controller is imposed upon us. So, whether it is possible to choose ρ_0 in a given practical situation is determined by case-specific and rather subjective considerations.

The remaining items are determined strictly by the performance criterion: $T_d(z)$, λ and Φ_r. Once the choice of performance criterion has been made, these variables are fixed; changing them would imply minimizing another criterion, not the one the user has chosen. Nevertheless, if our criterion of choice is too hard to optimize starting from our given controller ρ_0, we may consider minimizing an easier criterion as an intermediate task. Then, taking the new controller resulting from this optimization as the initial controller, it might be easier to optimize the desired criterion. Actually, we can think of inserting more than one intermediate task, optimizing at each time a criterion that is closer to the desired one, and guaranteeing that each one of these intermediate optimization tasks will converge. This is the central idea of what we have called "cost function shaping": to manipulate one or more of the variables $T_d(z)$, λ and Φ_r stepwise so that the resulting intermediate cost functions have a larger domain of attraction to their global optimum, in such a way as to eventually minimize the desired cost function. As will be seen in what follows, each one of the cited variables has its own way of influencing the convergence properties of the optimization.

4.2. Cautious control

Starting from an initial controller which delivers a given performance – say $T_0(z)$ – which we consider poor, let us choose a first intermediate reference model $T_d^1(z)$. This reference model should not require at once the achievement of a performance that is much better than the one we already have with $T_0(z)$. Instead, $T_d^1(z)$ should be *cautious*, aiming at a modest performance improvement, one which is closer to the (poor) performance $T_0(z)$ than the real reference model of interest, namely $T_d(z)$. Once the global optimum of this new criterion (ρ_*^1) has been attained, we can pick a second, more ambitious, reference model $T_d^2(z)$ (i.e. one closer to $T_d(z)$), and optimize it starting from ρ_*^1 as the initial controller. This argument can be used successively, with several intermediate reference models, until the desired reference model $T_d(z)$ is achieved. This concept of cautious control is a familiar one in data-based control design (Hjalmarsson et al., 1994; Kammer, 2005) and in iterative identification and control design (Zang, Bitmead, & Gevers, 1995); it is also a fundamental precept of the windsurfer approach to adaptive control (Lee, Anderson, Kosut, & Mareels, 1993; Lee, Anderson, Mareels, & Kosut, 1995). In Anderson and Gevers (1998), the problem of impractical control objective was recognized and it was suggested performing successive iterates of the criterion achieving designs which approach the optimum. We illustrate the procedure by means of an example.

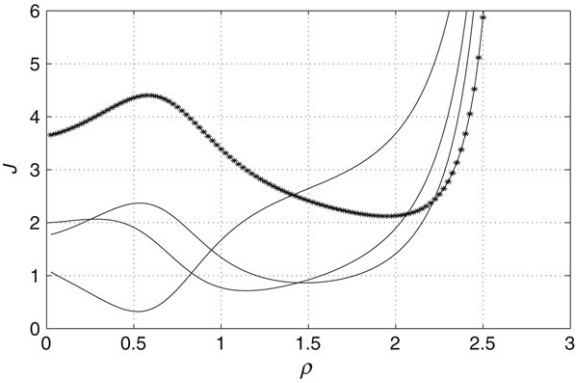

Fig. 3. Example 4.1 — cautious control: The H_2 cost $J(\rho)$ for different reference models — at each intermediate step the global optimum becomes larger.

Example 4.1. Consider the data in Example 3.2, a cost function $J(\rho)$ as in (7) with $\lambda = 0.75$, and that the noise can be described by $H(z) = \frac{z^2-1.15z+0.45}{z^2-1.4z+0.45}$ and $\sigma_e = 0.1$. This system is excited by a square-wave reference with period $T = 6$ s. The behavior of the cost function $J(\rho)$ is shown by the starred line in Fig. 3 — it presents a local maximum at $\rho \approx 0.6$. If the initial controller is one with very low gain (corresponding to $\rho < 0.6$), then convergence via gradient descent to the global optimum $\rho^* \approx 2$ is impossible. Consider the use of three intermediate reference models, starting from $T_d^1(z) = \frac{0.6z}{z^2-0.8z+0.45}$, and going successively to $T_d^2(z) = \frac{1.2z}{z^2-0.2z+0.45}$, $T_d^3(z) = \frac{1.8z}{z^2+0.4z+0.45}$ and finally $T_d(z) = \frac{2.4z}{z^2+z+0.45}$.

Then the intermediate cost functions behave as shown in Fig. 3. There, the final cost function is marked with $*$ and each intermediate cost function can be identified visually by the fact that each new intermediate cost presents a larger global optimum than the previous one ($\rho^* > \rho_3^* > \rho_2^* > \rho_1^*$). For the first cost function ($T_d^1(z)$) Γ is a candidate DOA and for each intermediate cost function the global optimum belongs to a candidate DOA of the next one. ◇

4.3. Manipulation of the reference spectrum

It has been proven that the SPR property of a particular transfer function is sufficient for uniqueness of extrema within a given set. However, the SPR condition is not a necessary condition; it can be circumvented by a proper manipulation of the reference $r(t)$. To realize how to do that, we first explore the properties of the sensitivity functions that enter the SPR condition (17).

4.3.1. Properties of the sensitivity

Lemma 4.1. *Let $S(z, \rho) = (1 + \rho^{\mathrm{T}}\bar{C}(z)G(z))^{-1}$ and let Γ be the set of all parameter values such that the closed loop is stable. For all $\rho_1, \rho_2 \in \Gamma$:*

$$\angle S(1, \rho_1) = \angle S(1, \rho_2) \qquad \angle S(-1, \rho_1) = \angle S(-1, \rho_2).$$

Proof.

$$\angle S(\mathrm{e}^{\jmath\omega}, \rho) = \sum_{i=1}^{n} \angle(\mathrm{e}^{\jmath\omega} - \mathrm{b}_i) - \sum_{i=1}^{n} \angle(\mathrm{e}^{\jmath\omega} - \mathrm{a}_i(\rho))$$

where b_i are the poles of the loop transfer function $\bar{C}(\mathrm{e}^{\jmath\omega})G(\mathrm{e}^{\jmath\omega})$ and $\mathrm{a}_i(\rho)$ are the closed-loop poles. For $\omega = 0$ we have

$$\angle S(1, \rho) = \sum_{i=1}^{n} \angle(1 - \mathrm{b}_i) - \sum_{i=1}^{n} \angle(1 - \mathrm{a}_i(\rho)).$$

But $\sum_{i=1}^{n} \angle(1-b_i)$ does not depend on ρ and $\sum_{i=1}^{n} \angle(1-a_i(\rho)) = 0\ \forall i,\ \rho \in \Gamma$ because $a_i(\rho)$ belong to the unit disc for all $\rho \in \Gamma$. The same argument is valid for $\omega = \pi$. ◇

From the property above and the continuity of $S(e^{j\omega}, \rho)$ we can also conclude that the phase difference between two sensitivity functions is small for frequencies close to $\omega = 0$ and $\omega = \pi$.

Lemma 4.2. *For all $\rho \in \Gamma$, $\exists \omega_l, \omega_h$ such that:*

$$|\angle S(e^{j\omega}, \rho) - \angle S_d(e^{j\omega})| < \frac{\pi}{2} \quad \forall \omega \le \omega_l$$

$$|\angle S(e^{j\omega}, \rho) - \angle S_d(e^{j\omega})| < \frac{\pi}{2} \quad \forall \omega \ge \omega_h. \quad \diamond$$

According to the previous results, if only those frequencies for which the phase difference $|\angle S(e^{j\omega}, \rho) - \angle S_d(e^{j\omega})|$ is small are contained in the reference spectrum, then the integral in (15) will still be bounded away from zero, even if the phase difference exceeds $\pi/2$ in some frequency ranges. For instance, if the reference spectrum is concentrated at the borders of the frequency spectrum – where the phase difference is always small – then $M_s(\rho)$ will be positive definite and the minimum is unique in the stability set Γ. This is formalized in the following theorem.

Theorem 4.1. *Let $\bar{C}(e^{j\omega})$ be full-rank and let $r(t)$ be PEp. Then $\exists \omega_l$, ω_h such that $\Phi_r = 0\ \forall \omega \in (\omega_l, \omega_h)$ implies that ρ_d is the unique extremum of $J_y(\rho)$ in Γ.*[3] ◇

4.3.2. Applying a different reference

Theorem 4.1 tells us that we can always find a reference for which the candidate DOA of ρ_d is the entire stability set Γ. So, whatever reference we have, we can always choose another reference for which the optimization can be performed from any initial controller and that will yield the same final result. In other words, if our cost function is difficult to minimize, we minimize instead another one, which is easier and has the same global minimum. This is also the central idea in VRFT (Campi et al., 2002) and some particular designs in MRAC (Åström & Wittenmark, 1995). Note, however, that this argument no longer holds for the combined cost $J_y(\rho) + J_e(\rho)$, which is the one to be minimized in the presence of noise. Nevertheless, we could expect that if $J_e(\rho)$ was significantly smaller than $J_y(\rho)$ away from ρ_d, the approach would be effective.

Example 4.2. Consider again Example 4.1 and use the tracking of square waves with larger periods as intermediate objective functions. In making the period larger, the reference spectrum becomes more concentrated at low frequencies. The result of optimizing the cost $J(\rho)$ successively for three different square waves is shown in Fig. 4. It can be observed that for the initial cost, given by the reference with the largest period ($T = 30$ s), the global optimum is at $\rho_*^1 = 0.65$ and Γ is a candidate DOA. The cost with the second reference ($T = 20$ s) has a minimum around $\rho_*^2 = 1$ and there are no other extrema in the set (ρ_*^1, ρ_*^2). Finally, ρ_*^2 is within a candidate DOA for the desired cost function. ◇

Similar theoretical results can be established for J_e and J_u, though these are of limited use for cost function shaping, since we cannot manipulate Φ_v.

Proposition 4.1. *Let $\bar{C}(e^{j\omega})$ be full-rank. Then:*

- *$\exists \omega_l^v, \omega_h^v$ such that $\Phi_v \approx 0\ \forall \omega \in (\omega_l^v, \omega_h^v)$ implies that ρ_e is the unique extremum of $J_e(\rho)$ in Γ;*
- *$\exists \omega_l^u, \omega_h^u$ such that $\Phi_r + \Phi_v \approx 0\ \forall \omega \in (\omega_l^u, \omega_h^u)$ implies that $\mathbf{0}$ is the unique extremum of $J_u(\rho)$ in Γ.* ◇

[3] Or, in case Γ is not a connected set, its whole connected subset which contains the global optimum.

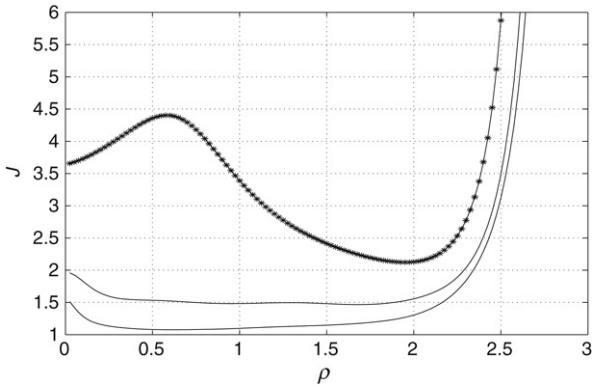

Fig. 4. Example 4.2 – H_2 performance criterion $J(\rho)$ for different reference spectra: Square wave, $T = 30, 20, 6$ s; as the period is decreased the global optimum increases.

4.3.3. Choosing the data window

We have shown that convergence can be achieved by applying to the process a properly chosen reference. But the procedure above requires that we actually apply to the process a reference that is different from the one it is supposed to track. This is something that will not always be allowed in practice. Alternatively the cost function can be manipulated such that it "sees" a different reference spectrum even though the reference is not actually changed. To realize this, notice that the cost $J_y(\rho)$ as originally defined is not computable in practice. What can be computed is the quantity

$$\hat{J}_y(\rho, N) = \frac{1}{N} \sum_{t=1}^{N} (y(t, \rho) - y_d(t))^2. \tag{27}$$

If the signal-to-noise ratio is large, then

$$\hat{J}_y(\rho, N) \approx \frac{1}{N} \sum_{t=1}^{N} [(T(z, \rho) - T_d(z))r(t)]^2.$$

Under the standing assumption that all signals are quasi-stationary, it is a standard result that the sum above converges to $J_y(\rho)$ as the data window size N grows:

$$\lim_{N\to\infty} \hat{J}_y(\rho, N) = J_y(\rho).$$

It is also clear that $\hat{J}_y(\rho, N)$ is a quadratic function of $\rho - \rho_d$. Hence ρ_d is an isolated global minimum of $\hat{J}_y(\rho, N)$ provided that $N > p$ (recall that p is the dimension of the parameter vector).

We would like to have a reference whose spectrum is concentrated either at very low or at very high frequencies. But the spectrum of the same reference is computed differently when measured under different time windows. Indeed, define $e(t) = (T(z, \rho) - T_d(z))r(t)$ and the signal $e^N(t)$ obtained as the periodic repetition of a truncation at $t = N$ of $e(t)$, that is

$$e^N(t + kN) = e(t) \quad t = 1, \ldots, N$$

for all integers k. Now calculate the associated cost:

$$J^N(\rho) = E[e^N(t)]^2 = \lim_{m\to\infty} \frac{1}{m} \sum_{t=1}^{m} [e^N(t)]^2$$

$$= \lim_{k\to\infty} \frac{1}{kN} k \sum_{t=1}^{N} [e^N(t)]^2$$

$$= \frac{1}{N} \sum_{t=1}^{N} [e^N(t)]^2 = \frac{1}{N} \sum_{t=1}^{N} [e(t)]^2 = \hat{J}_y(\rho, N).$$

The approximated cost $\hat{J}_y(\rho, N)$ equals the exact cost that would have been obtained should $r^N(t) \triangleq [T(z, \rho) - T_d(z)]^{-1} e^N(t)$ have

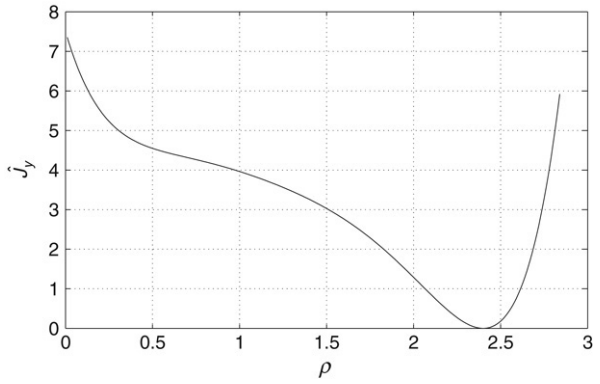

Fig. 5. Example 4.3 — performance criterion $\hat{J}_y(\rho, 4)$ for square-wave reference, $T = 6$ s.

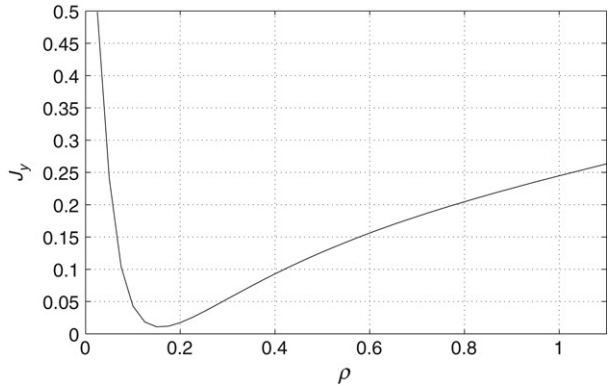

Fig. 6. Example 5.1 — reference tracking criterion.

been applied to the system. Hence, using a different N for the calculations has the same effect on the cost function as applying a different reference signal. For instance, if the reference is a step, taking smaller N is equivalent to applying a reference with more energy at higher frequencies. On the other hand, taking data after the transient will make the spectrum more concentrated at low frequencies. The following example illustrates the procedure.

Example 4.3. Reconsider once again Example 4.2. Take the original reference, with $T = 6$ s, but include only four data in the cost function, that is, consider the minimization of $\hat{J}_y(\rho, 4)$. This cost function presents the desired quasi-convexity in Γ, as shown in Fig. 5. ◇

5. The unmatched case

Let us analyze now the "unmatched control class" case, that is, the case where the ideal controller does not belong to the control class. We perform the analysis for the reference tracking criterion J_y; similar results apply to the two other cost components $-J_e$ and J_u. We thus want to analyze the behavior of the cost function J_y when assumption B_y is replaced by another, much weaker one:

Assumption $\mathbf{C_y}$ — The cost function J_y presents a unique global minimum ρ_*, that is, $\exists \rho_* \in \Gamma : J(\rho) > J(\rho_*) \forall \rho \neq \rho_*$; moreover, the global minimum satisfies

$$\nabla J(\rho_*) = \mathbf{0}, \quad \frac{\partial^2 J(\rho)}{\partial \rho^2}|_{\rho_*} \succ 0.$$

When Assumption B_y is satisfied, ρ_d is this global minimum ($\rho_* = \rho_d$) and $C(z, \rho_*) = C_d(z)$ — the ideal controller can be achieved. When Assumption B_y is not satisfied, the best controller that can be obtained is $C(z, \rho_*) \neq C_d(z)$ and Assumption C_y replaces Assumption B_y in our analysis. Assumption C_y rules out the occurrence of global minima at infinity or at the border of the stability set Γ, which is quite reasonable: a meaningful performance criterion should not require infinite control gains or operation of the system at the verge of instability, where it would not be robust.

The mismatch between the best controller allowed by the controller class under consideration and the ideal controller is defined by the following transfer function

$$K(z) \triangleq C_d(z) - C(z, \rho_*). \tag{28}$$

Now substitute (28) into (A.6) to get

$$\begin{aligned}\nabla J_y(\rho) &= \frac{1}{\pi}\int_{-\pi}^{\pi} \Phi_r |GS(\rho)|^2 \\ &\quad \times \Re\{(C(\rho) - C(\rho_*) - K)^* S_d^* S(\rho)\bar{C}\}\mathrm{d}\omega \\ &= M(\rho)(\rho - \rho_*) - m(\rho)\end{aligned} \tag{29}$$

where $M(\rho)$ is as defined previously in (15) and we have also defined

$$m(\rho) = \frac{1}{\pi}\int_{-\pi}^{\pi} \Phi_r |GS(\rho)|^2 \Re\{S_d^* S(\rho)\bar{C}K^*\}\mathrm{d}\omega. \tag{30}$$

Eq. (29) is similar to (13), but perturbed by the vector function $m(\rho)$. This perturbation is unknown, continuous and satisfies $m(\rho_*) = \mathbf{0}$. In addition, $m(\rho)$ is bounded for all $\rho \in \Gamma$. Thus, $|m(\rho)|$ can be linearly bounded, that is, for any given set $\Upsilon \subseteq \Gamma$ containing the global optimum ρ_*:

$$\exists \alpha_\Upsilon \in \Re^+ : |m(\rho)| < \alpha_\Upsilon |\rho - \rho_*| \quad \forall \rho \in \Upsilon. \tag{31}$$

Theorem 5.1. *Let $\bar{C}(e^{j\omega})$ be full-rank, $r(t)$ be PEp and α_Υ as defined in (31). Let $\Upsilon \subseteq \Gamma$ be a connected set such that $\rho_* \in \Upsilon$ and, for all $\rho \in \Upsilon$:*

$$\Re\{S_d^*(e^{j\omega})S(e^{j\omega}, \rho)\} > 0 \quad \forall \omega.$$

If, in addition, the perturbation term $m(\rho)$ is such that its bound α_Υ in (31) satisfies $M_s(\rho) \succ \alpha_\Upsilon I \; \forall \rho \in \Upsilon$, then

$$(\rho - \rho_*)^{\mathrm{T}} \nabla J_y(\rho) > 0 \quad \forall \rho \in \Upsilon, \rho \neq \rho_*.$$

Proof. Using (29) we have

$$(\rho - \rho_*)^{\mathrm{T}} \nabla J_y(\rho) = (\rho - \rho_*)^{\mathrm{T}} [M(\rho)(\rho - \rho_*) - m(\rho)]. \tag{32}$$

Then the result is proven by simple substitution of the assumptions of the theorem into (32). ◇

Clearly, computing estimates of the mismatch bound α_Υ requires some rough information about the process, which could be used directly in (30) for this purpose. How to get the best estimate with minimum prior information is an important topic of future research.

Example 5.1. Let $G(z) = \frac{1}{z-0.5}$ and consider a reference tracking performance criterion with $T_d(z) = \frac{0.3}{z-0.7}$; then $C_d(z) = \frac{0.3z-0.15}{z-1}$. Consider the class of all delay-free integral controllers: $\mathcal{C} = \{C(z) : C(z) = \rho\bar{C}(z), \; \rho \in \Re, \; \bar{C}(z) = \frac{z}{z-1}\}$; then $C_d(z) \notin \mathcal{C}$. The global minimum is $\rho_* = 0.15$. The SPR condition (19) yields

$$\frac{S_d(z)}{S(z, \rho)} = \frac{z^2 + (\rho - 1.5)z + 0.5}{(z - 0.7)(z - 0.5)}$$

which is SPR for all $0 < \rho < 1.1$. This implies that this interval is a candidate DOA provided that $K(z)$ – and hence $m(\rho)$ – is small enough. Indeed, it can be seen in Fig. 6 that the corresponding cost function is quasi-convex in the predicted interval. ◇

6. Concluding remarks

The typical way around convergence problems in iterative minimization of H_2 performance criteria is to propose more sophisticated or information-demanding algorithms. Instead, we have analyzed the cost function itself with a view to manipulating some design variables without compromising the final performance. Our main concern has been to determine whether or not given domains in parameter space can be made a DOA for a gradient descent optimization algorithm. We have established that this is determined by the maximum-*phase* difference, over all frequencies and for all parameter values in a given domain, between the sensitivity functions obtained for the different parameter values and the desired sensitivity function (the one obtained at the global optimum). A given domain can be made a DOA for a gradient descent algorithm if this maximum difference is smaller than $\pi/2$. It may happen that the whole set of stabilizing controllers is such a domain. Then a gradient descent algorithm with any "reasonable" initialization – one which provides a BIBO-stable closed loop – will converge to the global optimum.

From these analytical results it is possible to derive means to shape the cost function such that it becomes well-behaved as desired. This cost function shaping makes use of the variables that the designer has at his/her disposal: the reference signal, the intermediate reference models, the data window and the initial controller. Regarding the reference signal, applying references with frequency content sufficiently constrained to low and/or high frequencies is appropriate. Different reference spectra can also be simulated by taking different data windows with the same applied reference. Concerning the intermediate reference models, they should be sufficiently close to each other so that the global minimum of one intermediate criterion is within the DOA of the next, but not too close so that too many intermediate steps are required to reach the desired performance. A procedure to safely and effectively generate intermediate models, introducing an adequate but not excessive dose of caution into the control design, would be very welcome, and not only in the context of data-based design. Hence, an important topic for future research is to provide quantitative guidelines for cost function shaping as well as for estimation of the mismatch bound α_Υ. In this research we foresee the application of ν-gap stability margin measures (as in Kammer (2005)) and of overbounds on the process characteristics that can be directly estimated from the input–output data: transfer function order and relative degree, L_2 gain, etc.

Appendix. Gradient calculations of the cost function

In this section, in the interest of brevity, we drop the dependence on z and ω for all transfer functions and signals and define the following notation:

$$\mathcal{I}\{X(e^{j\omega})\} \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$$

A.1. J_y

$$\begin{aligned}\frac{\partial J_y(\rho)}{\partial\rho} &= \mathcal{I}\{[T(\rho)-T_d]^*\frac{\partial}{\partial\rho}[T(\rho)-T_d]\\ &\quad +[T(\rho)-T_d]\frac{\partial}{\partial\rho}[T^*(\rho)-T_d^*]\Phi_r\}\end{aligned} \tag{A.1}$$

$$\begin{aligned}T(\rho)-T_d &= \frac{C(\rho)G}{1+C(\rho)G}-\frac{C_dG}{1+C_dG}\\ &= (C(\rho)-C_d)GS_dS(\rho)\end{aligned} \tag{A.2}$$

$$(T(\rho)-T_d)^* = (C(\rho)-C_d)^*G^*S_d^*S^*(\rho) \tag{A.3}$$

$$\frac{\partial T(\rho)}{\partial\rho} = \frac{\partial}{\partial\rho}\frac{C(\rho)G}{1+C(\rho)G} = GS^2(\rho)\frac{\partial C(\rho)}{\partial\rho} \tag{A.4}$$

$$\frac{\partial T^*(\rho)}{\partial\rho} = G^*S^{*2}(\rho)\frac{\partial C^*(\rho)}{\partial\rho}. \tag{A.5}$$

Inserting (A.2)–(A.5) in (A.1):

$$\begin{aligned}\frac{\partial J_y(\rho)}{\partial\rho} &= \mathcal{I}\left\{\Phi_r\left[(C(\rho)-C_d)^*G^*S_d^*S^*(\rho)GS^2(\rho)\frac{\partial C(\rho)}{\partial\rho}\right.\right.\\ &\quad \left.\left.+\ (C(\rho)-C_d)GS_dS(\rho)G^*S^{*2}(\rho)\frac{\partial C^*(\rho)}{\partial\rho}\right]\right\}\\ &= \mathcal{I}\left\{\Phi_r|GS(\rho)|^2 2\Re\left\{(C(\rho)-C_d)S_dS^*(\rho)\frac{\partial C^*(\rho)}{\partial\rho}\right\}\right\}\end{aligned}$$

where $\Re\{\cdot\}$ indicates the real part of a complex quantity.

Let us use Assumption A from now on. Then

$$\begin{aligned}\frac{\partial J_y(\rho)}{\partial\rho} &= \mathcal{I}\left\{\Phi_r|GS(\rho)|^2\left[(C(\rho)-C_d)^*S_d^*S(\rho)\bar{C}\right.\right.\\ &\quad \left.\left.+\ (C(\rho)-C_d)S_dS^*(\rho)\bar{C}^{*T}\right]\right\}\\ &= 2\mathcal{I}\left\{\Phi_r|GS(\rho)|^2\Re\left\{(C(\rho)-C_d)^*S_d^*S(\rho)\bar{C}\right\}\right\}.\end{aligned} \tag{A.6}$$

Finally, if Assumption B_y is also satisfied:

$$\frac{\partial J_y(\rho)}{\partial\rho} = 2\mathcal{I}\{\Phi_r|GS(\rho)|^2\Re\{S_d^*S(\rho)\bar{C}\bar{C}^*\}\}(\rho-\rho_d). \tag{A.7}$$

A.2. J_u

As is clear from (8) and (10), the ρ-dependent part of the integrand in J_u is the same as in J_y, which allows determining the gradient based on the results above.

$$\begin{aligned}\frac{\partial J_u(\rho)}{\partial\rho} &= 2\mathcal{I}\left\{(\Phi_r+\Phi_v)|S(\rho)|^2\Re\left\{C^*(\rho)S(\rho)\frac{\partial C(\rho)}{\partial\rho}\right\}\right\}\\ &= \mathcal{I}\left\{(\Phi_r+\Phi_v)|S(\rho)|^2\left\{C^*(\rho)S(\rho)\frac{\partial C(\rho)}{\partial\rho}\right.\right.\\ &\quad \left.\left.+\ C(\rho)S^*(\rho)\frac{\partial C^*(\rho)}{\partial\rho}\right\}\right\}d\omega.\end{aligned} \tag{A.8}$$

Under Assumption A we get

$$\frac{\partial J_u(\rho)}{\partial\rho} = \mathcal{I}\{(\Phi_r+\Phi_v)|S(\rho)|^2\Re\{S(\rho)\bar{C}\bar{C}^*\}\}\rho.$$

A.3. J_e

Since $e(t)$ is white noise with variance σ_e^2:

$$J_e(\rho) = \sigma_e^2\mathcal{I}\{|S(\rho)H|^2\}.$$

Let $S_e = \frac{1}{H}$. Then

$$\begin{aligned}J_e(\rho) &= \sigma_e^2\mathcal{I}\{|HS(\rho)|^2\} = \sigma_e^2\mathcal{I}\{|1+HS(\rho)-1|^2\}\\ &= \sigma_e^2[1+\mathcal{I}\{|H|^2|S(\rho)-S_e|^2\}]\end{aligned}$$

where we have used the fact that $HS(\rho)-1$ is a strictly proper transfer function, which implies that the integral of its real part is zero. Then

$$\frac{\partial J_e(\rho)}{\partial\rho} = \sigma_e^2\mathcal{I}\left\{|H|^2\frac{\partial}{\partial\rho}|S(\rho)-S_e|^2\right\} \tag{A.9}$$

$$\frac{\partial}{\partial\rho}S(\rho) = \frac{\partial}{\partial\rho}\frac{1}{1+C(\rho)G} = -GS(\rho)^2\frac{\partial C(\rho)}{\partial\rho}. \tag{A.10}$$

From (A.10) we get:

$$\frac{\partial}{\partial\rho}|S(\rho)-S_e|^2 = -\left[S^*(\rho)S_e(C_e-C(\rho))\frac{\partial C^*(\rho)}{\partial\rho} + S(\rho)S_e^*(C_e-C(\rho))^*\frac{\partial C(\rho)}{\partial\rho}\right]|S(\rho)|^2|G|^2.$$

Now, under Assumptions A and B_e:

$$\frac{\partial}{\partial\rho}|S(\rho)-S_e|^2 = |S(\rho)|^2|G|^2[S^*(\rho)S_e\bar{C}\bar{C}^* + S(\rho)S_e^*(\bar{C}\bar{C}^*)^{\mathrm{T}}](\rho-\rho_e). \quad (A.11)$$

Substituting (A.11) into (A.9) gives

$$\frac{\partial J_e(\rho)}{\partial\rho} = M_e(\rho)(\rho-\rho_e) \quad (A.12)$$

$$M_e(\rho) = \sigma_e^2 \mathcal{I}\{|H|^2|G|^2|S(\rho)|^2\Re\{S^*(\rho)S_e\bar{C}\bar{C}^*\}\}.$$

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